B8.2 Continuous Martingales and Stochastic Calculus

UNIVERSITY OF OXFORD
Mathematical Institute

Introduction

Samuel Cohen
Hilary Term 2021

Oxford Mathematics

- Our topic is part of the huge field devoted to the study of *stochastic processes*.
- Since first year, you've had the notion of a *random variable*. In this course, we want to think of processes changing through time.

- When we model deterministic quantitities that evolve with (continuous) time, we often appeal to ordinary differential equations as models.
- In this course we develop the 'calculus' necessary to develop an analogous theory of *stochastic* (ordinary) *differential equations*.

An ordinary differential equation might take the form

$$dX(t) = a(t, X(t))dt,$$

for a suitably nice function a. A stochastic equation is often formally written as

$$dX(t) = a(t, X(t))dt + b(t, X(t))dB_t,$$

where the second term on the right models ‘noise’ or fluctuations. Equivalently, can write this in an integral form:

$$X(t) = X(0) + \int_0^t a(s, X(s))ds + \int_0^t b(s, X(s))dB_s.$$

- Here $(B_t)_{t\geq 0}$ is an object that we call Brownian motion.
- We shall consider what appear to be more general driving noises, but the punchline of the course is that under rather general conditions they can all be built from Brownian motion.
- Indeed if we added possible (random) 'jumps' in $X(t)$, we'd capture essentially the most general theory.
- We are not going to allow jumps, so we'll be thinking of settings in which our stochastic equation has a continuous solution $t \mapsto X_t$, and Brownian motion will be a fundamental object.

- Lectures and slides will follow the course notes fairly closely.
- Lectures are broken up into sections in the notes
- Theorem numbering etc is as in the notes, but equation references may be within a set of slides.