

C4.6: Fixed point methods for Nonlinear PDEs.

Course diary.

Week 0:

- To warm-up and be ready to start with the lectures, do the Exercise sheet 0.

Week 1: Introduction and Banach's fixed point theorem

- See Lecture 1 and Lecture 2.
- Lecture notes: introduction and Chap. 1.1.

Week 2: some basics of calculus of variations and Brouwer's fixed point theorem

- See Lecture 3 (first and second parts) and Lecture 4
- Lecture notes: Chap. 1.2.
- After watching the 4 lectures and studying up to Chap 1.2 you should be able to solve the Problem sheet 1.

Week 3: Schauder's fixed point theorem and Peano's existence Theorem for ODEs

- See Lecture 5 (Schauder FPT, version 1 and 2) and Lecture 6 (compact operators, Schauder FPT version 3, application to Peano's Theorem on ODE with continuous vector field)
- Lecture notes: Chap. 1.3 and 1.4.
- At the end of week 3 you should be able to solve Questions 1,2,3 of the Problem Sheet 2

Week 4: Recap on Sobolev spaces, linear elliptic PDES, first applications to existence of solutions to non-linear elliptic PDEs

- See Lecture 7-first part (recap on Sobolev spaces), Lecture 7-second part (recap on linear elliptic PDEs), Lecture 8 (application to non-linear elliptic PDEs)
- Lecture notes: Chap. 2.1, 2.2 and 2.3. For more details about Lecture 7 (which is a recap) see the lecture notes of C4.3.
- At the end of week 4 you should be able to fully solve Problem sheet 2, and to answer the first 3 questions of Problem sheet 3.

Week 5: Weak maximum principle, Sub and Super solutions (Perron's method), Differentiation in Banach spaces and (begin) Monotone operators

- See Lecture 9 (weak maximum principle, sub and super solutions), Lecture 10-first part (differentiation in Banach spaces) and Lecture 10-second part (Monotone operators)
- Lecture notes: Chap. 2.4, 3.1 and 3.2.
- At the end of week 5 you should be able to fully solve Problem sheet 3.

Week 6: Variational inequalities

- See Lecture 11 (introduction to variational inequalities), Lecture 12 first part (statements of main results about variational inequalities and proof of Prop 3.4) and Lecture 12 second part (proof of Proposition 3.7 and corollary 3.8). Notice that the proof of Theorem 3.5 will be done in Lecture 13-first part.

- Lecture notes: Chap. 3.3 and 3.4.
- At the end of week 6 you should be able to fully solve Problem sheet 4.

Week 7: Applications to quasi-linear problems in $W^{1,p}_0$

- See Lecture 13-first part (conclusion of Proof of Thm 3.5), Lecture 13-second part (proof of Theorem 4.1), and lecture 14 (p -Laplacian and growth conditions)
- Lecture notes: Chap. 3.4, 4.1, 4.2

Week 8: Second round of applications: elastic beam and stationary Navier-Stokes

- See Lecture 15 (elastic beam), Lecture 16 first and second parts (stationary Navier-Stokes)
- Lecture notes: Chap. 4.3 and 4.4