ELECTROMAGNETISM (Part B)

Lecture 1

Electromagnetism → study of behaviour of electric charges & currents

$\boxed{0}$ Electromagnetism : introduction & overview

Electromagnetism → study of behaviour of electric charges & currents

Mathematically : EM phenomena are described by a set of PDE's (Maxwell equations)

Basic objects :

1. charged particles (sources)
2. EM fields (interactions!)

(1) charged particles

* the electric charge is an <u>intrinsic property</u> of elementary particles

* charge is discrete: the charges of all known particles are integral multiples of a basic unit

$$q = 1.6 \times 10^{-19} \text{ Coulombs}$$

where: electron has charge $-q$ and the proton has charge $+q$

A neutral object has zero charge (e.i.e. the neutrons in the nucleus of an atom)

* conservation of charge: in an isolated region V of space, the total amount of charge is conserved

Mathematically:

point charge : finite non-zero charge q localized at a point.

One can have a discrete distribution of N point charges with charges q_i , $i = 1, \dots, N$, in a region V of space.

The total charge Q in this region V is

$$Q = \sum_{i=1}^{N} q_i$$

Mathematical idealization:

represent charge by a real function

$\rho(t, x, y, z) = \rho(t, \vec{r})$ charge density

with the property that the total charge Q in a region V of space is

$$Q = \int_V \rho \, dV = \int_V \rho \, dx \, dy \, dz$$

There are other idealizations:

* charges on a surface ie a 2dim region S

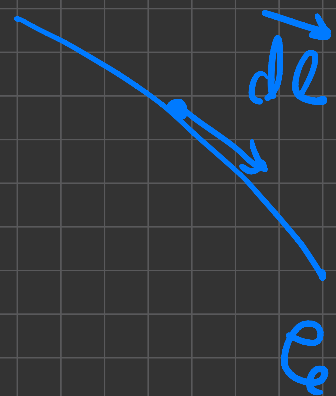
charge surface density $\sigma(t,x,y)$

Total charge on S: $Q = \int_S \sigma \, dS$

* one dimensional distribution of charge along a curve $\mathcal{C}$

charge line density λ = charge per unit length

Total charge: $Q = \int_{\mathcal{C}} \lambda \, dl$



Currents: materials (such as conductors) can carry currents ie charged particles in motion

Given a collection of N point charges q_i $(i=1,\dots,N)$ in motion with velocities $\underline{v}_i$ we define the corresponding electric current as

$$\sum_{i=1}^{n} q_i \vec{v}_i$$

We introduce a current density

$$\vec{J}(t,x,y,z) = \underline{J}(t,\vec{r})$$

s.t. the total electric current in a region V is $\int_V \vec{J}\, dV$

Recall

Basic objects :

1. charged particles
2. EM fields

② The electric & magnetic fields $\vec{E}$ & $\vec{B}$

$\vec{E}$ & $\vec{B}$ are vector valued functions of $(t, \vec{r})$

Idea: a distribution of charge generates $\vec{E}$ & $\vec{B}$

- We know the electromagnetic fields are there by the force excerted on a test particle with charge q moving with velocity $\vec{v}$

$$\boxed{\vec{F} = q\,(\vec{E} + \vec{v} \wedge \vec{B})}$$

Lorentz force law

$$\boxed{\vec{F} = q\,(\vec{E} + \vec{v} \wedge \vec{B})}$$

So: in practice one can use this force law to measure $\vec{E}$ & $\vec{B}$ by measuring the force on the test particle

$\vec{E}$ ~ <u>electric force</u> on the particle per unit charge

$\vec{B}$ ~ <u>magnetic force</u> on the particle per unit current

units:

$[\vec{E}]$ = Newton/Coul $\qquad [\vec{B}]$ = Tesla = Newton $\dfrac{\text{sec}}{\text{coul}\cdot\text{mt}}$

<u>Remark</u>: consider the analogy with the gravitational field

$\vec{F} = m\vec{g}$ = gravitational force on a test particle of mass m in the gravitational field $\vec{g}$

Recall

Basic objects :

① charged particles

② EM fields

The charges generate (sources) EM fields

There are three basic experimental principles

(A) <u>Coulomb law</u>: a stationary point charge generates an electric field $\vec{E}$ at all points $\vec{r}$ in space

(B) <u>Biot-Savart law</u>: magnetic field at a point $\vec{r}$ in space generated by a charge moving with velocity $\vec{v}$ (a static charge generates no $\vec{B}$)

(c) Principle of superposition: given a number N of charge & current distributions, the resultant electric and magnetic fields $\vec{E}$ & $\vec{B}$ is the linear sum of the fields that each distribution generates:

$$\vec{E} = \sum_{i=1}^{N} \vec{E}_i \qquad \vec{B} = \sum_{i=1}^{N} \vec{B}_i$$

Chapters 1 & 2: Electrostatics
Chapter 3: Magnetostatics

We can pass from Coulomb & Biot-Savart laws to Maxwell eqs which are PDEs relating $\vec{E}$ & $\vec{B}$ to the charge & current densities ρ & $\vec{J}$ (the sources). (Chapter 4)

$$\nabla \cdot \vec{E} = \frac{1}{\epsilon_0} \rho \qquad \nabla \cdot \vec{B} = 0$$

$$\nabla \wedge \vec{E} - \frac{\partial}{\partial t} \vec{B} = 0 \qquad \nabla \wedge \vec{B} - \frac{1}{c^2} \frac{\partial \vec{E}}{\partial t} = \mu_0 \vec{J}$$

Maxwell eqs together with Lorentz Force Law form the basis of modern EM

(verified experimentally to almost incredible accuracy)

In this course we will also discuss:

Chapter 5

- electromagnetic waves (these exist even when there are no sources)
  - → $\boxed{\text{light}}$ (radiation)

Chapter 6

- electromagnetism & special relativity
  - ↳ EM is compatible with special relativity (Lorentz transformations)