B1 Set Theory: Problem sheet 0

Course website: `www.maths.ox.ac.uk/ ~knight/lectures/b1st.html`

All course materials, plus additional material, can be found on the course website; you might want to consider bookmarking it.

This is an introductory problem sheet, and solutions to this sheet should not be handed in. The only set theory you've encountered so far may have been in the Mods *Introduction to Pure Mathematics* course, so some of this will be revision from a long way back!

Solutions will be posted in due course on the course website.

1. Prove that $\varnothing \neq \{\varnothing\}$.

2. (i) Prove that if A, B and C are sets, then $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

(ii) Prove that if A, B and C are sets, then $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

(iii) Prove that if X is a set and A and B are subsets of X, then $X \setminus (A \cup B) = (X \setminus A) \cap (X \setminus B)$.

(iv) Prove that if X is a set and A and B are subsets of X, then $X \setminus (A \cap B) = (X \setminus A) \cup (X \setminus B)$.

3. Let X and Y be sets, let A and B be subsets of X and let C and D be subsets of Y, and let f be a function from X to Y. Which of the following statements are always true?

(i) $f(A) \cup f(B) = f(A \cup B)$,

(ii) $f(A) \cap f(B) = f(A \cap B)$,

(iii) $f^{-1}(C) \cup f^{-1}(D) = f^{-1}(C \cup D)$,

(iv) $f^{-1}(C) \cap f^{-1}(D) = f^{-1}(C \cap D)$,

(v) $f(f^{-1}(C)) \subseteq C$,

(vi) $f^{-1}(f(A)) \subseteq A$,

(vii) $f(f^{-1}(C)) \supseteq C$,

(viii) $f^{-1}(f(A)) \supseteq A$.

4. For the statements in the previous question which are not always true, what conditions on f will ensure that they are true? (eg. f being one-to-one or onto).

5. (i) Prove that if A and B are countable sets, then $A \times B$ is countable.

(ii) Prove that $\mathbb{Z}$ and $\mathbb{Q}$ are countable.

(iii) Prove that the set of finite subsets of $\mathbb{N}$ is countable.

(iv) Prove that $\wp\mathbb{N}$ is uncountable.