

Problem Sheet 2

- (1)** Prove that for any $\phi, \phi_i, \psi, \chi \in \text{Form}(\mathcal{L})$ and any $\Gamma \subseteq \text{Form}(\mathcal{L})$
- (i) $((\phi \rightarrow \psi) \rightarrow ((\phi \rightarrow (\psi \rightarrow \chi)) \rightarrow (\phi \rightarrow \chi)))$ is a tautology
 - (ii) $\Gamma \cup \{\psi\} \models \phi$ if and only if $\Gamma \models (\psi \rightarrow \phi)$
 - (iii) $\neg \bigwedge_{i=1}^n \phi_i$ is logically equivalent to $\bigvee_{i=1}^n \neg \phi_i$.
 - (iv) $(\phi \vee \psi)$ is logically equivalent to $((\phi \rightarrow \psi) \rightarrow \psi)$.
- (2)** Let ϕ be the formula $((p_0 \rightarrow p_1) \wedge (p_1 \rightarrow p_2))$. Find a formula in dnf logically equivalent to ϕ which is a disjunct of just three conjunctive clauses.
- (3)** (i) Prove that every formula is logically equivalent to one in conjunctive normal form.
- (ii) Let v_0 be the valuation that assigns the value T to every propositional variable. Prove that a formula ϕ is logically equivalent to one built up from propositional variables using just the connectives $\wedge$ and $\rightarrow$ if and only if $\tilde{v}_0(\phi) = T$.
- (4)** (i) Find the truth tables for all binary connectives $\star$ having the property that $\{\star\}$ is adequate. Justify your answer.
- (ii) Show that there is no adequate unary connective.
- (5)** Prove that for any formulas α, β of $\mathcal{L}_0$, the following formulas are theorems of the system L_0 . You may use the deduction theorem and, that for any α, β ,

if $\vdash (\alpha \rightarrow \beta)$ and $\vdash (\neg \alpha \rightarrow \beta)$ then $\vdash \beta$.

- (i) $(\neg \alpha \rightarrow (\alpha \rightarrow \beta))$
- (ii) $(\neg \neg \alpha \rightarrow \alpha)$
- (iii) $(\alpha \rightarrow \neg \neg \alpha)$
- (iv) $((\neg \alpha \rightarrow \alpha) \rightarrow \alpha)$