STRING THEORY I

Lecture 1

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- Course largely based on C. Beem 's lecture notes from 2020 (2020 lecture notes → MI course management pages → archive courses tab!)

  I will upload the slides of the recorded lectures
- Problem sheets → course web pages (PS1 ✓, PS2 early W2)
- Recommended book list → String Theory 1 webpages
- Teams room for notifications & QA session

  You can put questions you have during a lecture in this room.

$\boxed{\text{Chapter 1}}$ Introduction

$\boxed{1.1}$ What is string theory?

(a few words about what string theory is and some motivation)

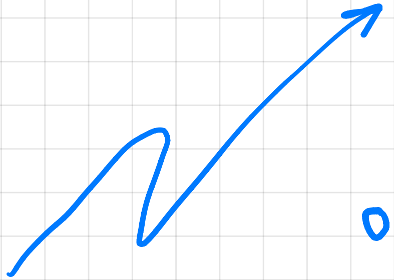
$\boxed{1.2}$ Historical introduction

1.1 What is string theory?

The starting point of string theory is that it is a theory of fundamental quantum mechanical strings

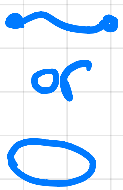
In QFT: fundamental particles point like objects

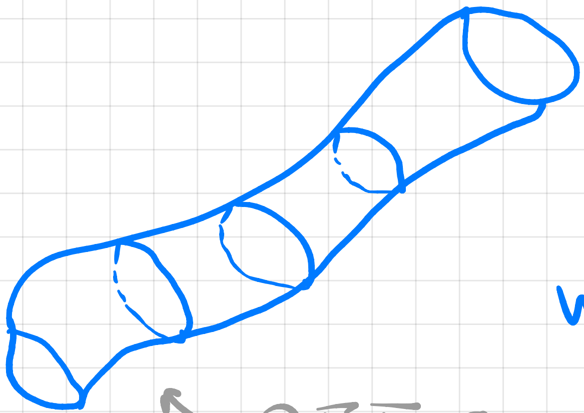
point like particles 



0+1 world line

Instead

strings 



1+1 dim world sheet

QFT on world sheet

$\underbrace{\text{Perturbative string theory}}_{\text{best developed formulation}}$ is $\underbrace{\text{first quantized}}_{\text{2nd quantized theory would be string field theory framework}}$ S-matrix theory

non perturbative insight
post early 1990's

not well understood

May see some in STII

Key features

- consistently incorporates gravity
  $\Rightarrow$ a theory of quantum gravity
- "unique" theory

- incorporates many other interesting & phenomenologically relevant ingredients from QFT & particle physics
  - non-Abelian gauge symmetries with chiral matter
  - spacetime supersymmetry
  - "unification"

This course:

- bosonic string theory
  - ↳ missing some of the features mentioned above and suffers from serious defects & inconsistencies
  - ↳ however illustrates key ideas & techniques

ST II: learn superstring theory which has been considered as a candidate to some day describe our world.

1.2 Historical motivation

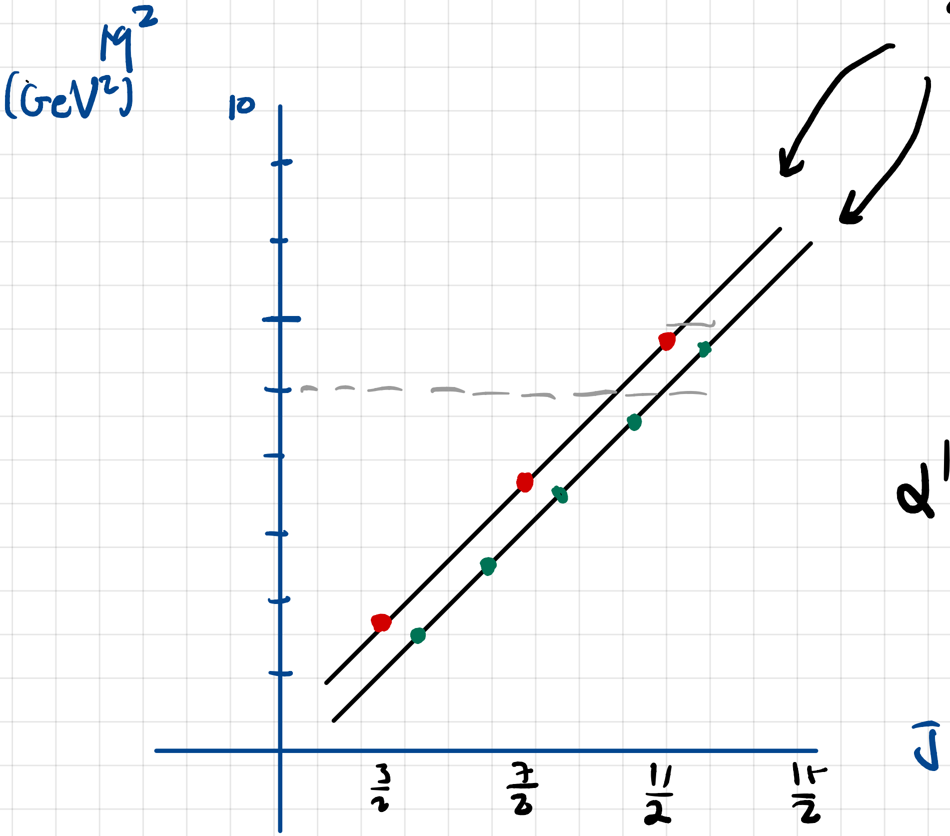
String theory appeared first in the 60's as a theory of strong interactions (the dual resonance models)

50's & 60's: QFT was unsatisfactory as a theory of strong interactions because

1. the experimental observation of the large proliferation of the number of hadrons with large masses & spins

2. UV (loop) divergences in the computation of perturbative scattering amplitudes particularly for high spin particles

① One of the most important observations was that hadronic resonances appeared in families



Chew-Frautschi plot

Regge trajectories

$$J = \alpha(0) + \alpha' M^2$$

$\alpha(0)$ Regge intercept

$\alpha' \simeq 1\ \mathrm{GeV}^{-2}$ slope

lightest scalar (J=0): $M^2 = -\frac{\alpha(0)}{\alpha'}$

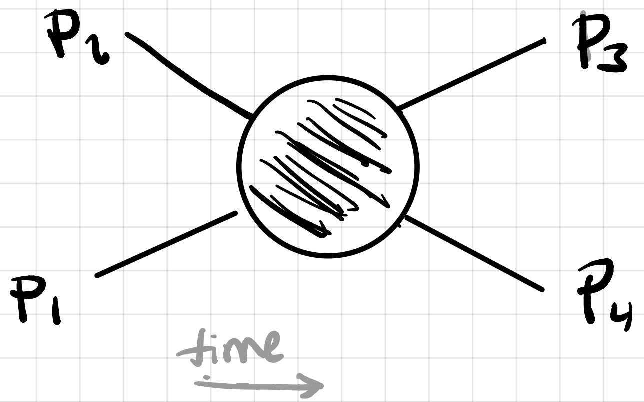
fixed $M^2 \to$ max spin $J(M^2)$

- There where doubts that all these particles where fundamental.
- renormalizable known QFTs: $J = 0, \frac{1}{2}, 1$

Instead people worked within the context of the S-matrix program: construct the S-matrix using a number of general principles like unitarity & analyticity, together with experimental data

② UV difficulties for high spin particles

Consider (for example) the elastic scattering



signature $(-1,+1,+1,\dots)$ so $M^2 = -p^2$

λ_i = flavour quantum numbers $\quad i = 1,2,3,4$

Comment: term in scattering amplitude $\propto \operatorname{tr}(\lambda_1 \lambda_2 \lambda_3 \lambda_4)$

cyclic symmetry $1234 \to 2341$

Mandelstam variables

$$s = -(p_1+p_2)^2 \quad (>0 \text{ for physical elastic scattering})$$
$$t = -(p_1+p_4)^2 \quad (<0 \quad \text{''} \quad)$$
$$u = -(p_1+p_3)^2 \quad (>0 \quad \text{''} \quad)$$

with $s+t+u = \sum m_i^2$

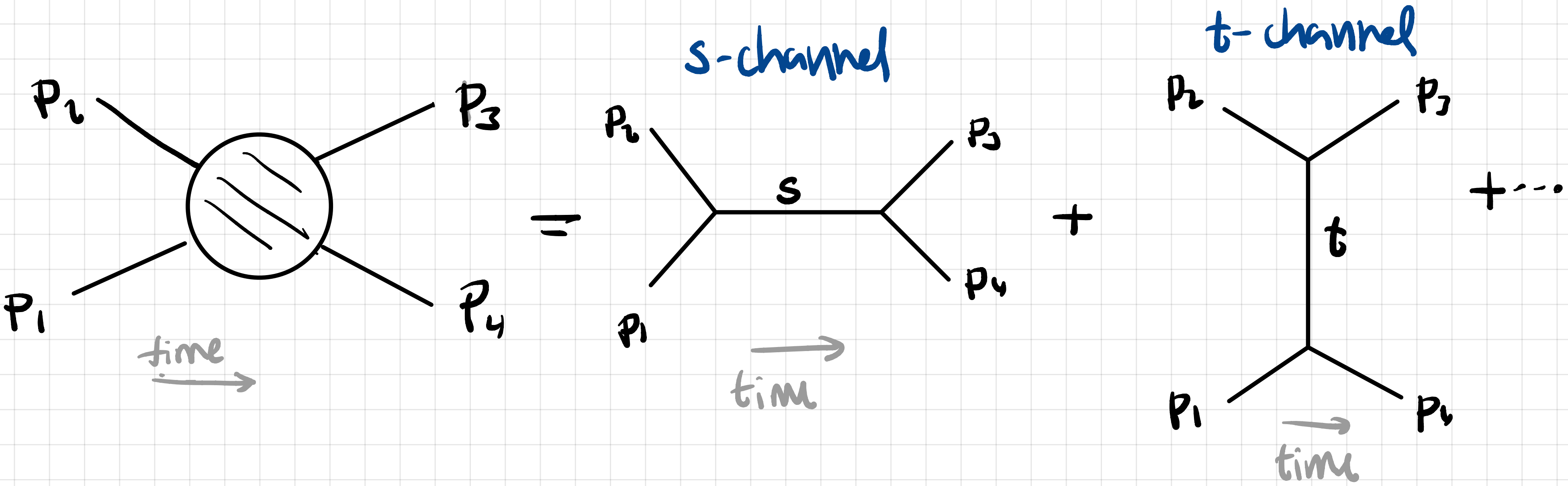
Amplitude $A(s,t)$ depends only on two of Mandelstam variables

Also: as cyclic symmetry of $\operatorname{tr}(\lambda_1 \lambda_2 \lambda_3 \lambda_4)$

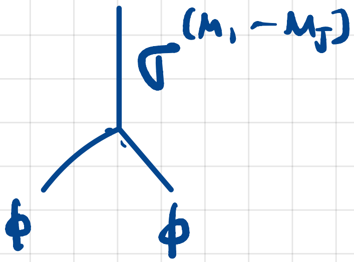
Bose statistics $\Longrightarrow$ $A(s,t)$ must have a cyclic symmetry $p_1 p_2 p_3 p_4 \to p_2 p_3 p_4 p_1$

$\therefore$ $A(s,t)$ invariant under $s \leftrightarrow t$

leading contributions



t-channel exchange of a spin J particle σ



cubic coupling

$$\sim g_J \left(\phi^* \overleftrightarrow{\partial}_{\mu_1} \cdots \overleftrightarrow{\partial}_{\mu_J} \phi\right) \sigma^{\mu_1 - \mu_J}$$

$$A(s,t) \sim -\frac{g_J^2 (-s)^J}{t - M_J^2}$$ for t fixed, s +ve, large

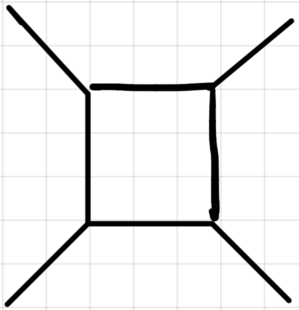
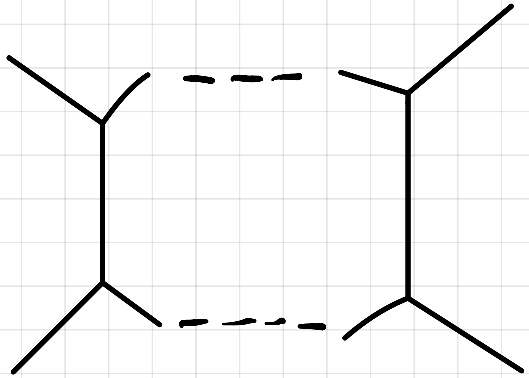
Remarks: • if $J=0$ cubic coupling $\sim g_0 (\phi^* \phi)\sigma$

$$\Rightarrow \quad A(s,t) \sim -\frac{g_0^2}{t - M_0^2} \longrightarrow 0 \text{ as } t \to \infty$$

• $J > 0$: A is more and more divergent for larger J (A grows too rapidly at high s)

This UV behaviour is not what was observed in for example pion scattering!

Even worse in loop diagrams:

 $\sim$  $\sim \displaystyle\int \frac{d^D p}{(2\pi)^D} \frac{p^{4J}}{(p^2)^4}$

4 dims:

- $J=0$ safe for scalars
- $J=1$ log divergence potentially renormalizable
- $J>1$ badly divergent ie not renormalizable

If there are particles of various spins exchanged in t-channel

$$A(s,t) \sim -\sum_{J=0}^{J_{max}} \frac{g_J\,(-s)^J}{t - M_J^2} \underset{\substack{s \to \infty \\ t \text{ fixed}}}{\sim} s^{J_{max}}$$

high energy behavior dominated by particle with J_{max}

Bad:
- very different from observations
- no s-channel poles

The story might be different if there are infinitely many exchange diagrams:

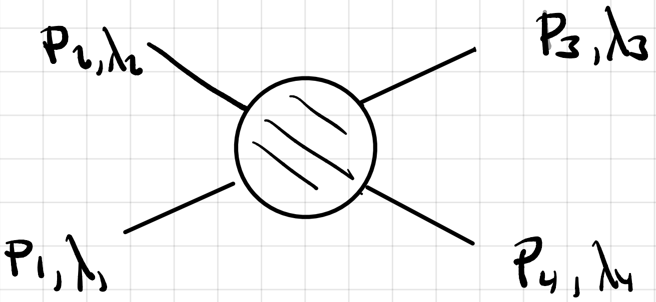
$$A_t(s,t) = -\sum_{J=0}^{\infty} \frac{g_J (-s)^J}{t - M_J^2} \sim ?$$

GSW eg e^{-x} smaller for $x \to \infty$ than individual terms in $e^{-x} = \sum_{n=0}^{\infty} \frac{1}{n!} (-x)^n$

As the sum is infinite perhaps s-channel poles arise automatically?

$\boxed{\text{Dolen-Horn-Schmid duality}}$ (1968)

In QFT: need $\boxed{\text{both}}$ s & t channel contributions



$$\sim \; A(s,t)\, \varpi(\lambda_1, \lambda_2, \lambda_3, \lambda_4) + \ldots$$

together with $\quad A(s,t) = A_t(s,t) + A_s(s,t)$ $\quad$ both s & t channel contributions

We have $\quad A_t(s,t) = -\sum_J \frac{g_J^2\,(-s)^J}{t-M_J^2}$, $A_s(s,t) = \sum_J \frac{g_J^2\,(-t)^J}{s-M_J^2}$

due to the $s \leftrightarrow t$ symmetry

Infinite sums: $A_t(s,t)$ might have divergences at some finite values of s

$\Rightarrow$ poles in s-channel

$\Rightarrow$ not obvious that $A_s(s,t)$ needs to be added separately

Instead DHS proposed

Dual model

$$A(s,t) = A_t(s,t) = A_s(s,t)$$

dual description of same physics

In 1968 Veneziano: using the channel duality postulated

$$A(s,t) = \frac{\Gamma(-\alpha(s))\,\Gamma(-\alpha(t))}{\Gamma(-\alpha(s)-\alpha(t))} = B(-\alpha(s), -\alpha(t))$$

- $\alpha(s)$ Regge trajectory

  Veneziano postulated $\alpha(s) = \alpha(0) + \alpha' s$

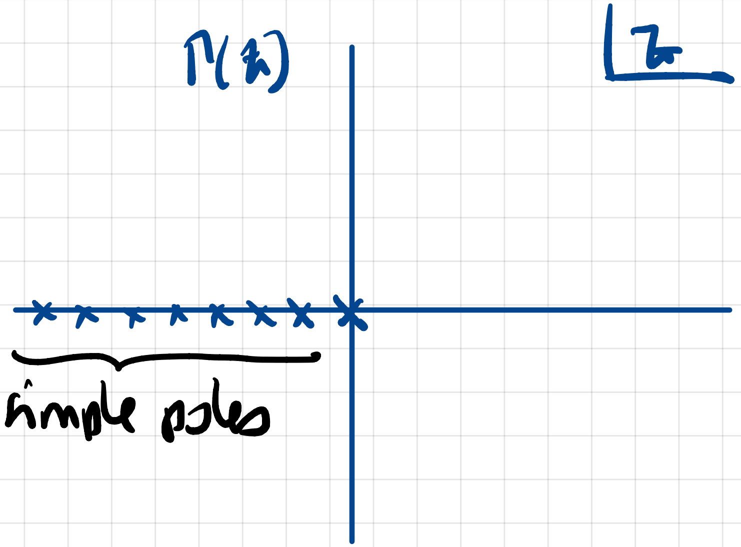
- $\Gamma(z) = \int_0^\infty t^{z-1} e^{-t}\, dt, \quad (\mathrm{Re}(z) > 0)$ Euler Γ-function

- $B(z, w) = \dfrac{\Gamma(z)\,\Gamma(w)}{\Gamma(z+w)}$ Euler beta-function

Consider the singularities:

$$A(s,t) = \frac{\Gamma(-\alpha(s))\,\Gamma(-\alpha(t))}{\Gamma(-\alpha(s)-\alpha(t))} = B(-\alpha(s), -\alpha(t))$$

$$\Gamma(z) = \int_0^\infty t^{z-1} e^{-t}\, dt, \quad \mathrm{Re}(z) > 0$$



- $\Gamma(z+1) = z\,\Gamma(z)$
- $\Gamma(z)$ has no zeros

Behaviour near singularities: near $z = -n$, n non-negative integer

$$\Gamma(z) = \frac{\Gamma(z+n+1)}{z(z+1)\cdots(z+n-1)(z+n)} \sim \frac{(-1)^n}{n!}\,\frac{1}{z+n}$$

$$A(s,t) = \frac{\Gamma(-\alpha(s))\,\Gamma(-\alpha(t))}{\Gamma(-\alpha(s)-\alpha(t))} = B(-\alpha(s), -\alpha(t))$$

$$B(z,w) = \frac{\Gamma(z)\,\Gamma(w)}{\Gamma(z+w)}$$

has simple poles only (at $z=-n$ or $w=-m$ where n,m are +ve integers) so far we have:

$$A(s,t) = \frac{\Gamma(-\alpha(s))\,\Gamma(-\alpha(t))}{\Gamma(-\alpha(s)-\alpha(t))} = B(-\alpha(s), \alpha(s))$$

with $\alpha(t) = \alpha(0) + \alpha' t$

t-channel poles: $t = \frac{1}{\alpha'}(-\alpha(0)+n)$ $\quad n = 0, 1, 2, \ldots$

s-channel poles: $s = \frac{1}{\alpha'}(-\alpha(0)+n)$ $\quad n = 0, 1, 2, \ldots$

Does $A(s,t)$ satisfy the DHS duality? Yes

$$A(s,t) = \frac{\Gamma(-\alpha(s))\,\Gamma(-\alpha(t))}{\Gamma(-\alpha(s)-\alpha(t))} = B(-\alpha(s), -\alpha(t))$$

consider $\quad B(z,w) = \dfrac{\Gamma(z)\Gamma(w)}{\Gamma(z+w)}$

Then near a singularity at $w=-n$

$$B(z,w) \sim \frac{1}{z+n} \frac{(-1)^n}{n!} (w-1)(w-2)\cdots(w-n)$$

Claim: $\quad B(z,w) = \displaystyle\sum_{n=0}^{\infty} \frac{1}{z+n} \frac{(-1)^n}{n!} (w-1)(w-2)\cdots(w-n)$

as sum reproduces all the singularities of B but it precisely B!

[from the fact that $\quad B(z,w) = \displaystyle\int_0^1 dx\, x^{z-1} (1-x)^{w-1}$]
(see GSW)

Hence:

$$A(s,t) = -\sum_{n=0}^{\infty} \frac{1}{n!} (\alpha(s)+1)(\alpha(s)+2)\cdots(\alpha(s)+n)\, \frac{1}{\alpha(t)-n} \qquad (*)$$

& DHS duality means

$$* \quad \underline{A(s,t) = A(t,s)} = -\sum_{n=0}^{\infty} \frac{1}{n!} (\alpha(t)+1)(\alpha(t)+2)\cdots(\alpha(t)+n)\, \frac{1}{\alpha(s)-n}$$

For the t-channel exchange

$$A(s,t) = -\sum_{n=0}^{\infty} \frac{1}{n!} \underbrace{(\alpha(s)+1)(\alpha(s)+2)\cdots(\alpha(s)+n)} \frac{1}{\alpha(t)-n}$$

$$(\alpha(t) = \alpha(0) + \alpha' t)$$

- singularities are simple poles $\alpha(t)=n$ $\rightsquigarrow$ t channel exchange of particles with mass $M^2 = \frac{1}{\alpha'}(-\alpha(0)+n)$
- residue at the pole $\alpha(t)=n$: n-th order polynomial in s
  $\rightsquigarrow$ particles of mass $M^2 = \frac{1}{\alpha'}(-\alpha(0)+n)$
  & max spin $J=n$

High energy behaviour of $A(s,t)$: does this solve the UV problem?

$A(s,t)$ in the Regge limit $(s \gg 1,\ t<0$ fixed$)$

Using Stirling formula $\quad \Gamma(z) \sim \sqrt{2\pi}\, z^{z-1/2}\, e^{-z}$

$$A(s,t) = \frac{\Gamma(-\alpha(s))\,\Gamma(-\alpha(t))}{\Gamma(-\alpha(s)-\alpha(t))} \sim \Gamma(-\alpha(t))\,(-\alpha(s))^{\alpha(t)} \sim C(t)\, s^{\alpha(t)},$$

$\alpha(t) = \alpha(0) + \alpha' t$

suppressed like $s^{-\alpha'|t|}$

Compare with $A_J(s,t) = \dfrac{g^2(-s)^J}{t-M^2} \sim s^J$, $\quad J = \alpha(t)$ (large s, fixed t)

Hence: infinite number of particle exchanges in t-channel $\leftrightsquigarrow$ like a single particle with negative spin $J = \alpha(t)$

"Regge-on"

Consider now the Veneziano amplitude at high energies $s \gg 1$ for a fixed scattering angle (so t/s fixed).

$$A(s,t) \sim \left[\underbrace{F(\theta_s)}_{} \right]^{-\alpha(s)}$$

└ function of the scattering angle θ_s

so falls off exponentially fast with s !

✗ Veneziano model

- softer UV behaviour than any QFT
- incorporates particles of high spin without UV divergences

Consider now the Veneziano amplitude at high energies for a fixed scattering angle and $s \gg 1$ with $\frac{t}{s}$ fixed

$$A(s,t) \sim [\underbrace{F(\theta_s)}]^{-\alpha(s)}$$

└ function of the scattering angle θ_s

falls off exponentially fast with s !

* Veneziano model has softer UV behaviour than any QFT and incorporated particles of high spin without UV divergences

Virasoro (69), Shapiro (70) model

$$A(s,t,u) = \frac{\Gamma(-\alpha_c(s))\,\Gamma(-\alpha_c(t))\,\Gamma(-\alpha_c(u))}{\Gamma(-\alpha_c(s)-\alpha_c(t))\,\Gamma(-\alpha_c(t)-\alpha_c(u))\,\Gamma(-\alpha_c(u)-\alpha_c(s))}$$

$$\alpha_c(x) = \alpha(0) + \frac{1}{4}\alpha' x \qquad \alpha'(s+t+u) = -16\,\alpha(0)$$

s-channel poles $\quad s = 4(-\alpha(0)+n), \quad n = 0,1,2,\ldots$

t-channel poles $\quad t =$ "

u-channel poles $\quad u =$ "

duality between $\underline{\text{all 3 channels}}$

max spin @ $m^2 = 4(-\alpha(0)+n)$ is $J = 2n$

Various generalizations

Veneziano model

Virasoro - Shapiro model 1969-1970

⋮

These included

- external particles other than the lightest scalar
- loop amplitudes

⋮

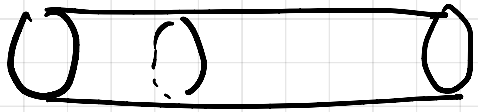
A theory of strings

1970 Nambu + Nielsen + Susskind realized that

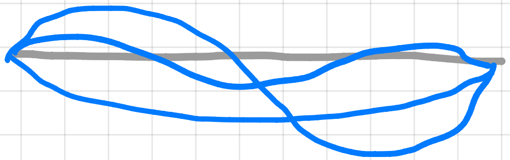
↳ Veneziano & Virasoro + Shapiro models (and their generalizations can be (re) interpreted in terms of a theory where elementary particles are replaced by vibrating relativistic strings

open strings

closed string



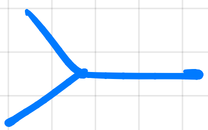
spectrum:



quantized fluctuations of a relativistic string

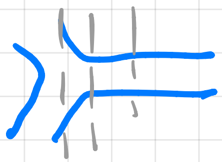
interactions:

For example



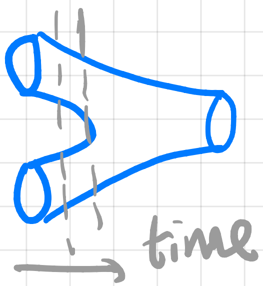
3-point field theoretic particle vertex

$\rightsquigarrow$



3 open-string vertex

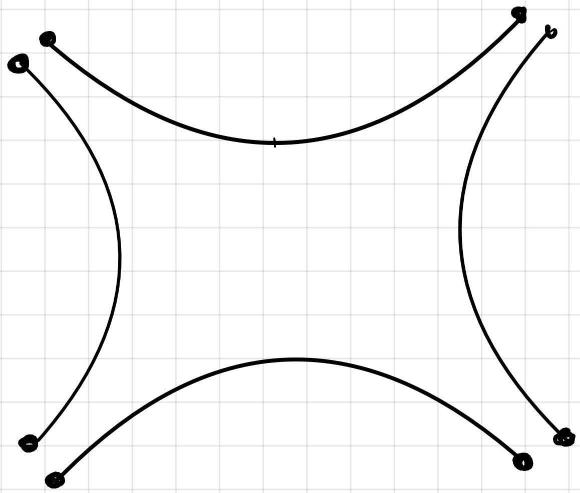
or



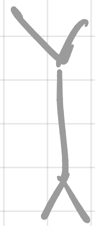
3 closed-string vertex

This gives a heuristic justification for the various good properties

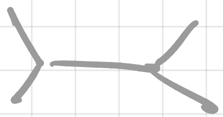
duality:



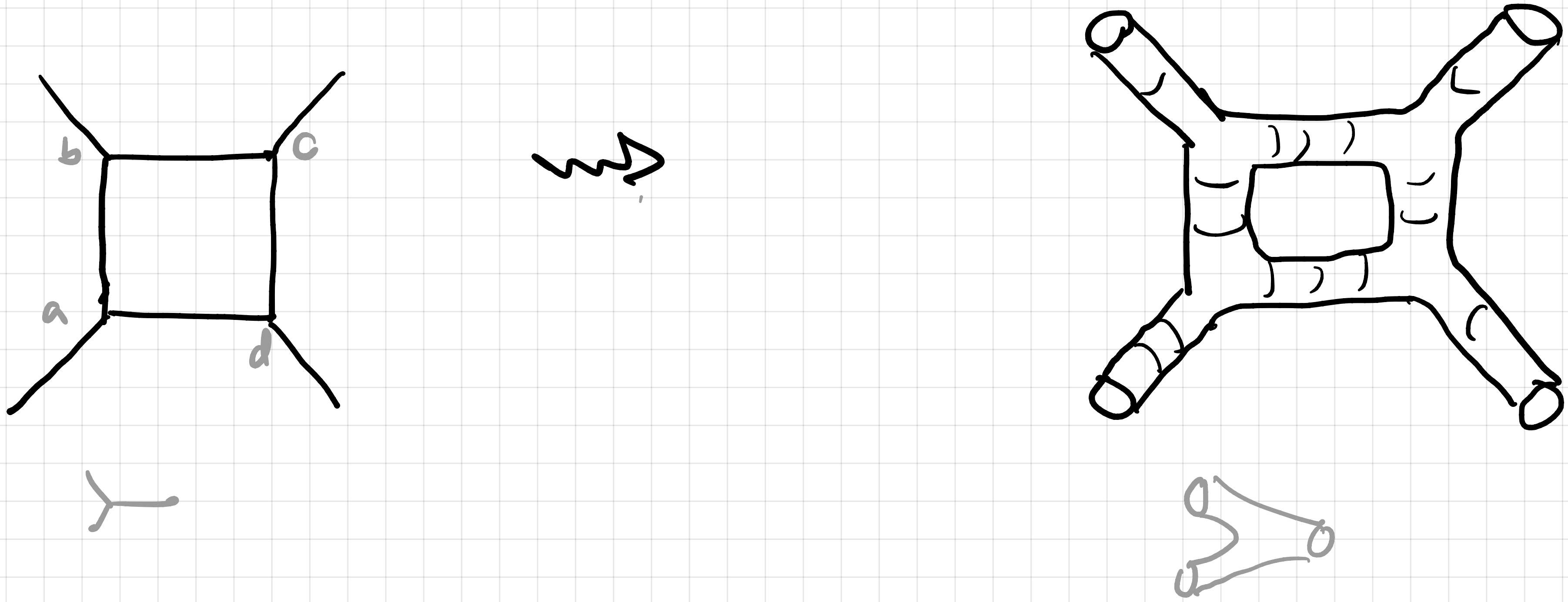
t-channel poles



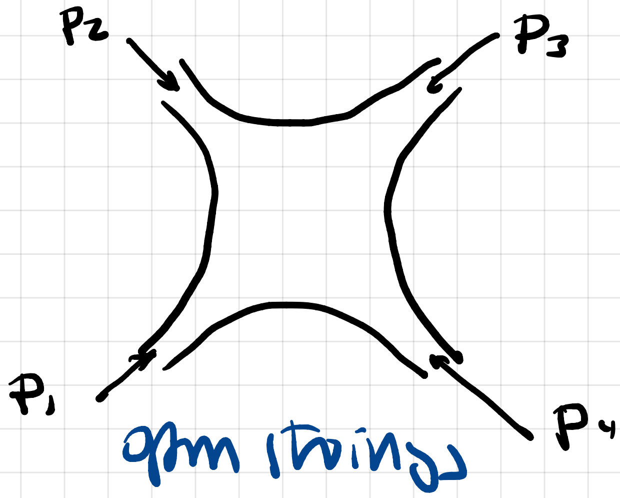
s-channel poles



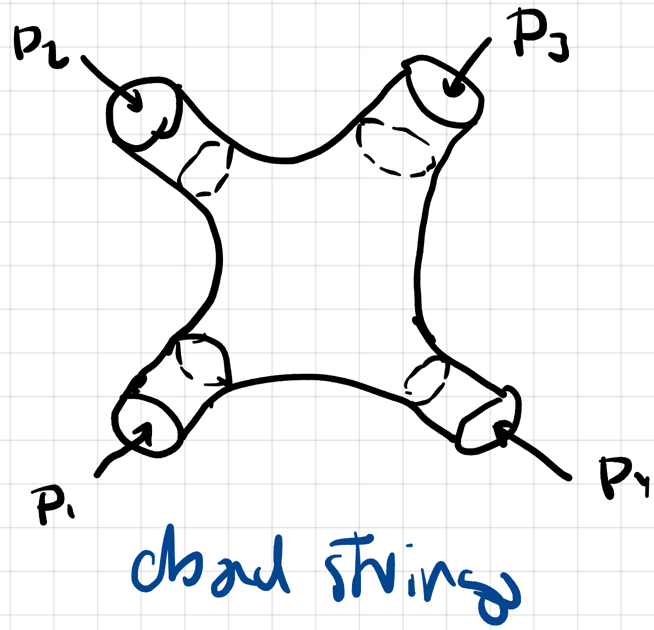
high energy behaviour :



Veneziano $\qquad A(s,t):$



Virasoro-Shapiro $\qquad A(s,t,u):$



Problems

* predicted massless particles
  - spin 1 Veneziano model
  - spin 2 Virasoro-Shapiro (closed strings)

* required more space-time dimensions
  - Veneziano model of bosons $\Rightarrow$ $D = 26$
  - Ramond-Neveau-Schwartz model of bosons & fermions (70) $\Longrightarrow$ $D=10$

* unitarity of the amplitude not manifest

Dual resonance models abandoned in the 70's in favour of QCD

QCD solves UV differently

Gravity and the string scale

Veneziano & Virasoro-Shapiro amplitudes depend on two parameters: $\alpha(0)$ & α' dimensionful
$[\alpha'] = [M]^{-2}$

original idea in dual resonance models

$\alpha' \sim 1(\mathrm{GeV})^{-2}$ (nuclear physics energy scale)

$-\dfrac{\alpha(0)}{\alpha'} =$ mass^2 of lightest scalar $= m_\pi^2$

However: the fact that all closed strings contain a massless spin 2 particle suggested the idea that perhaps the theory strings was a theory of gravity as long as

$$\alpha' \sim (10^{19}\ \mathrm{GeV})^{-2}.$$

J Scherk & J Schwarz 1974

"Dual models for non-hadrons"

reinterpreted the theory of strings as a unified theory of (quantum) gravity (& other fundamental interactions)

$$S_{\text{zero-slope limit}} = -\int d^4x \sqrt{g} \left(\frac{1}{16\pi G_N} R + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \, \partial_\nu \phi \right)$$

with $\sqrt{8\pi G_N} = g_{str} \sqrt{\alpha'}$

zero slope limit ($\alpha' E^2 \ll 1$) of the VS model gives the tree level Einstein gravity + scalar theory.

<u>Next</u>: Chapter $\boxed{2}$ $\rightarrow$ Classical theory of strings