C8.5: Introduction to Schramm-Loewner Evolution
A few comments

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At the moment you should have access to all four problem sheets, lecture notes, video recordings of the lectures and a copy of everything I wrote on a board while recording videos. This can be viewed as an alternative lecture notes.

The lecture notes are new and I am sure there are some typos. If you find any, I would appreciate if you let me know. I will fix them and update the notes.

At the moment, there are 24 videos covering all material. A bit later, I will upload one more about other lattice models and a quick overview of the development of the field in the past few years.

For the first problem sheet you will need videos 1-4. For the second videos 5-9, for sheet 4: videos 10-14, for sheet 4: videos 15-16.

Videos do not follow the usual structure of 50 minute lectures. They range in length from 10 minutes to one hour. They also assume that you remember the previous material, in particular, I do not recall the material of previous lectures. If there is something you don't remember, you can always look it up in the previous videos, lecture notes or board recordings. Below is a brief description of videos and how they relate to the lecture notes.

1. **Introduction** Section 1.1: General introduction, convergence of random walk, conformal invariance of Brownian Motion.

2. **Complex Analysis background** Sections 2.1 and 2.2: Quick introduction about univalent maps, Beurling estimates, Poisson and Schwarz formulas.

3. **Half-plane capacity** Section 2.3: Definition and basic properties of the half-plane capacity.

4. **Properties of g_K** Section 2.4: main properties of the mapping-out functions.

5. **Loewner Evolution** Section 3.1: deriving the Loewner evolution for a simple curve.

6. **Solving Loewner Evolution** Section 3.2: we show that the Loewner evolution can be solved for a general driving function/measure.

7. **Is it a curve?** Section 3.3: we discuss when the Loewner Evolution produces a curve and when it does not.

8. **Properties of LE** Section 3.4: basic properties of the Loewner Evolution.

9. **Definition of SLE** Sections 4.1 and 4.2: introduce Schramm's principle, define SLE and study its basic properties.

10. **Properties of SLE** Section 4.3: Introduce the phase transition.

11. **Proof of a technical lemma** Proof of Lemma 4.9.

12. **Proof of the phase transition** Proof of Lemma 4.8 and sketch of the proof of Theorem 4.7.

13. **Locality** Section 4.4: show that SLE(6) satisfies locality. Connection to percolation.

14. **Restriction** Section 4.5: show that SLE(8/3) satisfies the restriction property. Connection to self-avoiding random walk.

15. **Schramm's formula** Section 4.6.1: probability that SLE curve passes to the right or to the left of a given curve.

16. **One-arm exponent** Section 4.6.2: probability that SLE curve does not make a counter-clockwise loop by time t. Asymptotic analysis of SLE.

17. **Towards conformal invariance** Section 5.1: general strategy of proving that there is a conformally invariant scaling limit.

18. **Percolation crossing probability** Section 5.1: Cardy's formula for SLE(6).

19. **Russo-Seymour-Welsh estimate**: Section 5.3.4: a comment about a priori tightness estimates.

20. **Crossing probability implies convergence to SLE** Section 5.2: using the percolation example we show that convergence of one observable implies convergence to SLE.

21. **Loop representation** Section 5.3.2: percolation can be reformulated in terms of a loop model.

22. **Discrete holomorphicity** Proof of Lemma 5.1: discrete version of the Cauchy-Riemann equations.

23. **Discrete Cauchy Theorem** Proof of Corollary 5.2: discrete Cauchy-Riemann equations implies a discrete version of the Cauchy theorem.

24. **Scaling limit** Section 5.3.4: proof that the crossing probability converges to Cardy's formula.