Office hours 6

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What can change the trajectory of a particle?

- An external force (then $f = ma$, where f is the force and a is the four-acceleration).
- Gravitational waves "change" the trajectory, but it is not clear exactly what this means – a particle following a geodesic will still move along a geodesic, but now in a spacetime including a gravitional wave (so there is still no force – the trajectory is changed relative to what?). Nevertheless, the proper distance between two freely falling objects is changed.

Why do we look at extrema of the effective potential?

When studying circular orbits we reduced the equation of motion for r to something of the form

$$\frac{1}{2}\dot{r}^2 + V(r) = E.$$

Then we said that circular orbits (where r is constant) are at extrema of V. Why?
We can either use the Euler-Lagrange equations for r directly, or we can differentiate this equation to obtain

$$\dot{r}\left(\ddot{r} + V'(r)\right) = 0,$$

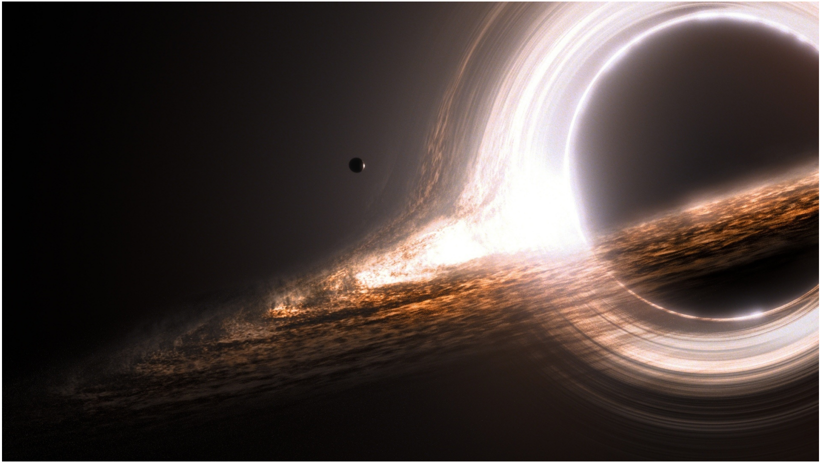
so either $\dot{r} = 0$ or $\ddot{r} = -V'(r)$. In fact, looking at the Euler-Lagrange equations we see that $\ddot{r} = -V'(r)$, so this is a shortcut.

Why use a large impact parameter?

The radius of the sun is around 700,000 km, while the surface $r = 2M$ is around 1km from the centre of the sun. When we say that we need the impact parameter b to be large, we really mean that we need $b/2M$ to be large: for the if the light rays just graze the surface of the sun then $b/2M \sim 700,000$.

In calculating the bending of light, we do not really "take the Newtonian limit"; really we just expand in powers of $\epsilon = 2m/b$.

Is this a light ring?



Weak solutions

Consider the wave equation in two dimensions

$$-\frac{\partial^2}{\partial t^2}\phi + \frac{\partial^2}{\partial x^2}\phi = 0.$$

D'Alembert's formula gives the general solution as a sum of two functions

$$\phi = F(x-t) + G(x+t).$$

But what if F and G are not smooth? What if they are not twice differentiable? Can we still think of this as a solution?

Yes, using *weak solutions*. We can call ϕ a solution if, for all smooth, compactly supported functions ψ, we have

$$\iint \left(\psi\left(-\partial_t^2\phi + \partial_x^2\phi\right)\right) \mathrm{d}x\mathrm{d}t = 0$$

$$\Leftrightarrow \iint \left(\phi\left(-\partial_t^2\psi + \partial_x^2\psi\right)\right) \mathrm{d}x\mathrm{d}t = 0,$$

where we have integrated by parts. In the second formula, we only have to differentiate the smooth function ψ, and not the solution ϕ – we can use this to *define* the 'derivatives' of a non-smooth function (*distributional derivatives*), e.g. $\partial_x\phi$ is an operator which acts on smooth, compactly supported functions ψ via

$$\langle \partial_x\phi, \psi\rangle := \iint -\phi(\partial_x\psi)\mathrm{d}x\mathrm{d}t.$$

Black hole or collapsing star?