Guide to Videos

February 19, 2021

1 A guide to the videos

The later parts of this guide is subject to change.

Update: As expected, the plan for weeks 5-8 has changed very slightly.

1.1 Week 1

I suggest you watch the first five videos (C1.4 lecture style, Prerequisites and Books, Overview of the course, Classes and Relativization, Absoluteness) quickly and don't worry about understanding everything in them.

Then watch The Axioms and go back to the videos Classes and Relativization and Absoluteness. You should now understand them better.

You should now be able to to Q1-5 on the first problem sheet. If you have trouble with bits of Q2, come back to it later.

For extra practice do as much of Q8 as you need.

1.2 Week 2

Watch Ordinals - Basics (feel free to skip some of the technical proofs as long as you're confident you could do them) and Recursion on Ordinals (again, if you're comfortable with this you can skip its proof) and the Addendum to Ordinals.

Now do question 6 on Sheet 1 - this will really test your understanding of the slightly confusing new parts of this course.

Do more of question 8 and think about explicit non-transitive classes in which these concepts are not absolute.

After the class on Sheet 1 you should be confident about relativization and absoluteness (although your understanding will further improve during the remainder of the course).

1.3 Week 3

Watch The Cumulative von Neumann Hierarchy - Definition and Basic Properties, V is a model of ZF and Addendum to the study of V.

You should now be able to do Sheet 2.

Make sure you really understand how you typically prove that a class is a model of some set theory.

Go back and rewatch the videos Overview, Classes and Relativization and Absoluteness. Make sure you understand them and how they are used really well.

1.4 Week 4

Watch Satisfaction (somewhat optional), Goedels Constructible Universe, L satisfies **ZF** Part 1.

You should be able to attempt most questions on Sheet 3, but some might be still out of reach.

Watch Levy's Reflection Principle briefly without aiming for full understanding.

1.5 Week 5

Watch L satisfies **ZF** Part 2, rewatch Levy's Reflection principle in the light of this. Watch V=L and L satisfies Choice.

You should now be able complete Sheet 3.

1.6 Week 6

Watch General Recursion Theorem, Mostwoski Collapse, Cardinals and Koenig's Lemma.

You should be able to do Q3-5 on Sheet 4.

1.7 Week 7

Watch the remaining videos (Towards GCH and L models GCH).

You now should be able to complete Sheet 4.

If you can complete the non-starred part of Question 1, you have understood the main points of the course.

1.8 Week 8

Review the course material to appreciate how it all hangs together.

Optionally, lightly (i.e. without aiming to fully understand everything) read section I.14-16 and section II of 'Set Theory' by Kunen to get a much better understanding of what this course is about.

Celebrate that you have finally understood the consistency of GCH.

2 The Videos and their intended Learning Outcomes

2.1 Introduction

The videos 'C1.4 lecture style', 'Prerequisites and Books' contain no relevant mathematics.

2.2 Overview of the course

Learning outcome:

- know the aim of the course;
- understand the distinction between the meta-theory and the theory and the distinction between meta-theorems and theorems.

2.3 Classes and Relativization

Learning outcome:

- understand what a class is and be able;
- understand how to relativize a formula to a class;
- understand the meaning of $A \models \phi$ for a class A;
- understand how relativization can be used to give relative consistency proofs;

2.4 Absoluteness

Learning outcome:

- understand the definition of absoluteness;

2.5 The Axioms

Learning outcome:

- be familiar with the axioms of **ZF** and their relativizations to a class;
- understand how various defined notions relativize;
- transitivity of classes;
- Δ_0 formulae and their absoluteness for transitive classes;

2.6 Ordinals - Basics

Learning outcome:

- basic fact about ordinals;
- the class of ordinals and its well-orderedness by $\in$;
- the structure of the class of ordinals, including the concepts of successor ordinals, limit ordinals, ω;
- induction on ordinals;

2.7 Recursion on Ordinals

Learning outcome:

- informal statement of the Recursion Theorem (on **On**);
- formal statement of the Recursion Theorem (on **On**);
- proof of the Recursion Theorem (on **On**);

2.8 Addendum to Ordinals

Learning outcome:

- variants of recursion on **On**;
- use of recursion on **On** to define ordinal addition and multiplication;

2.9 The Cumulative von Neumann Hierarchy - Definition and Basic Properties

Learning outcome:

- the definition of the class function V and the class V;
- the class function V is an increasing class function on **On** with $\alpha \in V_{\alpha+1}$; V is transitive;

2.10 V is a model of ZF

Learning outcome:

- understanding the general principle of showing that a subclass of U is a model of some set theory;
- being able to show that V is a model of **ZF**
- being able to show that if **ZF** is inconsistent then so is **ZF**$^{-}$;

2.11 Addendum to study of V

Learning outcome:

- relativizing classes, understanding $V^U = V^V$, $\mathbf{On}^V = \mathbf{On}^U$;
- the rank function $rk_V(x)$;
- axioms satisfied by V_α;

2.12 Introducing Gödel's Constructible Universe

Learning outcome:

- informally defining the definable subsets of x (in x) [with parameters in x];
- an idea on how to formalize the definable subsets via internalizing the $\models$ relation (formal details are non-examinable);
- some definable subsets of x ($\emptyset$, x, y, $\{y, z\}$, $\bigcup y$ for $y, z \in x$)
- definition of L and its basic properties;

2.13 Optional: Satisfaction

Learning outcome [non-examinable]:

- the details of internalizing the $\models$ relation;

2.14 L models ZF I

Learning outcome:

- showing that L models **Extensionality**, **Pairing**, **Union**, **Infinity**, **Foundation** (essentially the same as for V);
- showing that L models **Powerset** and the difference of this proof compared to $V \models$ **Powerset**;
- understanding the extra difficulty (compared to the situation in V) of showing $L \models$ **Separation**, **Replacement**;
- understanding which of the axioms may hold in L_γ;

2.15 Levy Reflection Principle

Learning outcome:

- the formal statement of the Levy Reflection Principle;
- Tarski-Vaught Criterion
- the proof of the Levy Relfection Principle

2.16 L models ZF II

Learning outcome:

- how to use LRP to show $L \models \textbf{Separation}, \textbf{Pairing}$
- the same for L_γ provided the $L|_\gamma$ satsifies the LRP;

2.17 V=L and L models V=L

Learning outcome:

- understanding the axiom $V = L$;
- absoluteness of $x^{<\omega}$
- absoluteness of the class function L (the absoluteness of $\models$ may be assumed)
- $L \models V = L$ and $L_\gamma \models V = L$

2.18 L models Choice

Learning outcome:

- understanding the idea behind $L \models \textbf{Choice}$;
- how to obtain a well-order of $x^{<\omega}$ from a well-order of x;

2.19 General Recursion Theorem

Learning outcome:

- properties of relations: set-like, well-founded
- definition of the 'transitive closure' $R^\star$ of a relation R;
- if R is set-like and well-founded then so is $R^\star$;
- using $R^\star$ to give recursive definitions along set-like, well-founded relations;

2.20 Mostowski Collapse

Learning outcome:

- informal definition of the Mostowski collapsing function along a set-like, well-founded relation;
- $mos_R[A]$ is transitive;
- if the relation R is extensional, then mos_R is injective;
- if $R =\in$ and x is transitive then $mos_\in$ is the identity on x;

2.21 Cardinals and König's Lemma

Learning outcome:

- definition of cardinals;
- statement of Cantor-Schröder-Bernstein Theorem;
- cardinal addition, multiplication and exponentiation and their basic properties
- König's inequality: if $\kappa_i < \lambda_i$ then $\sum \kappa_i < \prod \lambda_i$;
- definition of cofinality
- basic properties of cofinality
- König's Lemma: $\kappa < cf(2^\kappa)$;

2.22 Towards GCH

Learning outcome:

- for infinite α, $|L_\alpha| = |\alpha|$;
- LRP for $L|_\kappa$ and $L_\kappa \models$ **ZFC-P+V=L** for κ regular uncountable;
- Downward Löwenheim Skolem theorem (only the statement is examinable);
- Condensation Lemma: if a transitive set M models **ZFC-P+V=L** then $M = L_\gamma$ for $\gamma = M \cap \mathbf{On}$;

2.23 L models GCH

Learning outcome:

- definition of $TC(x)$ and H_κ;
- **ZFC+V=L** implies $H_\kappa = L_\kappa$;
- $L \models$ **GCH**