STRING THEORY I

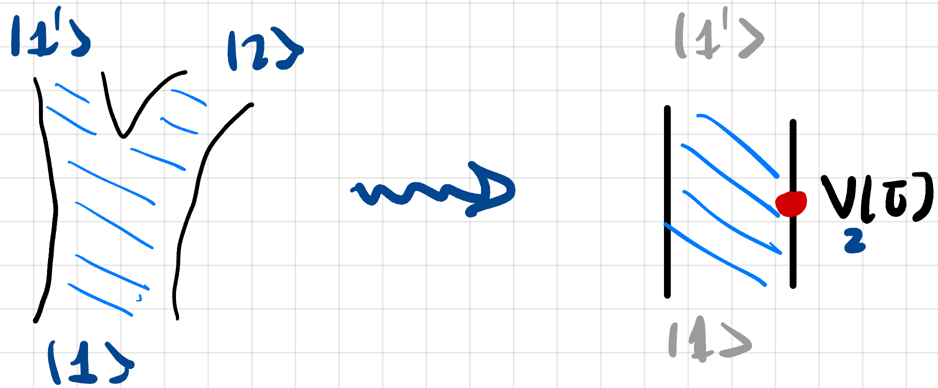
Lecture 9

4 Interactions

Last lecture:

We introduced vertex operators to describe the emission or absortion of physical string states from the perspective of a fixed string world sheet

For example:



branching of an open string in which an open string in state |1> branches into |1'> & |2>

process in which state |2> is emitted (or (absorbed) at the end point $\sigma=0$ of a fixed string with the vertex $V_2(\tau)$ describing the emission of state |2> from $\sigma=0$ at τ

As a consequence we have a map

state $\longrightarrow$ operator

in which $|\phi\rangle_{opm} \longmapsto V_\phi(\tau)$ conformal primary of dimension $h=1$

For example: noting that $:e^{iK\cdot X}:$ has $h=\frac{1}{2}(K\cdot K)$

level 0 tachyon $|0;K\rangle \longmapsto :e^{iK\cdot X(\tau)}:$

$K\cdot K=2$

level 1 photon $|\zeta;K\rangle \longmapsto :(\zeta\cdot\dot{X}(\tau))e^{iK\cdot X(\tau)}:$ $\quad \zeta\cdot K=0$

$K\cdot K=0$

level 2 massive spin 2 $|\gamma;K\rangle \longmapsto :\gamma_{\mu\nu}\dot{X}^\mu(\tau)\dot{X}^\nu(\tau)e^{iK\cdot X(\tau)}:$ $\quad \gamma^\mu{}_\mu=0$, $\quad K\cdot\gamma=0$

$K\cdot K=-2$

Gauge invariance:

level 1: for the photon vertex operator with $\xi \sim k$

(longitudinal mode)

$$\int d\tau \; k \cdot \dot{x} \, e^{ik\cdot x} = -i \int d\tau \; \partial_\tau \left(e^{ik\cdot x} \right) = 0$$ up to boundary terms

so longitudinal mode decouples.

It is not a coincidence that there is a similarity between states & vertex operators.

Note that if $V(\tau)$ is a vertex operator describing the emission (or absorption) from the end point $\sigma=0$ at some time τ, then

$$V(\tau) = e^{i\tau L_0} V(0) e^{-i\tau L_0}$$

because the Hamiltonian is $L_0 - a = L_0 - 1$.

For the tachyon:

$$V_T(K,\tau) = :e^{iK\cdot X(\tau)}: = e^{i\tau L_0} \underbrace{:e^{iK\cdot X(0)}:}_{V_T(K,0)} e^{-i\tau L_0}$$

Consider the action of this operator on the zero momentum string vacuum state $|0;0\rangle$

$L_0 = \frac{1}{2}p^2 + N$

$$V_T(K,\tau)\,|0;0\rangle = e^{i\tau L_0} :e^{iK\cdot X(0)}: e^{-i\tau L_0}\,|0;0\rangle$$

$$= e^{i\tau L_0} :e^{iK\cdot X(0)}: |0;0\rangle \qquad \text{as } L_0|0;0\rangle = 0$$

$$= e^{i\tau L_0}\, e^{K\cdot \sum_{n=1}^{\infty} \frac{\alpha_{-m}}{m}} :e^{iK\cdot x(0)}: e^{-K\sum_{n=1}^{\infty}\frac{\alpha_n}{n}}\,|0;0\rangle$$

$$= e^{i\tau L_0}\, e^{K\cdot\sum_{m\geq 1}\frac{1}{m}\alpha_{-m}}\, \underbrace{:e^{iK\cdot x(0)}:|0;0\rangle}_{|0;K\rangle} \qquad \text{as } \alpha_n|0;0\rangle = 0 \;\; \forall\, n\geq 1$$

$$V_T(K,\tau)\,|0;0\rangle = e^{i\tau L_0}\, e^{K\cdot\sum_{m\geq 1}\frac{1}{m}\alpha_{-m}}\,|0;K\rangle$$

Define $z = e^{i\tau} = e^{t}$, where $\tau = -it$ (so t is Euclidean world sheet time). Then

$L_0 = \frac{1}{2}p^2 + N$

$$V_T(K,\tau)\,|0;0\rangle = z^{L_0}\, e^{K\cdot\sum_{m\geq 1}\frac{1}{m}\alpha_{-m}}\,|0;K\rangle$$

$$= z^{1+N}\left(1 + K\cdot\alpha_{-1} + \frac{1}{2}\left((K\cdot\alpha_{-2}) + (K\cdot\alpha_{-1})^2\right) + \cdots\right)|0;K\rangle$$

$$z^{L_0}(\cdots)|0;K\rangle = e^{\frac{1}{2}K^2 + N}(\cdots)|0;K\rangle, \quad \frac{1}{2}K^2 = 1$$

$$V_T(K,\tau)|0;0\rangle = z\left(1 + z\,K\cdot\alpha_{-1} + \frac{1}{2}z^2\left((K\cdot\alpha_{-2}) + (K\cdot\alpha_{-1})^2\right) + \mathcal{O}(z^3)\right)|0;K\rangle$$

Then we can recover the state $|0;K\rangle$ from V_T by taking

$$|0;K\rangle = \lim_{z\to 0}\frac{1}{z}V_T(K;\tau)|0;0\rangle = \lim_{t\to -\infty} e^{-t}\, V_T(K;it)\,|0;K\rangle$$

For the photon

$$V_\varsigma(K,\tau) = \varsigma\cdot\dot{X}\, e^{iK\cdot X} \quad |0;0\rangle = z^N \sum_{m>0} (\varsigma\cdot\alpha_{-m})\, e^{\sum_{n>0}\frac{1}{n}K\cdot\alpha_{-n}} \quad |0;K\rangle$$

$$= \left(z\,(\varsigma\cdot\alpha_{-1}) + z^2\big((\varsigma\cdot\alpha_{-2}) + (\varsigma\cdot\alpha_{-1})(K\cdot\alpha_{-1})\big) + \cdots \right) |0;K\rangle$$

We recover the state $|\varsigma;K\rangle$ from

$$\lim_{z\to 0} \frac{1}{z} V_\varsigma(K;it)|0;0\rangle$$

Continuing like this, a pattern becomes clear: for a physical state $|\psi\rangle$ with vertex operator $V_\psi(\tau)$ we have

$$|\psi\rangle = \lim_{z\to 0} \frac{1}{z} V_\psi(it)\,|0;0\rangle$$

An analogous statement holds for "out" states. For example

$$\langle 0;0|\, V_T(K;it) = \langle 0;K|\, e^{\sum_{n>0} \frac{1}{n} \alpha_n \cdot K}\, z^{-L_0}$$

$$= \frac{1}{z} \langle 0;K| \left(1 + z^{-1}(\alpha_1 \cdot K) + \frac{1}{2} z^{-2}\left(\alpha_2 \cdot K + (\alpha_1 \cdot K)^2\right) + \dots \right)$$

$$\langle 0;K| = \lim_{z\to 0} z\, \langle 0;0|\, V_T(K,\tau) = \lim_{t\to\infty} e^{t} \langle 0;0|\, V_T(K;it)$$

$$\boxed{\begin{aligned} |\psi\rangle &= \lim_{z\to 0} \frac{1}{z} V_\psi(it)\,|0;0\rangle \qquad &\text{in} \\ \langle\psi| &= \lim_{z\to\infty} z\,\langle 0;0|\,V_\psi(it) \qquad &\text{out} \end{aligned}}$$

One can also use this to describe an incoming state $|\psi\rangle$ (or an outgoing state $\langle\psi|$) by acting with the vertex operator on the two momentum vacuum state in the infinite Euclidean past ie $t\to-\infty$ (resp Euclidean future, $t\to\infty$)

The picture presented here is part of a general treatment of the operator-state correspondence in conformal field theory where the general construction is

$$A(\tau) \longrightarrow |\psi_A\rangle = \lim_{t\to-\infty} z^{h_A} A(it)|\Omega\rangle$$

$$\longrightarrow \langle\psi_A| = \lim_{t\to\infty} z^{-h_A} \langle\Omega|A(it)$$

where $|\Omega\rangle$ is the vacuum state.

Closed strings : analogous to the open string.

Recall, a primary operator $A(\tau,\sigma)$ of dimension $(h,\tilde h)$ is an operator transforming under a conformal transformation as

$$A(\sigma_+,\sigma_-) \longrightarrow \tilde A(\tilde\sigma_+,\tilde\sigma_-) = \left(\frac{d\sigma_+}{d\tilde\sigma_+}\right)^{\tilde h}\left(\frac{d\sigma_-}{d\tilde\sigma_-}\right)^{h} A(\sigma_+,\sigma_-)$$

Infinitesimal transformations

$$\delta A(\sigma_+,\sigma_-) = -\partial_+(\tilde\epsilon A) - (\tilde h-1)(\partial_+\tilde\epsilon)A - \partial_-(\epsilon A) - (h-1)(\partial_-\epsilon)A$$

(total derivatives if $h=\tilde h=1$)

For $\epsilon = \frac{i}{2} e^{2im\sigma_+}$ $\left(\tilde\epsilon = \frac{i}{2} e^{2im\sigma_-}\right)$ these gives the action of L_m $(\tilde L_m)$

$$\begin{cases} [L_m, A(\sigma_\pm)] = \frac{1}{2} e^{2im\sigma_+}\left(-i\partial_+ + 2mh\right) A(\sigma_\pm) \\ [\tilde L_m, A(\sigma_\pm)] = \frac{1}{2} e^{2im\sigma_-}\left(-i\partial_- + 2m\tilde h\right) A(\sigma_\pm) \end{cases}$$

The vertex operator

$$:e^{ik\cdot X(\sigma_I)}:$$

is primary with $h=\tilde{h}=\frac{1}{8}k\cdot k$ so

$$V_T(k;\sigma_I)=:e^{ik\cdot X(\sigma_I)}: \quad , \quad k\cdot k=8$$

etc.

The map Vertex operators $\to$ states is given by

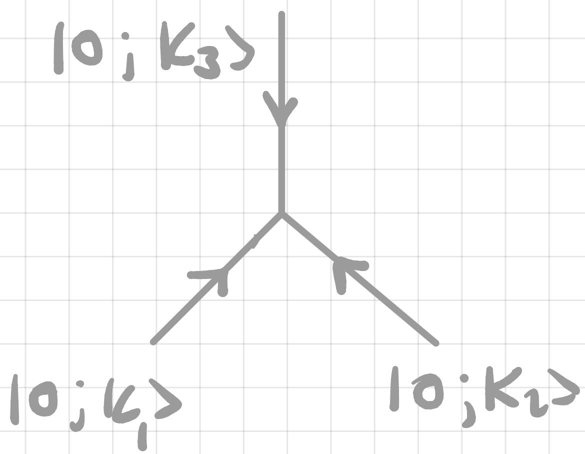
$$|\psi\rangle=\lim_{t\to\infty}(z\bar{z})^{-1}V_\psi(it,\sigma) \quad , \quad z=e^{2(t-i\sigma)}=e^{2i\sigma_-}$$

$$\bar{z}=e^{2(t+i\sigma)}=e^{-2i\sigma_+}$$

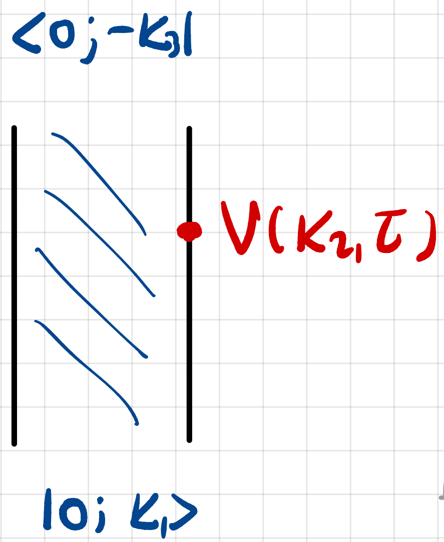
$$|z\bar{z}|=e^{t}$$

4.5 Three point interactions   Tree level

three point open string interaction eg tachyon absorbing a tachyon at $\sigma=0$, τ



$\longleftrightarrow$



$$\mathcal{A}_3^{o}(k_1,k_2,k_3) = g_o \int_{-\infty}^{\infty} d\tau \, \langle 0;-k_3|\, V_T(\tau;k_2)\,|0;k_1\rangle \Big/ \text{Vol(conf.)}$$

open string coupling constant

infinite volume of the gauge group

divide out by a divergent "volume" due to the residual gauge symmetry (τ-translations of V_T leaving past & future states invariant)

$L_0 = \frac{1}{2}p^2 + N \to 1 + 0$

$$A_3^{\infty}(k_1,k_2,k_3) = g_0 \int d\tau \, \underbrace{\langle 0; -k_3 | e^{i\tau L_0}}_{\langle 0;-k_3 | e^{i\tau}} V_T(0;k_2) \underbrace{e^{-i\tau L_0} | 0; k_1 \rangle}_{e^{-i\tau}|0;k_1\rangle} \Big/ \mathrm{Vol(conf.)}$$

$$= g_0 \int d\tau \, \langle 0; -k_3 | \underbrace{V_T(0;k_2)}_{e^{k\cdot\sum_{n=1}^{\infty}\frac{\alpha_{-m}}{m}} : e^{ik\cdot x} : e^{-k\sum_{n=1}^{\infty}\frac{\alpha_n}{n}}} | 0; k_1 \rangle \Big/ \mathrm{Vol(conf.)}$$

$$\underbrace{e^{k\cdot\sum_{n=1}^{\infty}\frac{\alpha_{-m}}{m}}}_{1 + \text{creation operators}} : e^{ik\cdot x} : \underbrace{e^{-k\sum_{n=1}^{\infty}\frac{\alpha_n}{n}}}_{1 + \text{annihilation operators}}$$

$$= g_0 \int d\tau \, \langle 0; -k_3 | \underbrace{: e^{ik_2\cdot x} : \, | 0; k_1 \rangle}_{} \Big/ \mathrm{Vol(conf.)}$$

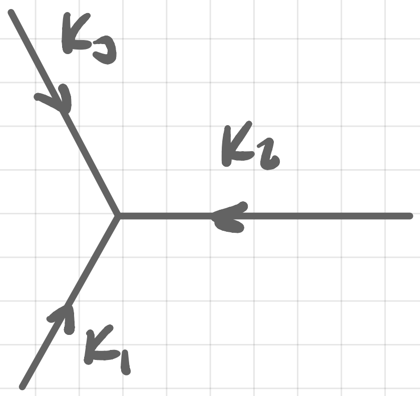
$$\mathcal{A}_3^{\infty}(k_1,k_2,k_3) = g_0 \int d\tau \, \langle 0;-k_3 \,|\, 0; k_1+k_2 \rangle \Big/ \mathrm{Vol(conf.)}$$

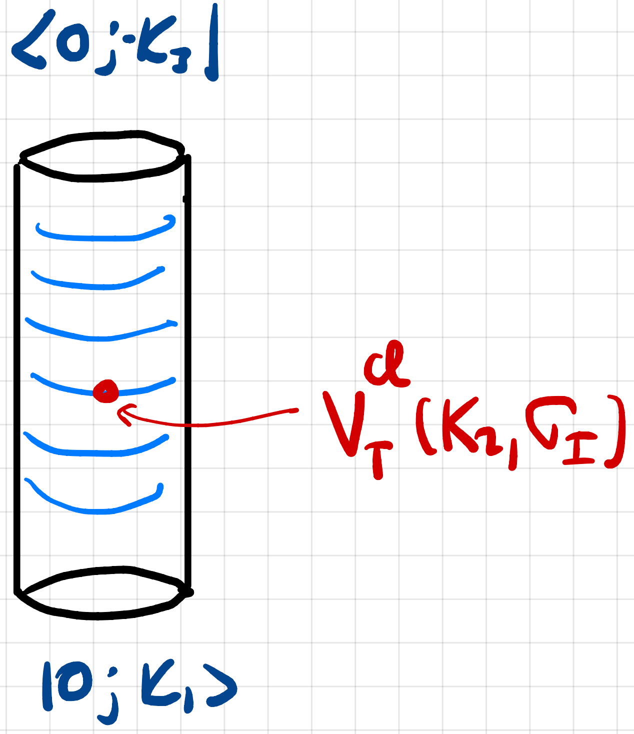
$$= g_0\, \delta(k_1+k_2+k_3) \left[\int_{-\infty}^{\infty} d\tau \Big/ \mathrm{Vol(conf.)} \right]$$

alternatively: gauge fix at $\tau = 0$!

$$\mathcal{A}_3^{\infty}(k_1,k_2,k_3) = g_0\, \delta(k_1+k_2+k_3)$$

three point tree level closed string diagrams
(tachyon absorbing a tachyon)





$$A_3^{cl}(k_1, k_2, k_3) = g_{cl} \int d^2\sigma_I \, \langle 0; -k_3 | \, V_T^{cl}(k_2, \sigma_I) \, | 0; k_1 \rangle \Big/ \mathrm{Vol(conf.)}$$

$$V_T^{cl}(k,\sigma_\pm) = e^{2i\sigma_- L_0 + 2i\sigma_+ \tilde{L}_0}\, V_T^{cl}(k,0)\, e^{-2i\sigma_- L_0 - 2i\sigma_+ \tilde{L}_0}$$

$$L_0 = \frac{\ell^2 p^2}{8} + N$$

$$L_0|0;k_1\rangle = |0;k_1\rangle$$

$$A_3^{cl}(k_1,k_2,k_3)$$

$$= g_{cl}\int d^2\sigma_\pm \langle 0;-k_3|\, e^{2i\sigma_- L_0 + 2i\sigma_+ \tilde{L}_0}\, V_T^{cl}(k_2,0)\, \underbrace{e^{-2i\sigma_- L_0 - 2i\sigma_+ \tilde{L}_0}}_{e^{-2i\sigma_- - 2i\sigma_+}} |0;k_1\rangle \Big/ \mathrm{Vol(conf.)}$$

$$= g_{cl}\int d^2\sigma_\pm \langle 0;-k_3|\, \underbrace{V_T^{cl}(k_2,0)}_{(1+\text{creation})\, e^{ik_2\cdot x}\, (1+\text{annihilation})} |0;k_1\rangle \Big/ \mathrm{Vol(conf.)}$$

$$\mathcal{A}_3^{cl}(k_1, k_2, k_3) = g_{cl} \int d^2\sigma_\pm \langle 0; -k_3 | 0; k_1 + k_2 \rangle \Big/ \mathrm{Vol(conf.)}$$

$$= g_{cl}\, \delta(k_1 + k_2 + k_3) \int d^2\sigma_\pm \Big/ \mathrm{Vol(conf.)}$$

$$\underline{\mathcal{A}_3^{cl} = g_{cl}\, \delta(k_1 + k_2 + k_3)}$$

Next:

4-point tachyon amplitude, etc.