CORRECTIONS TO VIDEO LECTURES

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There are a couple of errors in the lecture videos which I have been alerted to. I'll try and re-record these, but in the mean time I will note them here. Please let me know of any other errors or ambiguities you spot when watching the videos.

(1) Week 6, lecture 3: at 24:30:
I should have written
$$\mathrm{tr}\big([\mathrm{ad}(e_\alpha),\mathrm{ad}(f_\alpha)]\big)$$
and not $\mathrm{tr}(\mathrm{ad}(e_\alpha)\mathrm{ad}(f_\alpha))$. As ad is a homomorphism of Lie algebras, $[\mathrm{ad}(e_\alpha),\mathrm{ad}(f_\alpha)] = \mathrm{ad}([e_\alpha, f_\alpha]) = \mathrm{ad}(h_\alpha)$, which is the point being made in that line.

(2) Week 7, video 3:
Around 8:00 into the video: The definition of indecomposable roots in a positive system $\Phi^+(v)$ is not the simplest one to use in this situation. (In fact it is equivalent to the definition given here, but on the face of it, a root might be indecomposable in the sense given in the lecture video while still being decomposable in the sense given below.)

Definition 1.1. If $v \in V$ is regular, so that $\Phi^+(v) = \{\alpha \in \Phi : (\alpha, v) > 0\}$ gives a positive system of roots (that is, $\Phi = \Phi^+(v) \sqcup -\Phi^+(v)$), we say that a root $\alpha \in \Phi^+$ is *indecomposable* if whenever $\alpha = \sum_{i=1}^N n_i\alpha_i$, with $n_i \in \mathbb{Z}_{\geq 0}$ and $\Phi^+(v) = \{\alpha_1, \ldots, \alpha_N\}$, we have $n_i = 0$ for all but one i, say i_0, and hence the equation is just $\alpha = \alpha_{i_0}$.

The arguments in the video go through with very slight modifications:
At 16:00 in the video in the proof on the existence of bases for a root system, the argument should be modified as follows:

(a) The claim that, given any root γ in $\Phi^+(v)$, the expression $\gamma = \sum_{i=1}^N n_i\alpha_i$ which maximises $\sum_{i=1}^n n_i$ has $n_i \neq 0$ only if α_i is an indecomposable root then follows just as in the video: If there is some j with $n_j > 0$ and $\alpha_j = \sum_{i=1}^N m_i\alpha_i$ where more than one m_i is nonzero, replacing α_i by this sum gives:
$$\gamma = \sum_{i\neq j}^N n_i\alpha_i + \sum_{i=1}^N m_i\alpha_i,$$
and the sum of the coefficients of the roots in $\Phi^+(v)$ here is at least $\sum_{i\neq j} n_i + n_i\sum_{j=1}^N m_i$, which is clearly at least $\sum_{i=1}^N n_i + n_j$ since $\sum_{i=1}^N m_i \geq 2$.

(b) At around 37:00: The argument proving $\Delta(v) = \Delta$ (*i.e.* the arbitrary base Δ is of the form $\Delta(v)$ for the vector chosen vector v). This actually is immediate from the definition of indecomposable used above. (Whereas to apply the definition in the video one needs to show that the two notions are equivalent, which while true is not used anywhere else, so it is simpler to replace our definition with the one above).