C6.1 Numerical Linear Algebra

- SVD and its properties, applications
- Direct methods for linear systems and least-squares problems
- Direct methods for eigenvalue problems
- Krylov subspace methods for linear systems
- Krylov subspace methods for eigenvalue problems
- Other topics in numerical linear algebra (randomised algorithms etc)

References

- Trefethen-Bau (97): Numerical Linear Algebra
  - covers essentials, beautiful exposition
- Golub-Van Loan (12): Matrix Computations
  - classic, encyclopedic
- Horn and Johnson (12): Matrix Analysis (& topics (86))
  - excellent theoretical treatise, little numerical treatment
- J. Demmel (97): Applied Numerical Linear Algebra
  - impressive content, some niche
- N. J. Higham (02): Accuracy and Stability of Algorithms
  - bible for stability, conditioning
- H. C. Elman, D. J. Silvester, A. J. Wathen (14): Finite elements and fast iterative solvers
  - PDE applications of linear systems, preconditioning

What is numerical linear algebra?

The study of numerical algorithms for problems involving matrices

Two main (only!?) problems:

1. Linear system

$$Ax = b$$

2. Eigenvalue problem

$$Ax = \lambda x$$

λ: eigenvalue (eigval), x: eigenvector (eigvec)

Why numerical linear algebra?

- Many (in fact most) problems in scientific computing (and even machine learning) boil down to a linear problem
  - Because that's often the only way to deal with the scale of problems we face today! (and in future)
  - For linear problems, so much is understood and reliable algorithms available
- $Ax = b$: e.g. Newton's method for $F(x) = 0$, $F : \mathbb{R}^n \to \mathbb{R}^n$ nonlinear
  1. start with initial guess $x^{(0)} \in \mathbb{R}^n$
  2. find Jacobian matrix $J \in \mathbb{R}^{n\times n}$, $J_{ij} = \frac{\partial F_i(x)}{\partial x_j}|_{x=x^{(0)}}$
  3. update $x^{(1)} := x^{(0)} - J^{-1}F(x^{(0)})$, repeat
- $Ax = \lambda x$: e.g. Principal component analysis (PCA), data compression, Schrödinger eqn., Google pagerank,
- Other sources: differential equations, optimisation, regression, data analysis, ...