Problem Sheet 3

1. Exercise 4.5 on page 133 of the Lecture Notes.

2. Exercise 4.15 on page 136 of the Lecture Notes.

3. Exercise 6.1 on page 190 of the Lecture Notes.

4. Consider the velocity jump process (4.49)–(4.50) with initial conditions $X(0) = 0$ and $V(0) = v_0$. In this question you are asked to analyze this velocity jump process in the limit $\Delta t \to 0$. We use $\langle \cdot \rangle$ to denote the average (expected value) over many realizations of the velocity jump process.

(a) Find $\langle V(t) \rangle$, $\langle V^2(t) \rangle$ and $\langle V^3(t) \rangle$ as functions of time t, initial velocity v_0, diffusion constant D and friction coefficient β.

(b) Let $t_1 > 0$ and $t_2 > 0$. Compute $\langle V(t_1)V(t_2) \rangle$ as a function of times t_1 and t_2, initial velocity v_0 and parameters D and β.

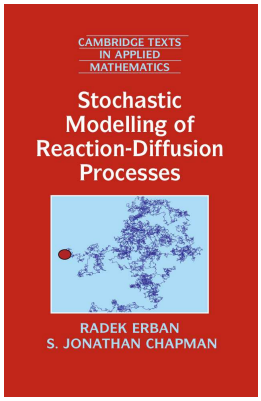
(c) Compute the mean square displacement $\langle X^2(t) \rangle$ as a function of time t, initial velocity v_0 and parameters D and β.

(d) Suppose that t is large. What is the probability that the velocity $V(t)$ is greater than 1?

(e) Choose $D = 0.44$ and $\beta = 0.74$. Write a computer code which uses 10^4 realizations of the velocity jump process (4.49)–(4.50) with initial conditions $X(0) = 0$ and $V(0) = 0.62$ to estimate $\langle X^2(t) \rangle$. Plot the estimated $\langle X^2(t) \rangle$ in the time interval $[0, 3]$ and compare it (in the same figure) with your analytical result obtained in Problem 4(c).

The Lecture Notes for course B5.1 are published by the Cambridge University Press:

R. Erban and S.J. Chapman, "Stochastic Modelling of Reaction Diffusion Processes", Cambridge Texts in Applied Mathematics, CUP (2020)



The online version is available to everyone at all times through SOLO (Search Oxford Libraries Online). College libraries also have physical copies.

Students wishing to further practice material covered in Chapters 4 and 6 can solve any exercise in the Lecture Notes accompanying Chapter 4 (exercises on pages 133-136) and Chapter 6 (exercises on pages 190-191).

Example Exam Questions: Students could solve Question 3 in the 2019 exam paper (Honour School of Mathematics Part B: Paper B5.1 & Honour School of Mathematics and Statistics Part B: Paper B5.1, Trinity Term 2019). Question 3 in this exam is a two-dimensional version of problems studied in Exercises 4.13, 4.14 and 4.15 (students can use a similar approach to solve Question 3, but there is a qualitative difference between the obtained 2D and 3D results).

Students should also be equipped by now with all necessary background material to solve Question 1 in the 2020 exam paper (Honour School of Mathematics Part B: Paper B5.1 & Honour School of Mathematics and Statistics Part B: Paper B5.1, Trinity Term 2020).