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Three ingredients of deep learning.

Theories of Deep Learning: C6.5, Video 1

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Three ingredients to deep learning

Architecture, data, and training

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- Architecture: There are a number of network architectures, with the most classical being *fully connected* and *convolutional* networks which are composed of *layers with non-linear activations of affine transformations*. There are exponential gains in *expressivity with increased numbers of layers, depth.*
- *Data: sufficient amount of data to learn the isometries contained within the data, such as translation or rotation invariant in images, to learn context in text, to learn accents in speech, to learn styles of paintings, etc...*
- *Training Algorithms: Networks need to be trained, to learn* many parameters*; this requires proper initialisation, effective optimisation algorithms, and computational hardware.*

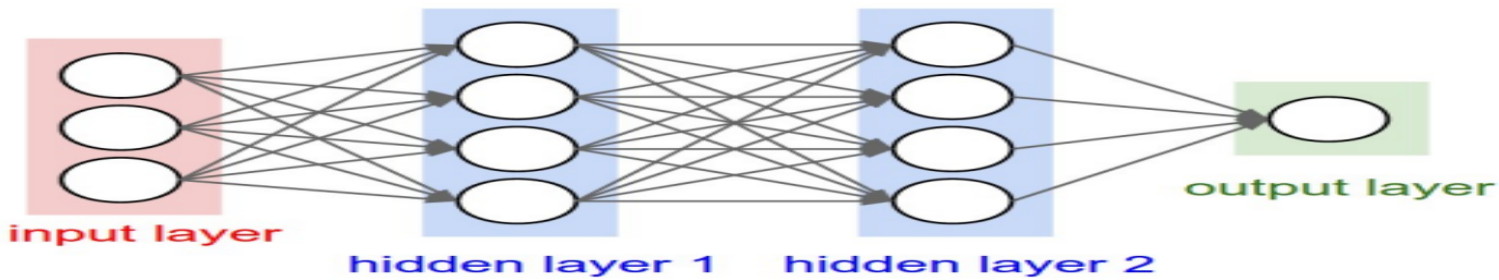
Example of a fully connected DNN:

Two layer fully connected neural net

Repeated affine transformation followed by a nonlinear action:

$$h_{i+1} = \sigma_i \left(W^{(i)} h_i + b^{(i)} \right) \qquad \text{for} \quad i = 1, \ldots, N-1$$

where $W^{(i)} \in \mathbb{R}^{n_{i+1} \times n_i}$ and $b^{(i)} \in \mathbb{R}^{n_{i+1}}$ and $\sigma(\cdot)$ is a nonlinear activation such as ReLU, $\sigma(z) := max(0, z) = z_+$.
The input is h_1, the output is h_N, and h_i for intermediate $i = 2, \cdot, N-1$ are referred to as "hidden" layers.
The number of layers N is the depth, $N \gg 1$ is called "deep."



https://cs.stanford.edu/people/eroberts/courses/soco/projects/neural-networks/Architecture/feedforward.html

Convolutional neural net (CNN):

Convolutional nets impose structure on weight matrices

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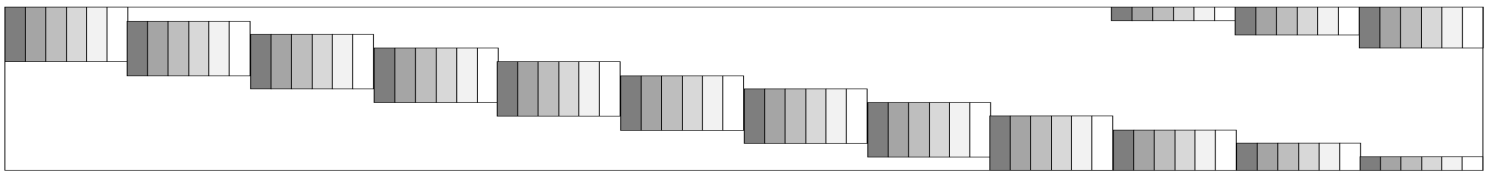
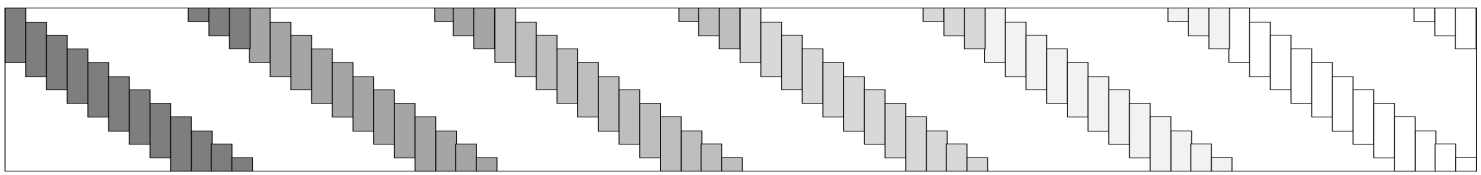
Convolutional neural network layers impose a structure on $W^{(i)}$:



(a) A convolutional matrix.



(b) A concatenation of banded and Circulant matrices.

$W^{(i)}$ is composed of a "mask" (usually of compact support, say just living on 9 pixels) translated by some amount which is referred to as "stride." These "masks" are sometimes referred to as "features."

`https://arxiv.org/pdf/1607.08194.pdf`

LeNET-5, an early Image processing DNN:

Network architectures often include fully connected and convolutional layers

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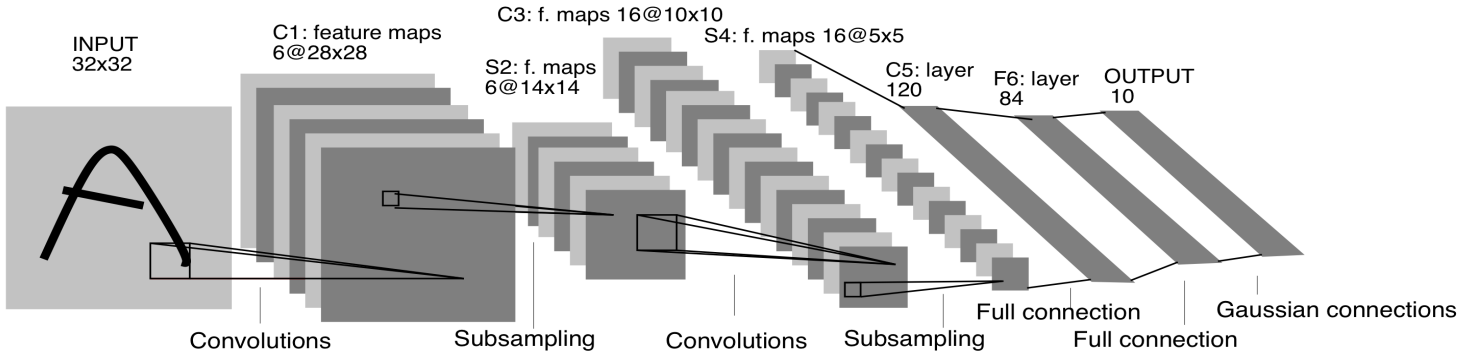


Fig. 2. Architecture of LeNet-5, a Convolutional Neural Network, here for digits recognition. Each plane is a feature map, i.e. a set of units whose weights are constrained to be identical.

C1: conv. layer with 6 feature maps, 5 by 5 support, stride 1.

S2 (and S4): non-overlapping 2 by 2 blocks which equally sum values, mult by weight and add bias.

C3: conv. layer with 16 features, 5 by 5 support, partial connected.

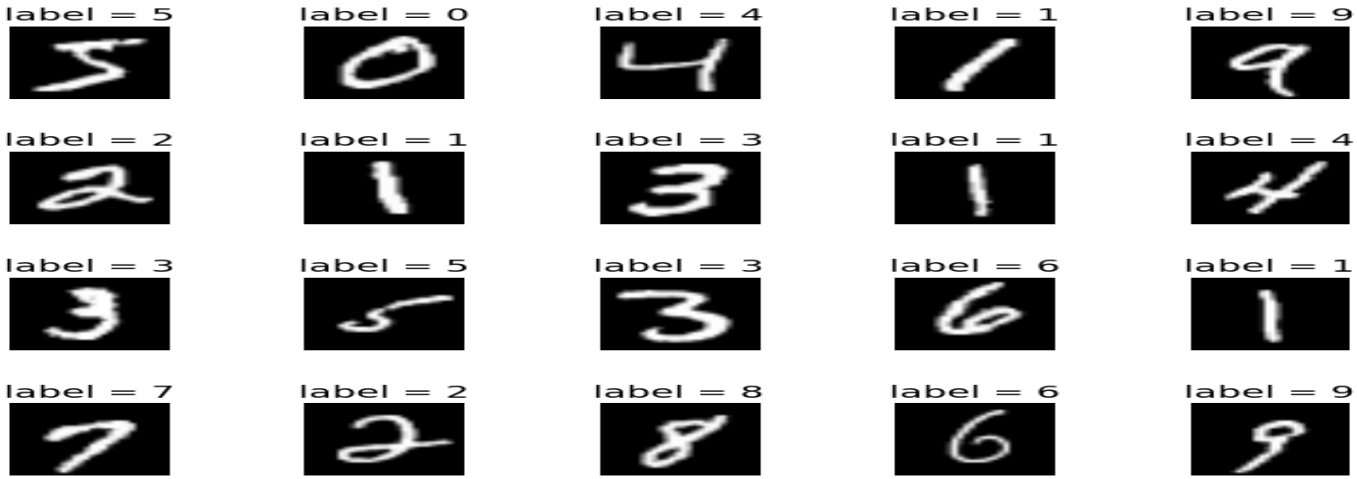
C5: 120 features, 5 by 5 support, no stride; i.e. fully connected.

F6: fully connected, $W \in \mathbb{R}^{84\times 120}$.

`http://yann.lecun.com/exdb/publis/pdf/lecun-98.pdf`

Dataset MNIST, simple exemplar dataset for classification:

MNIST collection of 70,000 labeled digitised gray scale hand written digits 0 to 9



https://corochann.com/mnist-dataset-introduction-1138.html

Vectorising each 28×28 image gives $x(j) \in \mathbb{R}^{784}$ and with ten output classes we set $y(j) \in \mathbb{R}^{10}$ where the index of $y(j)$ denotes the index; that is for an input $x(j)$ corresponding to digit 4 we set $y(j)(\ell) = 1$ for $\ell = 5$ and $y(j)(\ell) = 0$ for $\ell \neq 5$.

http://yann.lecun.com/exdb/mnist/

Data characteristics and isometries:

DNNs are learned functions for specific data types

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- DNN parameters $\theta := \left\{W^{(i)}, b^{(i)}\right\}_{i=1}^{N}$ are learned so that the DNN is the desired function from the data input ambient dimension to the task output dimension.
- Data generally has high correlations such as local smoothness in images, low-rank in consumer data, or phonemes for audio data. Though the data lives in a high-dimensional ambient dimension, the correlation typically causes the data to be approximately low dimensional; e.g. each MNIST digit class is contained on a locally less than 15 dimensional space.
- For classification tasks, much of the variation we observe are invariants which should be in the nullspace of the DNN; e.g. translation and rotation.

Data set size and complexity

The quantity of and richness of datasets has been central to the use of DNNs

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`http://image-net.org`

ImageNet was first presented in 2009 and was central to the development of image classification methods through the ImageNet Large Scale Visual Recognition Challenge (ILSVRC). In 2010 ImageNet included more than 1.2 million labeled images with each image class having approximately 1000 examples.

Datasets as random realisations

Datasets are

- The *training data* used to learn the network parmaters $\theta := \left\{W^{(i)}, b^{(i)}\right\}_{i=1}^{N}$ is only a small set of the data of the data class it is used to represent.
- The value of the DNN is its ability to *generalize* to unseen *testing data*. The ability to generalize demonstrates that the DNN is approximating the data manifold beyond those seen data, and has learned invariants which are considered to be unimportant for the task.
- Typically a dataset is "randomly" partitioned into disjoint training and testing sets. This and many other aspects of training a DNN introduce randomness in the DNN obtained and its performance; for these reasons DNNs are not trained to zero training accuracy.

Training loss function:

Training is formulated as optimizing a loss function

The network "Weights" $W^{(i)}$ (and biases $b^{(i)}$) are learnt to fit a task for a particular data set.

A "labeled" data set is a collection of input, $x(j) = h_1(j)$, and desired output $y(j)$, pairs $\{(x(j), y(j))\}_{j=1}^{m}$.

The net is trained by minimising a loss function $L(x(i), y(i); \theta)$ summed over all training data pairs; that is

$$\min_{\theta} \sum_{i=1}^{m} L(x(i), y(i); \theta).$$

where $\theta := \left\{ W^{(i)}, b^{(i)} \right\}_{i=1}^{N}$ and the resulting learned net is, $H(\cdot; \theta)$. The learned network depends on the choice of function $L(\cdot, \cdot; \theta)$.

Optimisation algorithms:

Loss function minimised through gradient descent

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Training ever larger number of parameters and larger data sets is possible in large part to improvements in optimisation algorithms:

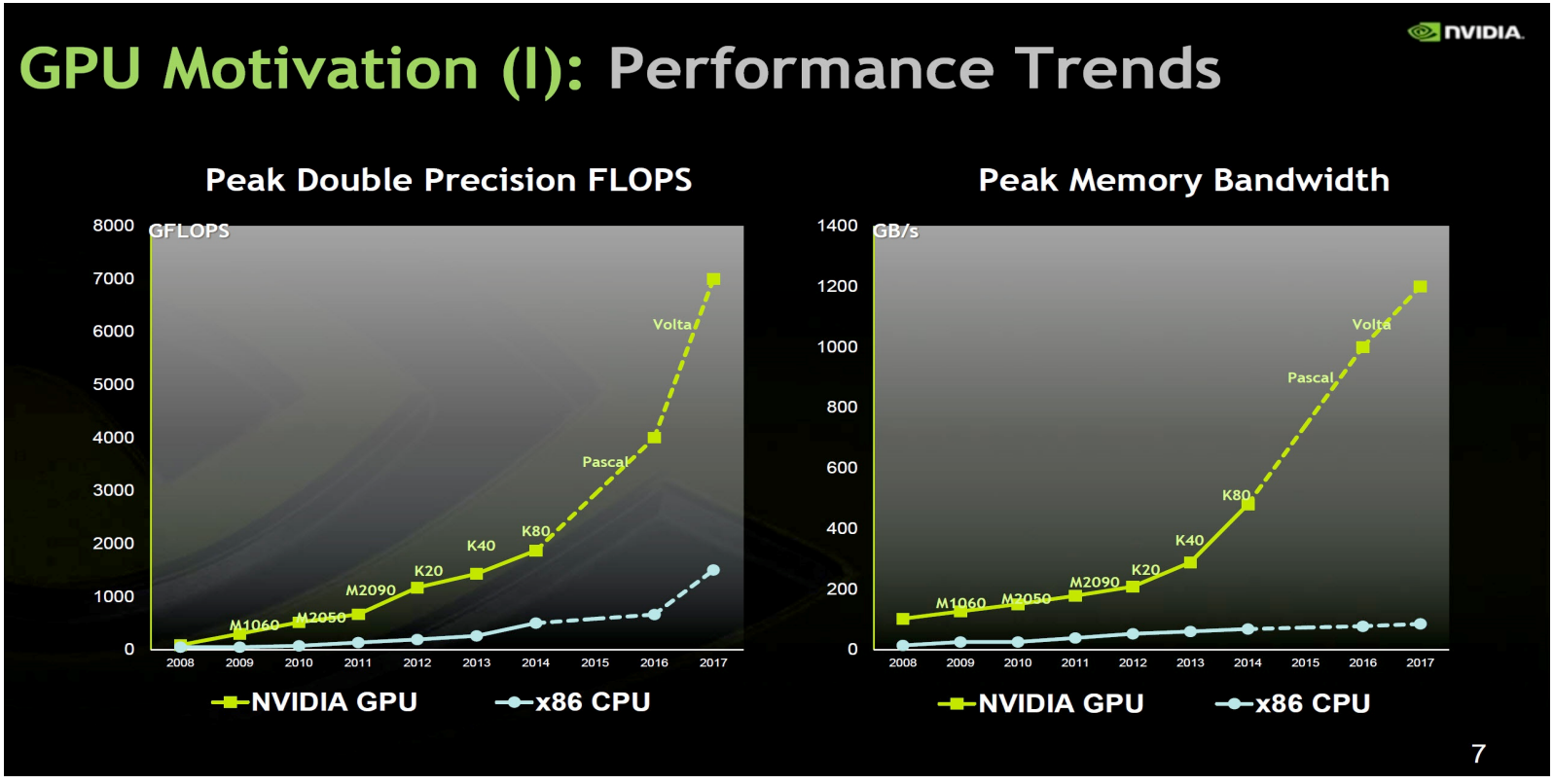
- Back-propagation is key advancement, giving an efficient way to compute the gradient of the training loss function
- Stochastic gradient descent (SGD), and advanced variants, such as Adagrad and Adam, have substantially reduced the computational time and ability to train networks.
- Due to the size of large networks, and inherent randomness due to the dataset construction, optimisation algorithms compute gradients using subset of the training data; this batch-normalisation, and stopping training before the training error is too low seems to *regularize* the network which aids in generalisation.

DNNs are applied to increasingly challenging tasks, which have required increased depth.

- The network hyperparameters, width and depth, as well as training loss function impact the shape of the *typically non-convex landscape* to be minimised. Non-convex landscapes often have many local minima which can result in wide variation in learned networks.
- In some cases, network parameters or architectures can be proven to result in *convex* landscapes which are inherently easier to train.
- Unless network weight initialisation is chosen appropriately, deep networks can suffer from gradients that either diverge or converge to zero with depth (vanishing or exploding).

Training of DNNs aided by hardware advances:

Hardware is now specifically designed to training DNNs



Cloud computing and software such as PyTorch and TensorFlow have made training the DNN parameters computationally tractable.

LeCun et. al. considered, amongst others, a 2 layer fully connected net with architecture $W^{(1)} \in \mathbb{R}^{100\times 728}$ and $W^{(2)} \in \mathbb{R}^{10\times 100}$ and sigmoid activation $\sigma(z) = \frac{1}{1+e^{-z}}$, applied to MNIST classification using the sum of squares training loss objective:

$$L(x(i), y(i); \theta) := (y(i) - H(x(i); \theta))^2$$

Backprop was introduced in this article, and gradient descent applied. The MNIST dataset is partitioned into 50,000 training images and 10,000 test images, each in $\mathbb{R}^{728}$.

The above two layer net has $73,910$ parameters and achieved a 4.7% classification accuracy on the test set.
`http://yann.lecun.com/exdb/publis/pdf/lecun-98.pdf`

Returning to MNIST classification: data

Exemplifying the three ingredients of deep learning: data augmentation

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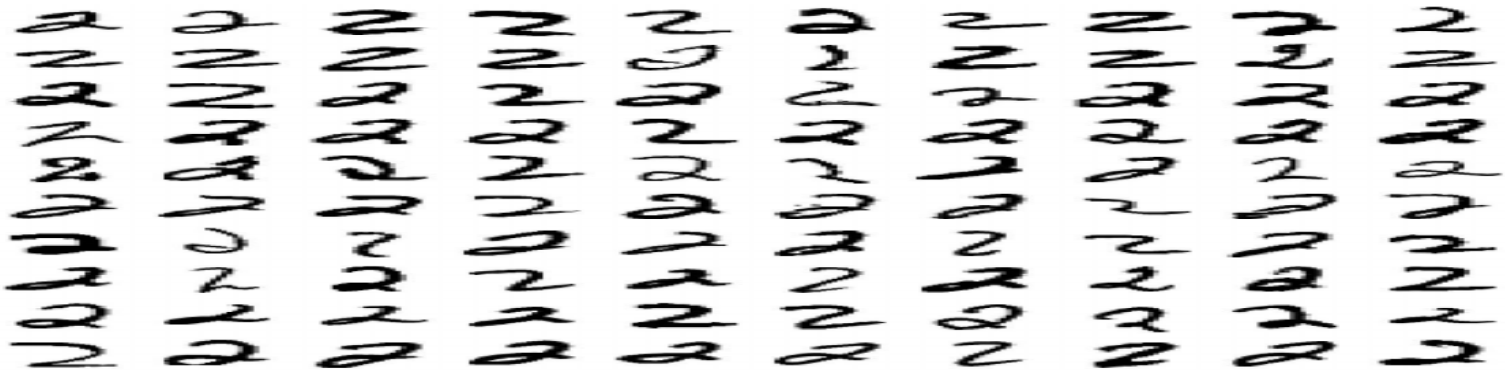
The resulting net $H(x;\theta)$ is a function from $\mathbb{R}^{728}$ to $\mathbb{R}^{10}$ whose goal is to map points from a given class to a single point; that is, all images of the digit 4 should be mapped to the single point $y(2)$ whose 3^{rd} entry is 1 and all other entries are zero.

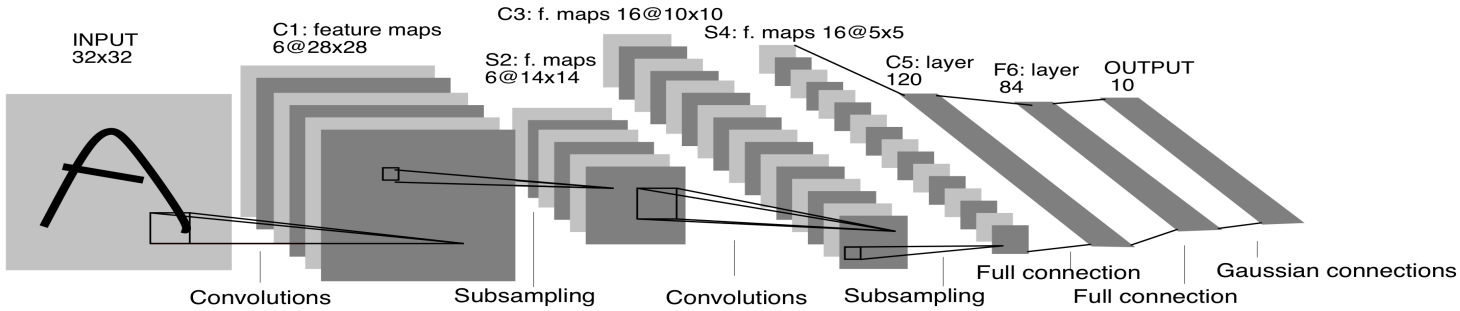


The function $H(x;\theta)$ trained on 50,000 examples achieves 4.7% error rate; augmenting the data by including artificially distorted examples decreased the error to 2.5%.

`http://yann.lecun.com/exdb/publis/pdf/lecun-98.pdf`

Returning to LeNet-5 for MNIST classification: architecture

Exemplifying the three ingredients of deep learning: architecture



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C5: 120 features, 5 by 5 support, no stride; i.e. fully connected.

F6: fully connected, $W \in \mathbb{R}^{84\times 120}$.

MNIST classification error rate from 4.5% 2 layer FFN to 0.95% error; or 0.8% when trained with artificially distorted data.

`http://yann.lecun.com/exdb/publis/pdf/lecun-98.pdf`

Examples of topics in this course:

Example course content

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- Approximation theory perspectives, characteristics of data, and generalisation error.
- Interpretability and structure in DNN weights
- Role of initialisation on information flow and ability to train very DNNs.
- Training landscape: non-convex and convexification.
- Optimisation algorithms used for training DNNs.
- Robustness of DNNs and adversarial perturbations.
- More recent architectures and advances.

Course practicalities:

Reading, tutorials, and assessment

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- We will not be following a textbook, but you might find Deep Learning by Goodfellow, Bengio, and Courville useful: `http://www.deeplearningbook.org`
- Lectures / videos will typically include links to conference proceedings or journal articles. You are encouraged to read some but not all of these; this week read: lecun-98.pdf.
- Tutorials will include some pen and paper questions, along with computational experiments. This course is focused on theoretical aspects of deep learning. Computational experiments help explore the theory, and are a valuable skill.
- The course is assessed entirely by individual mini-projects. You will have significant scope to select a topic of interest to you, but there are some restrictions; details to follow.