C5.12 Lecture 9 : Waves in 2 (or 3) dimensions

Waves propagate through the heart in $\approx$ 2 dimensions.

We provide two approximate theories for this.

I. Periodic wave propagation (this gets rather technical – don't worry)

Let $\underset{\sim}{w}$ be a vector of 'reactants' (e.g. V, m, n, h)

with kinetics

$$\underset{\sim}{w}_t = f(\underset{\sim}{w})$$ having limit cycle behaviour $\underset{\sim}{w} = \underset{\sim}{W}_0(t)$ of period T.

Now add weak diffusion

$$\underset{\sim}{w}_t = f(\underset{\sim}{w}) + \varepsilon \nabla^2 \underset{\sim}{w}$$ (assuming for convenience all reactants diffuse equally)

The weak coupling causes a slow variation in $\underset{\sim}{w}$ on a time scale $\tau = \varepsilon t$

We seek a multiple scale approximation $\underset{\sim}{w} = \underset{\sim}{w}(\underset{\sim}{x}, t, \tau)$

so $$\underset{\sim}{w}_t + \varepsilon \underset{\sim}{w}_\tau = f(\underset{\sim}{w}) + \varepsilon \nabla^2 \underset{\sim}{w}$$

and now $\underset{\sim}{w} \sim \underset{\sim}{w}_0 + \varepsilon \underset{\sim}{w}_1 + \dots$

$$\Rightarrow \underset{\sim}{w}_{0t} = f(\underset{\sim}{w}_0) \quad \Rightarrow \underset{\sim}{w}_0 = \underset{\sim}{W}_0(t+\psi), \quad \psi = \psi(\underset{\sim}{x}, \tau)$$

$$\underset{\sim}{w}_{1t} - J\underset{\sim}{w}_1 = -(\psi_\tau - \nabla^2\psi)\underset{\sim}{W}_0' + |\nabla\psi|^2 \underset{\sim}{W}_0''$$ linear inhomogeneous

Note : $\underset{\sim}{s} = \underset{\sim}{W}_0'$ satisfies $\underset{\sim}{s}_t - J\underset{\sim}{s} = 0$. $J = Df[\underset{\sim}{W}_0(t)]$ is Jacobian

Solution is $$\underset{\sim}{w}_1 = -t[\psi_\tau - \nabla^2\psi]\underset{\sim}{s} + |\nabla\psi|^2 \underset{\sim}{u}$$

and as $t \to \infty$, $\underset{\sim}{u} \approx \underset{\sim}{s}[\bar{\alpha} t + p(t)]$, p periodic

Secular terms are those $\propto t$: for $t \sim \frac{1}{\varepsilon}$ the expansion breaks down.

Therefore, choose

$$\boxed{\psi_\tau = \nabla^2\psi + \bar{\alpha}|\nabla\psi|^2}$$

eg: $\phi = e^{\bar{\alpha}\psi}$
$\Rightarrow \phi_\tau = \nabla^2\phi$

target patterns

$$\psi_\tau = \nabla^2\psi + \bar{\alpha}|\nabla\psi|^2 \qquad (\underset{\sim}{w} \sim \underset{\sim}{W}_0(t+\psi))$$

We suppose a boundary condition $\psi = \Omega\tau$ at $r = b$, say at impurity

Also radiation condition: waves move outwards $\Rightarrow \frac{\partial\psi}{\partial r} < 0$ at $r \to \infty$

Ansatz $\psi = \Omega\tau - f(r)$ ($f' > 0$ at ∞, if $\Omega > 0$) $f(b) = 0$

$\Rightarrow f'' + \frac{1}{r}f' - \bar{\alpha}f'^2 + \Omega = 0$ (or via diffusion equation for $e^{\bar{\alpha}\psi}$)

$\Rightarrow f = -\frac{1}{\bar{\alpha}}\ln\left[\frac{K_0(\sqrt{\bar{\alpha}\Omega}\,r)}{K_0(\sqrt{\bar{\alpha}\Omega}\,b)}\right]$ K_0 solution of modified Bessel equation for $g(z)$: $g'' + \frac{1}{z}g' - g = 0$

(Abramowitz + Stegun, Handbook of Mathematical Functions)

[suppress I_0 (for $\Omega > 0$) as for $z \to \infty$ $I_0 \sim \frac{e^z}{\sqrt{2\pi z}}$, $K_0 \sim \sqrt{\frac{\pi}{2z}}$ $\Rightarrow \ln \begin{matrix} I_0 \\ K_0 \end{matrix} \sim \begin{matrix} z \\ -z \end{matrix} \Rightarrow f \sim \begin{matrix} -\sqrt{\frac{\Omega}{\bar{\alpha}}}\,r \\ +\sqrt{\frac{\Omega}{\bar{\alpha}}}\,r \end{matrix}$ ← choose this]

If $\bar{\alpha}\Omega < 0$ solutions are Bessel functions $J_0, Y_0 \to f \to \infty$. X.

[Note: $\phi = e^{\bar{\alpha}\psi} = e^{\bar{\alpha}\Omega\tau}\Phi \Rightarrow \Phi_\tau = \nabla^2\Phi - \bar{\alpha}\Omega\Phi$, $\Phi = 1$ at $r = b$!? if $\bar{\alpha}\Omega < 0$]

Spiral waves

$\psi = \Omega\tau + m\theta - g(r)$, $\bar{\alpha}\Omega > 0$, $\psi = \Omega\tau + m\theta$ on $r = b$

($\underset{\sim}{w} = \underset{\sim}{W}_0(t + \Omega\tau + m\theta)$ on $r = b$ requires $m = \frac{2\pi n}{T}$ $n \in \mathbb{Z}$)

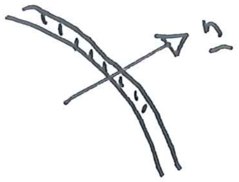
$$\to g = -\frac{1}{\bar{\alpha}}\ln\left[\frac{K_\nu(\sqrt{\bar{\alpha}\Omega}\,r)}{K_\nu(\sqrt{\bar{\alpha}\Omega}\,b)}\right]$$

$\nu = i\bar{\alpha}m$

& $\psi \sim \Omega\tau + m\theta - \sqrt{\frac{\Omega}{\bar{\alpha}}}\,r$ as $r \to \infty$

– Archimedean spiral

Curved fronts

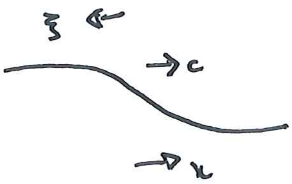


We assume a wave in which the phase (say) ψ of the potential varies rapidly in a thin region (the wave front), which may curve more slowly in other than normal directions.

To illustrate the method, suppose

$$v_t = f(v) + \nabla^2 v \qquad \text{(e.g. Fisher!)}$$

& it has a 1-D travelling wave solution $v = V(\xi)$, $\xi = ct - x$, $c > 0$



$V(\infty) = 1$
$V(-\infty) = 0$

& $cV' = f(V) + V''$

Let ψ denote the phase of the wave front – in 1-D this is basically ξ (or a function thereof) ~~.~~

Suppose a quasi-one-dimensional wave has wave front $\psi(\underline{x}, t) = 0$
[e.g. $\psi = ct - r$ is a target pattern]

Let ξ be distance along normal $\underline{n} = -\dfrac{\nabla\psi}{|\nabla\psi|}$ but inwards (as for ψ)

then $\delta\psi = \nabla\psi \cdot \delta\underline{x} = -|\nabla\psi|\,\underline{n}\cdot\delta\underline{x} = |\nabla\psi|\,\delta\xi$

$$\Rightarrow \psi_\xi = |\nabla\psi|$$

Look for a solution $V(\psi)$ (need to determine ψ..)

$$\Rightarrow \quad V'(\psi)\left(\psi_t - \nabla^2\psi\right) = f(V) + V''(\psi)\,|\nabla\psi|^2$$

cf familiar!

Quasi-one-dimensional

$$V_\xi \simeq |\nabla\psi|\,V'(\psi) \quad \ldots$$

$$\Rightarrow \quad V_{\xi\xi} + \frac{V_\xi}{|\nabla\psi|}\left\{\nabla^2\psi - \frac{\partial|\nabla\psi|}{\partial\xi} - \psi_t\right\} + f(V) = 0$$

$$\Rightarrow \quad \psi_t = \nabla^2\psi - \frac{\partial|\nabla\psi|}{\partial\xi} + c|\nabla\psi|$$

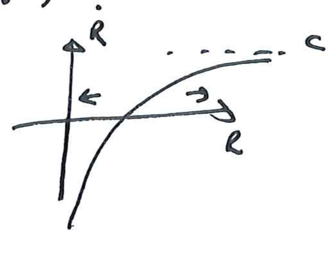
$$\text{or} \quad \psi_t = c|\nabla\psi| + |\nabla\psi|\,\nabla.\left\{\frac{\nabla\psi}{|\nabla\psi|}\right\}$$

$$\text{or} \quad v_n = c - \underbrace{\nabla.n}_{\text{curvature}} \qquad \text{– Eikonal equation}$$

In 2-D if the front is $r = R(\theta, t)$ we derive (ex.)

$$R_t = \frac{c\left(R^2 + R_\theta^2\right)^{1/2}}{R} - \frac{\left(R^2 + 2R_\theta^2 - RR_{\theta\theta}\right)}{R\left(R^2 + R_\theta^2\right)}$$

Target patterns $\quad R = \bar{R}(t)$ & $\dot{\bar{R}} = c - \dfrac{1}{\bar{R}}$



– causes curvature blockage – if $R(0)$ is too small.

Spiral waves?

– Possible, but less easy to analyse.

Lecture 9, addendum

Spiral waves

with $r = R(\theta, t)$, $R_t = \dfrac{c(R^2 + R_\theta^2)^{1/2}}{R} - \dfrac{(R^2 + 2R_\theta^2 - RR_{\theta\theta})}{R(R^2 + R_\theta^2)}$

Look for a solution $R(\eta)$, $\eta = \omega t - \theta$

$$\Rightarrow \quad \omega R' = c\left[1 + \frac{R'^2}{R^2}\right]^{1/2} - \frac{1}{R}\left[1 + \frac{2R'^2}{R^2} - \frac{R''}{R}\right]\left[1 + \frac{R'^2}{R^2}\right]^{-1}$$

Look for a solution at large R (thus large η)

$$R' = \frac{c}{\omega}\left[1 + \frac{R'^2}{2R^2}\cdots\right] - \frac{1}{\omega R}\left[1 + \frac{R'^2}{R^2} - \frac{R''}{R}\cdots\right]$$

$\Rightarrow R \simeq \dfrac{c\eta}{\omega}$ i.e. $r \simeq ct - \dfrac{c}{\omega}\theta$ Archimedean spiral

$\dfrac{R'}{R} \simeq \dfrac{1}{\eta}$

At next order

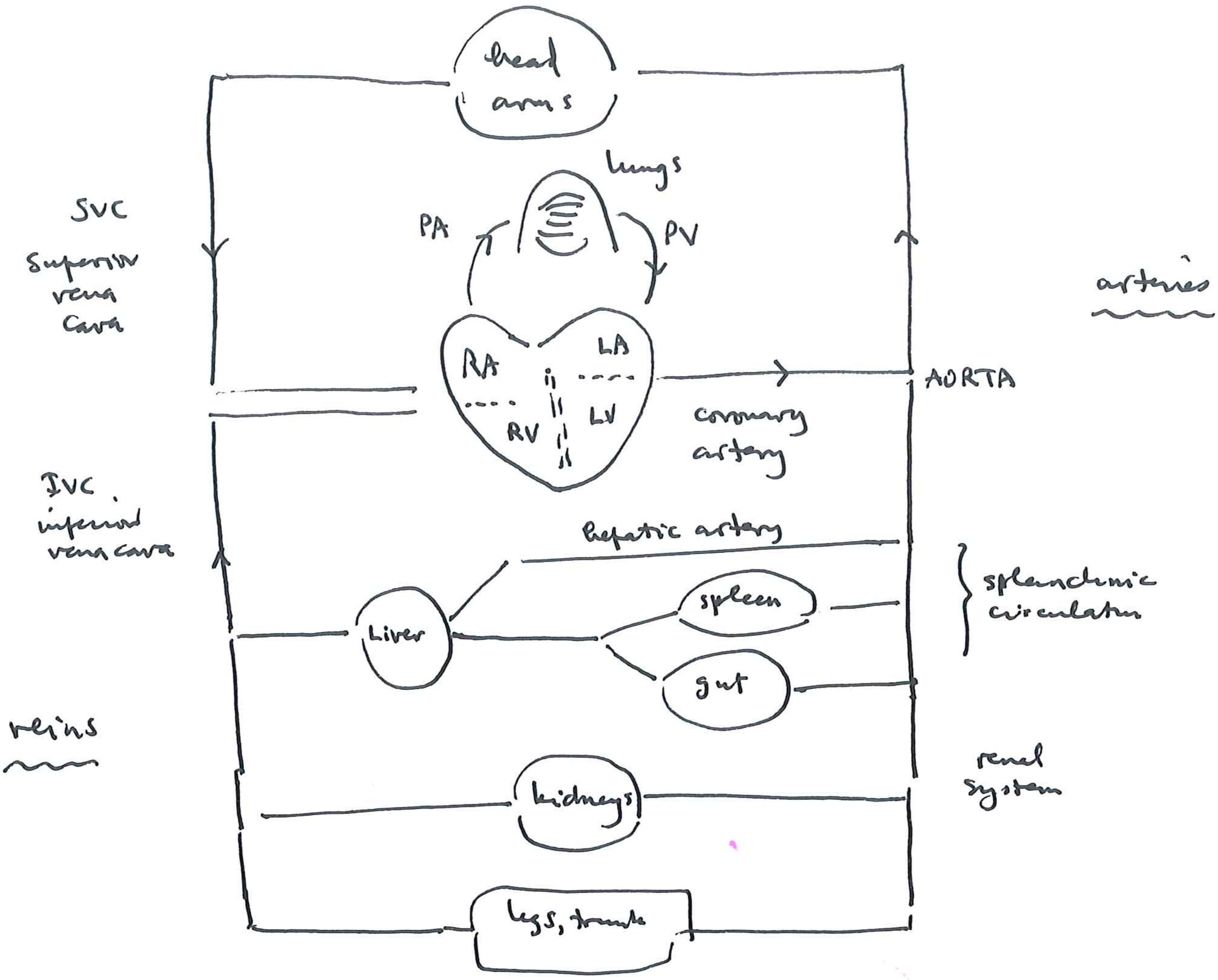
$$R' \simeq \frac{c}{\omega}\left[1 + \frac{1}{2\eta^2}\cdots\right] - \frac{1}{c\eta}\left[1 + \frac{1}{\eta^2}\cdots\right]$$

$$\Rightarrow R \simeq \frac{c\eta}{\omega} - \frac{1}{c}\ln\eta + \cdots$$

and an ansatz $R \sim \dfrac{c\eta}{\omega} - \dfrac{1}{c}\ln\eta + A_0 + \dfrac{A_1}{\eta} + \cdots$

probably might do
(but there may be other log terms)

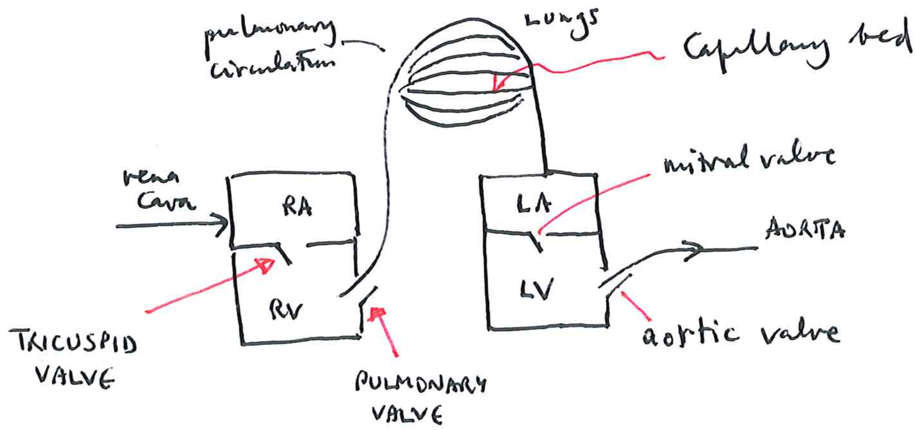
CS·12 Lecture 10 : The heart as a pump.



The circulation

Terminology
RA LA atria
RV LV ventricles
PA PV pulmonary artery, vein

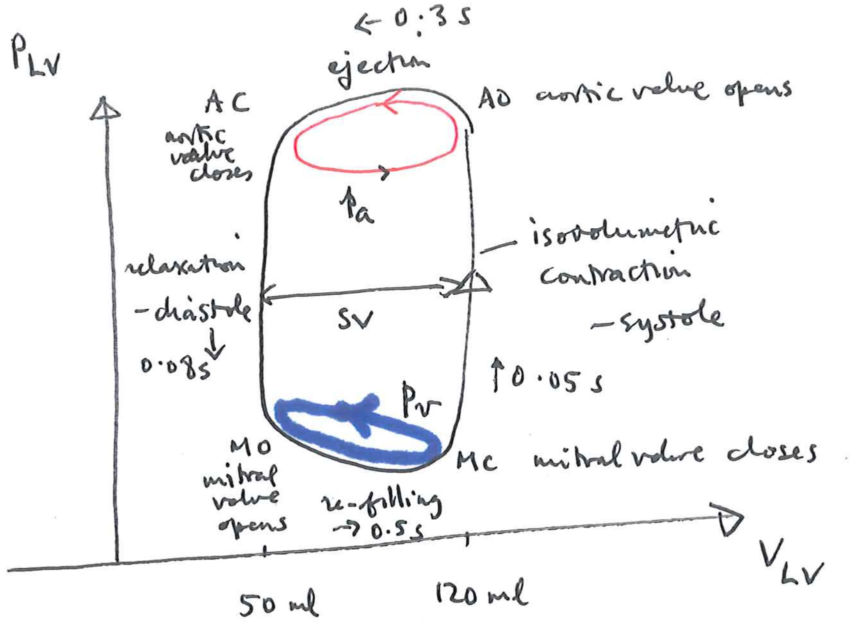
Blood collects O_2 from lungs and delivers to tissues
collects CO_2 from tissues and dumps to lungs (+ exhale)

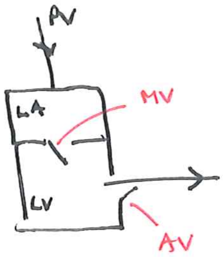


Pressure volume cycle of the left ventricle

P_a arterial pressure

P_v venous pressure

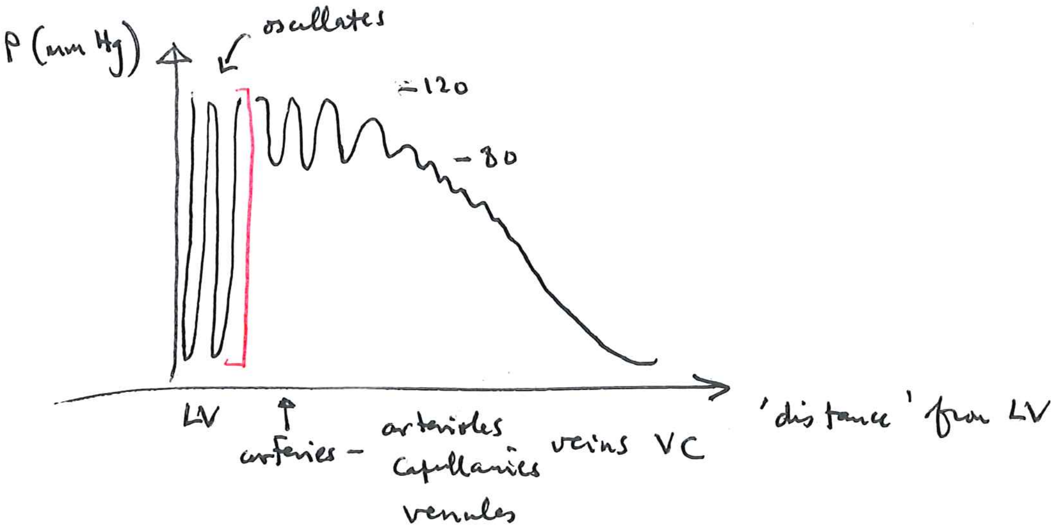




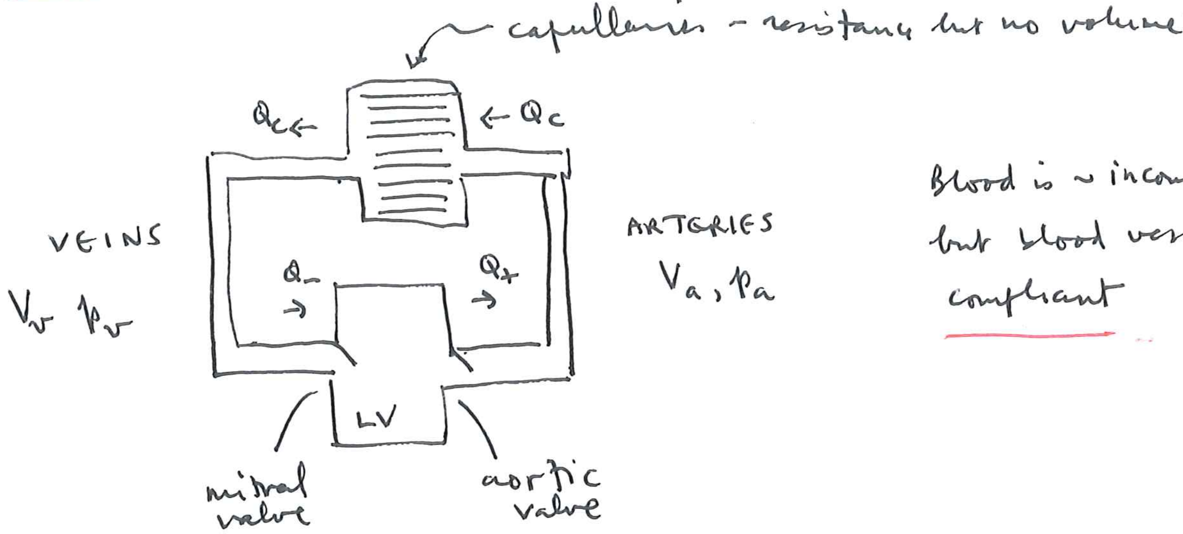
SV : Stroke volume - change of left ventricular volume on contraction

Heart rate HR : ~~period of~~ rate $\left(\frac{1}{\text{period}}\right)$ of SA node cycling

Cardiac output = SV × HR



A simple mechanical model of the circulation



Blood is ~ incompressible but blood vessels are compliant

Q_+, Q_-, Q_c : blood flows

V_a, V_v, V_{LV} : compartment volumes

Blood conservation

$$\dot{V}_a = Q_+ - Q_c$$
$$\dot{V}_v = Q_c - Q_-$$
$$\dot{V}_{LV} = Q_- - Q_+$$

Capillary resistance

we assume capillaries have no volume but provide resistance

$$Q_c = \frac{p_a - p_v}{R_c}$$

Ventricular valves resistance

$$Q_+ = \frac{[p_{LV} - p_a]_+}{R_a}, \quad Q_- = \frac{[p_v - p_{LV}]_+}{R_v}$$

AV MV

Note : $[x]_+ = \max(x, 0)$

Compliance

Increasing pressure distends blood vessels, thus

$$V_a = V_{a0} + C_a p_a$$

$$V_v = V_{v0} + C_v p_v$$

$$V_{LV} = V_{LV0} + C_{LV} p_{LV}$$

C : compliance

$= \frac{1}{E}$: elastance

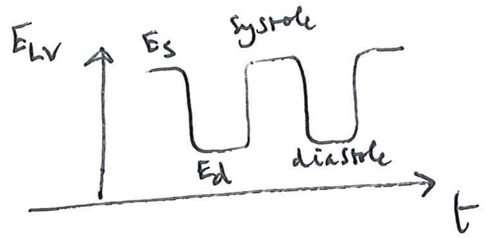
or $p_{LV} = E_{LV}(V_{LV} - V_{LV0})$

C low, E high $\leftrightarrow$ tight

C high E low $\leftrightarrow$ loose

Contraction

Periodic firing of SA node cells and the resulting contraction cause stiffening of the LV during systole



E_{LV} varies between systolic E_s > diastolic E_d

Lecture 11: Heart and blood flow models

I: Heartbeat

The model, written in terms of pressures is

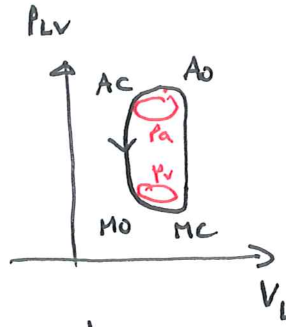
$$\dot{p}_a = \frac{\overset{\text{AO}\uparrow}{[p_{LV} - p_a]_+}}{R_a C_a \;\; 0.09\,s} - \frac{(p_a - p_v)}{R_c C_a \;\; 1.8\,s}$$

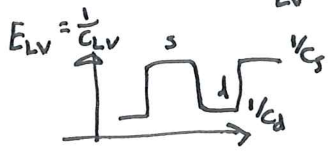
$$\dot{p}_v = \frac{p_a - p_v}{R_c C_v \;\; 60\,s} - \frac{\overset{\text{MO}\uparrow}{[p_v - p_{LV}]_+}}{R_v C_v \;\; 0.8\,s}$$

$$\dot{p}_{LV} = \frac{[p_v - p_{LV}]_+}{R_v C_{LV}} - \frac{[p_{LV} - p_a]_+}{R_a C_{LV}}$$

$R_v C_{LV}$: 0.005 s (S), 0.25 s (D) — $R_a C_{LV}$: 0.02 s (S), 1 s (D)

while C_{LV} is constant





We use values

$R_a C_a \sim 0.09\,s$

$R_c C_a \sim 1.8\,s$

$R_c C_v \sim 60\,s$

$R_v C_v \sim 0.8\,s$

$R_v C_s \sim 0.005\,s$

$R_a C_s \sim 0.02\,s$

$R_v C_d \sim 0.25\,s$

$R_a C_d \sim 1\,s$

$\leftarrow$

$R_a \sim 0.06$, $R_v \sim 0.016$ } mmHg s ml^{-1}

$R_c \sim 1.2$

$C_a \sim 1.5$, $C_v \sim 50$, $C_s \sim 0.3$, $C_d \sim 16$ } ml mmHg^{-1}

Also

$R_c C_s \sim 0.36\,s$

$R_c C_d \sim 19.2\,s$

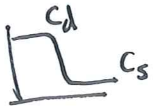
Note $p_a \sim 80 - 120$ mmHg

$p_v \sim 10$ mmHg $\ll p_a$ & we assume this

A. Isovolumetric contraction (0·05 s)

Start with both valves closed (end diastole/start systole)

$\Rightarrow \; \dot{(C_{LV} p_{LV})} = 0$ (V_{LV} constant) ; $p_v \simeq$ constant

& $C_d \to C_s$ 

so $p_{LV} \uparrow$: starting at p^0_{LV} aims for $p^0_{LV} \dfrac{C_d}{C_s} \simeq 50 p^0_{LV}$!!

B Ejection ($\Delta t_F \sim 0\cdot 3$ s)

p_{LV} increases above p_a, aortic valve opens

[This actually happens during contraction, so p_a jumps up.]
↑
via

$C_{LV} p_{LV} + C_a p_a$
$\simeq C_d p^0_{LV} + C_a p^0_a$
$\to (C_s + C_a) p_a$

$p_v \approx$ constant

$$\dot{p}_a = \frac{p_{LV} - p_a}{\underset{0\cdot 09}{R_a C_a}} - \frac{(p_a - p_v)}{\underset{1\cdot 8}{R_c C_a}}$$

$$\dot{p}_{LV} = \frac{-(p_{LV} - p_a)}{\underset{0\cdot 02}{R_a C_s}}$$

Ejection lasts $0\cdot 3\,s \gg 0\cdot 02\,s$ so $p_{LV} \simeq p_a$

To find evolution of each,

$$R_a C_s \dot{p}_{LV} = -(p_{LV} - p_a)$$

$$R_a C_a \dot{p}_a = p_{LV} - p_a - \frac{R_a}{R_c}(\overbrace{p_a - p_v}^{\simeq p_a})$$

Add $\quad R_a (C_s + C_a) \dot{p}_a \simeq -\dfrac{R_a}{R_c} p_a$

$$\dot{p}_a \simeq \frac{-p_a}{R_c (C_s + C_a)} \quad \sim 2\cdot 2\,s$$

& p_a declines by $\simeq \exp\left[\dfrac{-\Delta t_F}{R_c (C_s + C_a)}\right] \sim 0\cdot 87 \qquad \Delta t_F = 0\cdot 3\,s$

C Isovolumetric relaxation (0.08 s)

The elastance E_{LV} drops, compliance rises from C_s to C_d (rapidly)

As before $(\dot{C_{LV} P_{LV}}) \simeq 0$ when $p_{LV} < p_a$, $p_a \simeq$ constant

until $p_{LV} \leqslant p_v$, mitral valve opens, ~~flow~~ p_{LV} overshoots to $p_v - \Delta p$, say.

D Re-filling ($\Delta t_R \sim 0.5$ s)

Mitral valve open, aortic valve closed

$$\dot{p}_a = \frac{-(p_a - p_v)}{R_c C_a} \simeq \frac{-p_a}{R_c C_a}$$

, p_a further decline by $\exp\left[\frac{-\Delta t_R}{R_c C_a}\right] \simeq 0.76$

So total p_a decline $0.87 \times 0.72 \simeq 0.66$ (gives $120 \to 80$)

$$\dot{p}_{LV} = \frac{p_v - p_{LV}}{R_v C_d} \quad 1\text{s}$$

$$\dot{p}_v = \underbrace{\frac{p_a - p_v}{R_c C_v}}_{\text{small}} \; 60\text{s} - \frac{(p_v - p_{LV})}{R_v C_v} \; 0.8\text{s}$$

Approximately $(\dot{p_v - p_{LV}}) = -\frac{1}{R_v}\left(\frac{1}{C_v} + \frac{1}{C_d}\right)(p_v - p_{LV})$

$$\Rightarrow p_v - p_{LV} = \Delta p \exp\left[-\frac{1}{R_v}\left(\frac{1}{C_v} + \frac{1}{C_d}\right)t\right]$$

Δp from overshoot of p_{LV} at end relaxation

E Venous pressure

What actually is p_v?

It is the balance between capillary relaxation and filling:

$$\dot{p}_v \simeq \frac{p_a}{R_c C_v}\left[-\frac{\Delta p}{R_v C_v} \exp\left\{-\frac{1}{R_v}\left(\frac{1}{C_v} + \frac{1}{C_d}\right)t\right\}\right]_{\text{filling}}$$

And integrating over a heartbeat Δt_H leads to

mean

$$\Delta p = p_v - p_{LV}\Big|_{\substack{\text{end} \\ \text{relaxation}}} = \frac{\bar{p}_a \, \Delta t_H}{R_c} \frac{\left(\frac{1}{C_d} + \frac{1}{C_v}\right)}{\left[1 - \exp\left\{-\frac{1}{R_v}\left(\frac{1}{C_v} + \frac{1}{C_d}\right)\Delta t_R\right\}\right]}$$

Note

$$\frac{R_c}{\left(\frac{1}{C_d} + \frac{1}{C_v}\right)} \sim 15\,s$$

$$\frac{R_v}{\left(\frac{1}{C_d} + \frac{1}{C_v}\right)} \sim 0 \cdot 02\,s$$

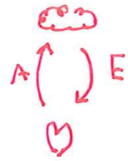
$$\text{So} \quad \Delta p \simeq \frac{\bar{p}_a \, \Delta t_H}{\left\{\frac{R_c}{\frac{1}{C_d} + \frac{1}{C_v}}\right\}} \sim \frac{100 \times 0 \cdot 9}{15} = 6 \text{ mmHg}$$

MP Lecture 12 : Nervous control of the heart

- Control of heart rate HR, stroke volume SV & arterial blood pressure (ABP)

Nervous control is effected by

- afferent nerves (to the brain)
- efferent nerves (from the brain)



There are 2 systems of nerves

- sympathetic system
  - α-sympathetic (peripheral vessels)
  - β-sympathetic (ventricular muscle)

The sympathetic system acts by release of neurotransmitters called catecholamines (noradrenaline + adrenaline)

→ ventricular muscle [β]
- chronotropic effect : increases SA firing rate → HR ↑
- inotropic effect : increases elastance (decreases compliance) → SV ↑

→ peripheral resistance [α]
- via vasoconstriction : tightens blood vessels → R_c ↑ (resistance)

The sympathetic system acts slowly ($t \sim 10$ s)

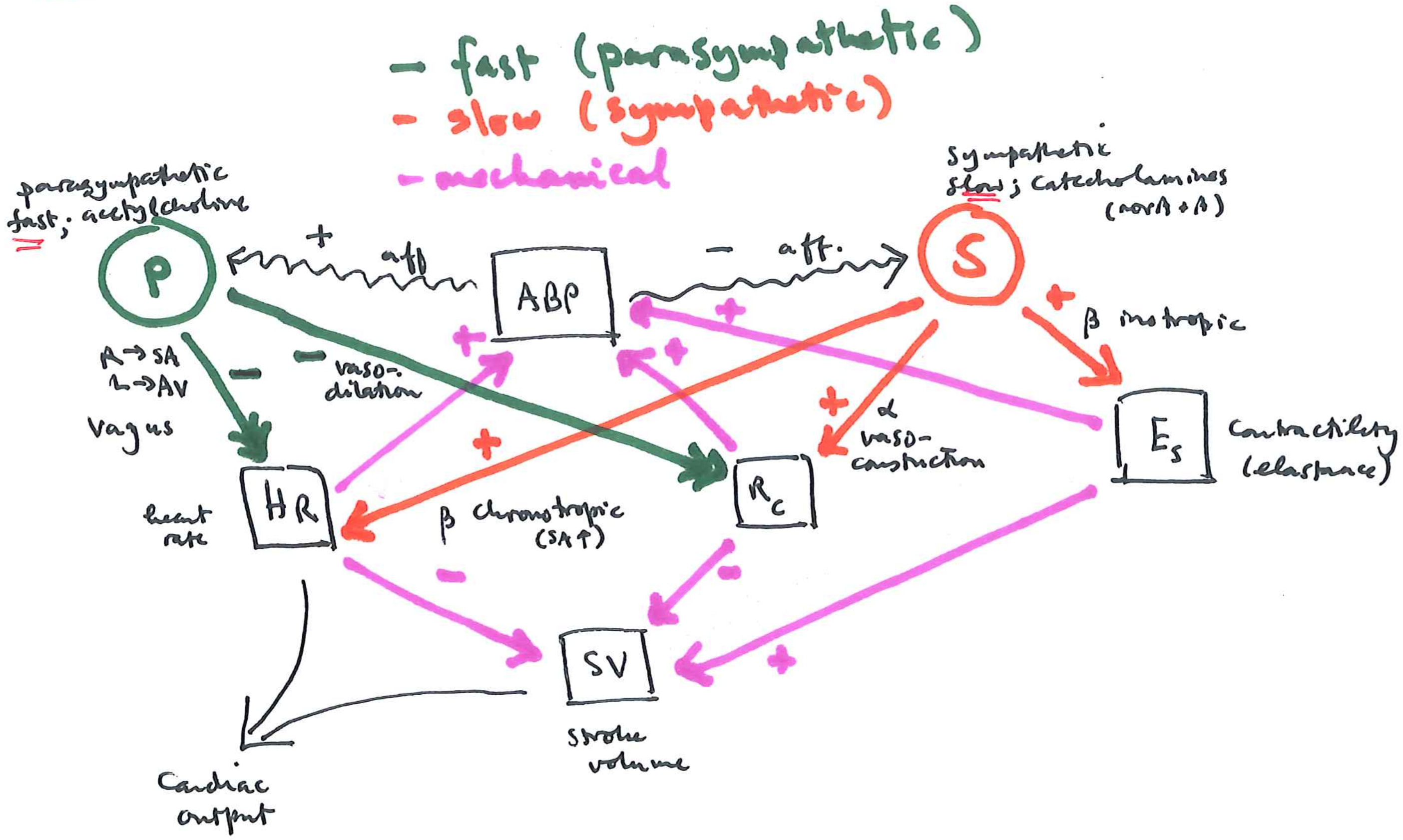
- Parasympathetic system
  - innervates heart via vagus nerves
    releases neurotransmitter (acetylcholine)
    decreases HR ↓ instantly
  - peripheral vessels R_c ↓ via vasodilation

Afferent nerves

Most important are arterial baroreceptors in the chest + neck which respond to arterial pressure via stretching.

The response of these is called the baroreflex.

Response diagram

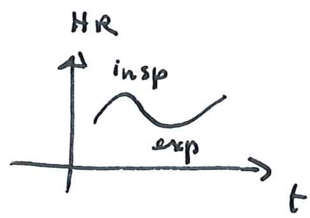


Oscillations

Respiratory sinus arrhythmia (RSA)

Period $\lesssim 5$ s forced by respiration

Inspiration → low pressure → HR↑ via vagus.



Mayer waves

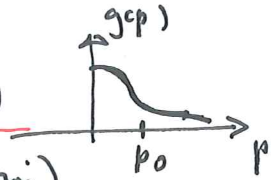
~10 s oscillations in blood pressure

– associated with sympathetic response.

The Ottesen model (modified)

This is a continuous time model (!) in which

sympathetic tone (firing frequency) $\underline{T_s = g(p_a^\tau)}$



(e.g. Hill function)

$p_a^\tau = p_a(t-\tau)$ delay

parasympathetic tone $\underline{T_p = 1 - g(p_a)}$ $\qquad \underline{g(p) = \dfrac{1}{1+\left(\frac{p}{p_0}\right)^n}}$

H = heart rate (taken as continuous variable!)

$$\dot{H} = \delta_H (H_0 - H) + \lambda_H T_s - \mu_H T_p$$

together with

$$C_a \dot{p}_a = -\frac{(p_a - p_v)}{R_c} + H\Delta V \qquad (cts)$$

ΔV = stroke volume

$$\left[C_v \dot{p}_v = \frac{p_a - p_v}{R_c} - \frac{p_v}{R_v} \right]$$

$p_v \ll p_a$ so uncouples

Assume $p_v \ll p_a$, non-dimensionalise $H = H_0 h$, $p_a = p_0 p$, $t \sim \tau$

$$\Rightarrow \boxed{\begin{aligned} \dot{p} &= \kappa(-p + \nu h) \\ \varepsilon \dot{h} &= \delta(1-h) + \lambda g(p_1) - \mu\{1 - g(p)\} \end{aligned}}$$

$\kappa \sim 5 \cdot 6$
$\nu \sim 1 \cdot 85$
$\varepsilon \sim 0 \cdot 06$
$\delta \sim 1$
$\lambda \sim 0 \cdot 29$
$\mu \sim 0 \cdot 4$

Here $p_1 = p(t-1)$: delayed

$g(p) = \dfrac{1}{1+p^n}$

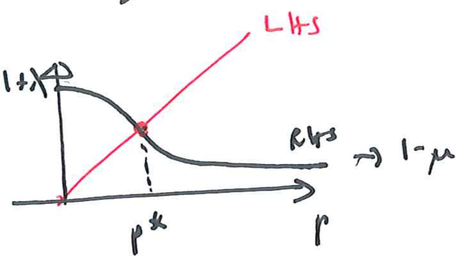
$n = 7$: sharp!

with $\varepsilon \ll 1$, take $\delta = 1$

$$\Rightarrow h = 1 + \lambda g(p_1) - \mu\{1 - g(p)\}$$

$$\Rightarrow \dot{p} = \kappa\left[\nu(1-\mu) - p + \nu\{\lambda g(p_1) + \mu g(p)\}\right]$$

Steady state $\quad \dfrac{p}{\nu} = 1 - \mu + (\lambda+\mu) g(p)$



Stability $\quad p = p^* + P, \quad \dot{P} \simeq \kappa\left[-P - \nu s\{\lambda P_1 + \mu P\}\right]$

where $s = -g'(p^*) > 0$

$$\begin{cases} B = \kappa(1 + \nu s \mu) & \sim 8 \\ G = \kappa \nu s \lambda & \sim 2 \end{cases}$$

$\Downarrow$

stable

Solutions $P = e^{\sigma t}$, $\quad \sigma = -B - Ge^{-\sigma}$

— ∞ # roots, $\sigma \to \infty$ (Picard)

— $B, G > 0$, fix B, $\mathrm{Re}\,\sigma < 0$ for $G < B$

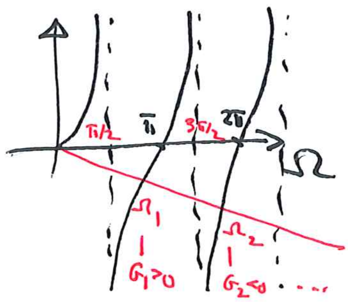
— instability if $\sigma = i\Omega \Rightarrow i\Omega = -B - G\cos\Omega + iG\sin\Omega$

$$\Rightarrow G = \frac{\Omega}{\sin\Omega} = \frac{-B}{\cos\Omega} \; : \; \tan\Omega = -\frac{\Omega}{B}$$

$\Rightarrow \Omega_1, \Omega_2 \ldots \; (>0)$

$\Omega_n \in \left((n-\tfrac{1}{2})\pi, n\pi\right)$

$\Rightarrow G_n(B)$



— transversality $\mathrm{Re}\,\sigma'(G) > 0$