B7.2 ELECTROMAGNETISM

Chapter 5 : Electromagnetic waves (part 2)

Lecture 16

5 Electromagnetic waves

5.1 Plane electromagnetic waves ✓

5.2 Polarization ✓

This lecture:

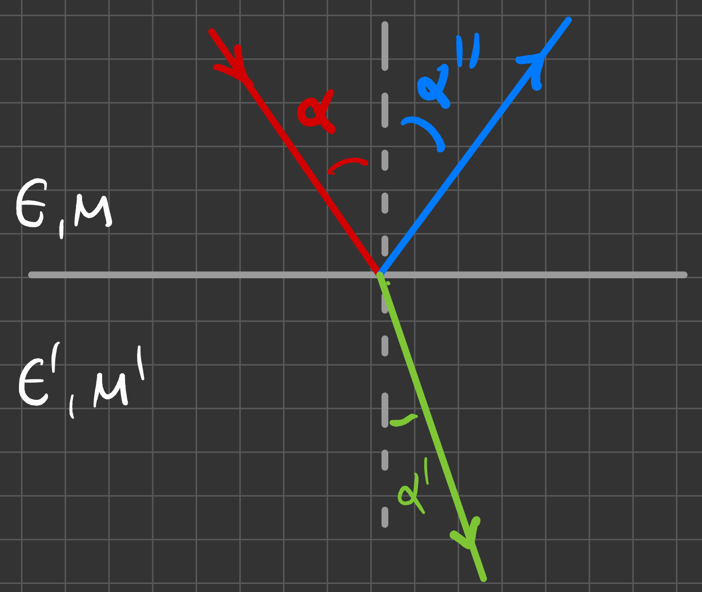
5.3 Reflection & refraction of electromagnetic waves

5.4 Comments on the electromagnetic spectrum

15.3 Reflection and refraction of electromagnetic waves

illustrate the wave-like behaviours of the solutions of the wave equation

Consider a light ray which crosses a boundary between two media. This ray can



- reflect on the surface boundary
  We will see that the angle of reflection equals the angle of incidence

$$\alpha = \alpha'' \qquad (i)$$

- refract through the surface boundary
  We will see that the ray bends as it enters the new medium

$$n \sin\alpha = n' \sin\alpha' \qquad (ii)$$

Snell's law

where $n = \frac{c}{v}$ refraction index of the material

[air $n \sim 1$ as $v \sim c$; water $n \sim 1.3$]

Laws (i) & (ii) can be deduced from Maxwell's equations

$$\left(v = \frac{1}{\sqrt{\epsilon \mu}} \right)$$

Let the plane $z=0$ be the surface boundary between the two media.

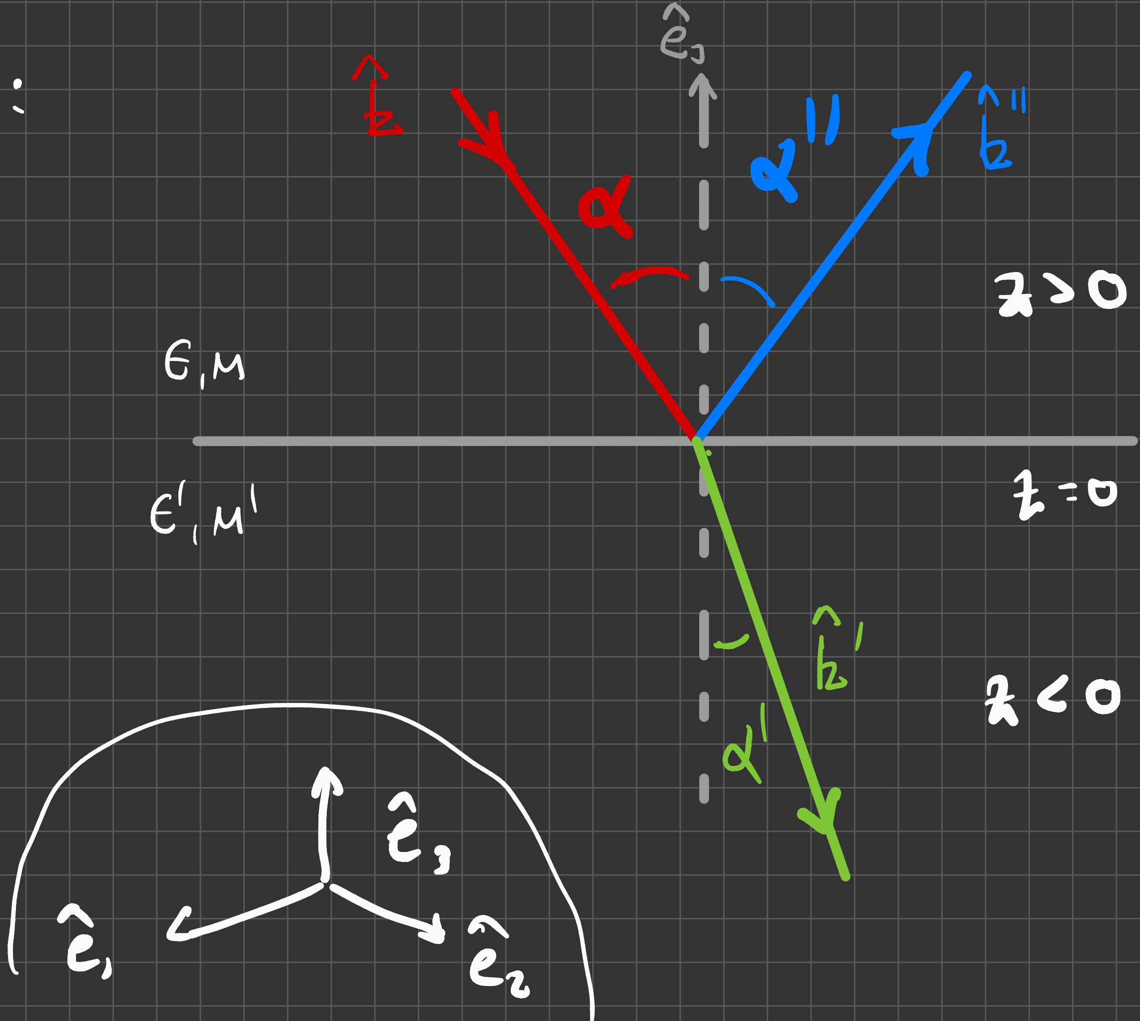
Let $\hat{k}$ be the direction of propagation of the incident wave be st it makes an angle α with $\hat{e}_3$

For the reflected wave we have:

$\hat{k}''$ = direction of propagation making an angle α'' with $\hat{e}_3$

For the refracted wave we have

$\hat{k}'$ = direction of propagation making an angle α' with $\hat{e}_3$



Experimentally: all waves have the same frequency ω of the incident wave.

So, in a medium with ϵ, μ

$$k = \omega v \quad , \quad v = \frac{1}{\sqrt{\epsilon \mu}} = \frac{1}{n} c$$

velocity of the electromagnetic waves in medium with ϵ, μ $(z > 0)$

Hence $$k'' = k = \frac{\omega}{v}$$

and $$k' = \frac{\omega}{v'} \quad , \quad v' = \frac{1}{\sqrt{\epsilon' \mu'}} = \frac{1}{n'} c$$

velocity of the electromagnetic waves in medium with ϵ', μ' $(z < 0)$

Electromagnetic fields

$\boxed{z>0}$ by the superposition principle

$$\vec{E} = \underbrace{\vec{E}_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)}}_{\vec{E}_{incident} = \vec{E}_I} + \underbrace{\vec{E}_0'' e^{i(\vec{k}''\cdot\vec{r}-\omega t)}}_{\vec{E}_{reflected} = \vec{E}''}$$

$$\vec{B} = \frac{1}{\omega}\left(\vec{k} \wedge \vec{E}_I + \vec{k}'' \wedge \vec{E}''\right)$$

$\boxed{z<0}$

$$\vec{E}' = \vec{E}_0' e^{i(\vec{k}'\cdot\vec{r}-\omega t)}$$ refracted wave

$$\vec{B}' = \frac{1}{\omega} \vec{k}' \wedge \vec{E}'$$

Want to find $\vec{k}'$, $\vec{k}''$, $\vec{E}_0'$, $\vec{E}_0''$ in terms of the given (arbitrary) quantities $\vec{k}$ & $\vec{E}_0$

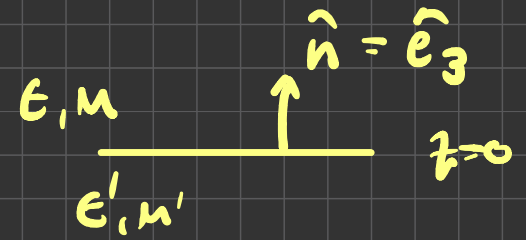
How? require the $\boxed{\text{correct}}$ boundary conditions at $z=0$

These are:

(A) normal components of $\epsilon \vec{E}$ and $\vec{B}$ are continuous across the boundary

(B) tangential components of $\vec{E}$ and $\frac{1}{\mu}\vec{B}$ are continuous across the boundary

Then at $\boxed{z=0}$



$$\text{(A)} \Rightarrow \begin{cases} \left(\epsilon\left(\vec{E}_I + \vec{E}''\right) - \epsilon' \vec{E}'\right) \cdot \hat{n} = 0 & (1) \\ \left(\vec{B}_I + \vec{B}'' - \vec{B}'\right) \cdot \hat{n} = 0 & (2) \end{cases}$$

$$\text{(B)} \Rightarrow \begin{cases} \left(\vec{E}_I + \vec{E}'' - \vec{E}'\right) \wedge \hat{n} = 0 & (3) \\ \left(\frac{1}{\mu}\left(\vec{B}_I + \vec{B}''\right) - \frac{1}{\mu'} \vec{B}'\right) \wedge \hat{n} = 0 & (4) \end{cases}$$

(1) – (4) must be true at $z=0$ **for all** (x, y)

$$\boxed{\text{At } z=0}$$

$$(1) \quad \left[\epsilon\left(\vec{E}_0 e^{i\vec{k}\cdot\vec{r}} + \vec{E}_0'' e^{i\vec{k}''\cdot\vec{r}}\right) - \epsilon' \vec{E}_0' e^{i\vec{k}'\cdot\vec{r}}\right]\cdot\hat{n} = 0$$

$$(2) \quad \left[\vec{k}\wedge\vec{E}_0 e^{i\vec{k}\cdot\vec{r}} + \vec{k}''\wedge\vec{E}_0'' e^{i\vec{k}''\cdot\vec{r}} - \vec{k}'\wedge\vec{E}_0' e^{i\vec{k}'\cdot\vec{r}}\right]\cdot\hat{n} = 0$$

$$(3) \quad \left[\vec{E}_0 e^{i\vec{k}\cdot\vec{r}} + \vec{E}_0'' e^{i\vec{k}''\cdot\vec{r}} - \vec{E}_0' e^{i\vec{k}'\cdot\vec{r}}\right]\wedge\hat{n} = 0$$

$$(4) \quad \left[\frac{1}{\mu}\left(\vec{k}\wedge\vec{E}_0 e^{i\vec{k}\cdot\vec{r}} + \vec{k}''\wedge\vec{E}_0'' e^{i\vec{k}''\cdot\vec{r}}\right) - \frac{1}{\mu'}\vec{k}'\wedge\vec{E}_0' e^{i\vec{k}'\cdot\vec{r}}\right]\wedge\hat{n} = 0$$

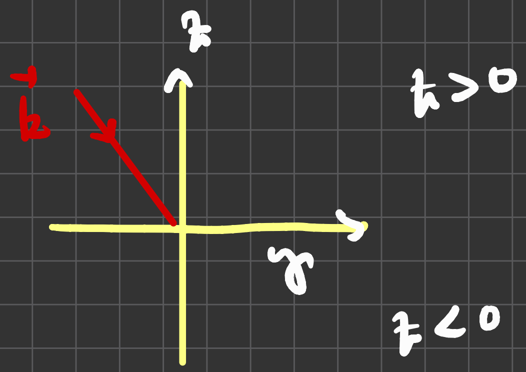
This is true at $z=0$ $\boxed{\forall (x,y)}$ only if

$$\vec{k}\cdot\vec{r}\,\Big|_{z=0} = \vec{k}'\cdot\vec{r}\,\Big|_{z=0} = \vec{k}''\cdot\vec{r}\,\Big|_{z=0}$$

WLOG

assume $\vec{k}$ lies on the (x, z) plane

ie $k_y = 0$



plane $y=0$

Then $\vec{r}\big|_{z=0} = x\hat{e}_1 + y\hat{e}_2$

$\vec{k} = k_x\hat{e}_1 + k_z\hat{e}_3$

$$\left.\begin{aligned}
\vec{k}\cdot\vec{r}\big|_{z=0} &= x k_x \\
\vec{k}''\cdot\vec{r}\big|_{z=0} &= x k''_x + y k''_y \\
\vec{k}'\cdot\vec{r}\big|_{z=0} &= x k'_x + y k'_y
\end{aligned}\right\} \Rightarrow \quad x k_x = x k''_x + y k''_y = x k'_x + y k'_y$$

$$\boxed{\forall (x, y)}$$

$$\Longrightarrow \quad k'_y = k''_y = 0$$

ie $\vec{k}, \vec{k}'$ & $\vec{k}''$ lie on the $\boxed{\text{same}}$ plane (the (x, z) plane)

and $\quad k_x = k''_x = k'_x$

$$\Longrightarrow \quad \left.\begin{aligned} \vec{k} &= k_x \hat{e}_1 + k_z \hat{e}_3 \\ \vec{k}'' &= k_x \hat{e}_1 + k''_z \hat{e}_3 \\ \vec{k}' &= k_x \hat{e}_1 + k'_z \hat{e}_3 \end{aligned}\right\} \quad \text{with} \quad k'' = k$$

$$k_x = k''_x = k'_x$$

$$\Longleftrightarrow \quad k \sin\alpha = k'' \sin\alpha'' = k' \sin\alpha'$$

$$\underset{k \sin\alpha''}{\overset{\parallel}{}}$$

$$\Longleftrightarrow \quad \sin\alpha = \sin\alpha''$$

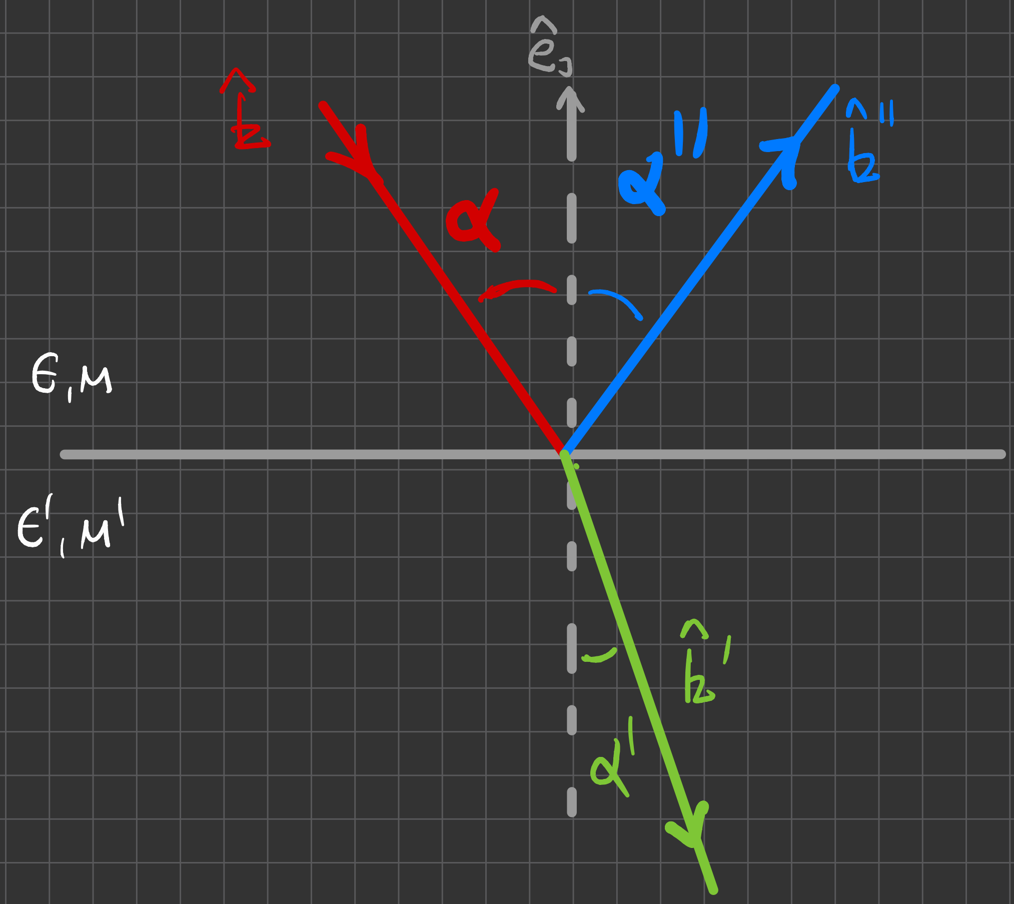
$$k \sin\alpha = k' \sin\alpha'$$

$$\left(k = \frac{\omega}{v}, \quad k' = \frac{\omega}{v'}\right)$$

$$\Longleftrightarrow \quad \boxed{\begin{gathered} \alpha = \alpha' \\ n \sin\alpha = n' \sin\alpha' \end{gathered}}$$

Snell's law

$$\left(n = \frac{c}{v}, \quad n' = \frac{c}{v'}\right)$$



Remark: The refractive index depends on the frequency of the light

You have observed the dispersion of light by a prism

separation of light into different colors (frequencies!)

waves with higher frequencies (blue) "bend" more than those with lower frequencies (red)

You have also observed rainbows:

(can you explain the <u>double</u> rainbow?)

(flooded Port Meadow)

Returning to the boundary conditions at $z=0$

(1) $$\left[\epsilon\left(\vec{E}_0 + \vec{E}_0''\right) - \epsilon' \vec{E}_0'\right]\cdot\hat{n} = 0$$

(2) $$\left[\vec{k}\wedge\vec{E}_0 + \vec{k}''\wedge\vec{E}_0'' - \vec{k}'\wedge\vec{E}_0'\right]\cdot\hat{n} = 0$$

(3) $$\left[\vec{E}_0 + \vec{E}_0'' - \vec{E}_0'\right]\wedge\hat{n} = 0$$

(4) $$\left[\frac{1}{\mu}\left(\vec{k}\wedge\vec{E}_0 + \vec{k}''\wedge\vec{E}_0''\right) - \frac{1}{\mu'}\vec{k}'\wedge\vec{E}_0'\right]\wedge\hat{n} = 0$$

with $\alpha = \alpha''$ & $n\sin\alpha = n'\sin\alpha'$ $\hat{n} = \hat{e}_3$

$k = k'' = \frac{\omega}{c}n$ $k' = \frac{\omega}{c}n'$

$\vec{k}$, $\vec{k}'$ & $\vec{k}''$ all lie on the same plane ((x,z)-plane)

Example: Suppose the electric field of the incident ray is linearly polarized with

$$\vec{E}_0 = E_0 \hat{e}_2 \quad (\text{perpendicular to } (x,z) \text{ plane})$$

Then $\vec{E}_0'$ & $\vec{E}_0''$ are also in the y-direction

(3) : $[\vec{E}_0 + \vec{E}_0'' - \vec{E}_0'] \wedge \hat{n} = 0 \qquad \hat{n} = \hat{e}_3$

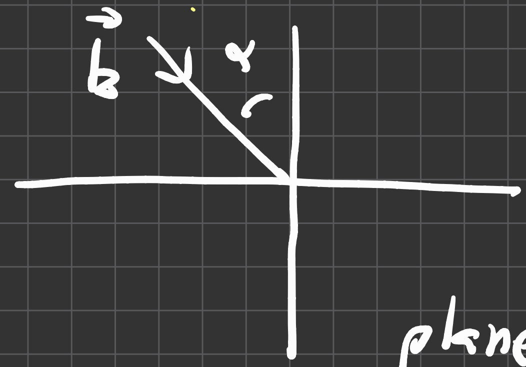
$$\Longrightarrow \quad E_0 + E_0'' - E_0' = 0$$

(4) : $\left[\frac{1}{\mu}\left(\vec{k} \wedge \vec{E}_0 + \vec{k}'' \wedge \vec{E}_0''\right) - \frac{1}{\mu'} \vec{k}' \wedge \vec{E}_0'\right] \wedge \hat{n} = 0$

$$(\vec{k} \wedge \hat{e}_2) \wedge \hat{e}_3 = -k_z \hat{e}_2$$

$$\therefore \frac{1}{\mu}\left(k_z E_0 + k_z'' E_0''\right) - \frac{1}{\mu'} k_z' E_0' = 0$$

$$k_z = -k\cos\alpha$$
$$k''_z = k\cos\alpha$$
$$k'_z = -k'\cos\alpha'$$



plane $y=0$

$$\frac{1}{\mu} k\cos\alpha\left(-E_0 + E''_0\right) + \frac{1}{\mu'} k'\cos\alpha' E'_0 = 0$$

$$\frac{k}{\mu} = \frac{\omega}{v\mu} = \omega\sqrt{\frac{\epsilon}{\mu}}$$

$$\Rightarrow \quad \sqrt{\frac{\epsilon}{\mu}}\cos\alpha\left(E''_0 - E_0\right) = \sqrt{\frac{\epsilon'}{\mu'}}\cos\alpha' E'_0$$

Assuming $\mu=\mu'$:

$$E'_0 = \frac{2\cos\alpha\sin\alpha'}{\sin(\alpha+\alpha')} E_0$$

$$E''_0 = -\frac{\sin(\alpha-\alpha')}{\sin(\alpha+\alpha')} E_0$$

$\boxed{15.4}$ Comments on the electromagnetic spectrum

Electromagnetic spectrum → "full" range of electro-magnetic frequencies

recall: · everything radiates electromagnetic energy (our own bodies, the stars ...)
the frequency of the electromagnetic waves depends on the temperature of the body

When you look at the night sky: what is the light being detected is telling you about the celestial bodies you are observing?

you can ask for example:

why is the sun "yellow" or some other star "red"?

red $\longrightarrow$ blue

$\downarrow$ $\downarrow$

colder hotter

higher frequencies $\rightarrow$

sun surface 5778 Kelvin $\leftarrow$

core 10^8 Kelvin

$\hookleftarrow$ hot, high pressure (due to gravitational pull)

$\Rightarrow$ nuclear fusion

$H \rightarrow He$ + photons of different frequencies

(also heat & energetic charged particles)

$\hookrightarrow$ solar wind

increasing frequency/ energy ↑

increasing wavelength ↓

γ-ray & X-ray astronomy
(study radiation of celestial bodies)

radiation expected warm bodies which contain very hot gases ($10^6 - 10^8$ Kelvin)

γ Ultra HE cosmic γ rays

⋮

γ rays

X rays — 1 nm - 10 pm

the funny names are historical

UV — 1 μm - 1 nm

visible light — 390 - 700 nm

thermal body radiation → IR — 1 m - 10 nm

think of the microwave oven: can transfer energy to heat food → microwaves — 1 cm - 10 m

radio waves — 100 m - 100 km

⋮

Next (and the last lecture)

electromagnetism & special relativity