

STRING THEORY I

Lecture 11



[4] Interactions

- 4.1 Generalities
- 4.2 Vertex operators: introduction
- 4.3 Vertex operators: open string
- 4.4 The state vertex correspondence
- 4.5 3-point interactions
- 4.6 4-point tachyon amplitude
- 4.7 Comments on the general picture



[4.7] Comments on the general picture

In string perturbation theory we are interested in the amplitude for the scattering of asymptotic in and out states (the S -matrix)

We have discussed a number of ideas and tools for computing amplitudes.

Wrap up this chapter on interactions with a number of comments on the lessons learned and on the general picture for scattering amplitudes

To study string amplitudes we use

state \longleftrightarrow vertex correspondence
 $|\psi\rangle \longleftrightarrow V_\psi$ operator of conformal
weight $\begin{cases} h=1 & \text{open strings} \\ h=\tilde{h}=1 & \text{closed strings} \end{cases}$

V_ψ represents emission/absorption of a physical string state $|\psi\rangle$ from a point on the worldsheet

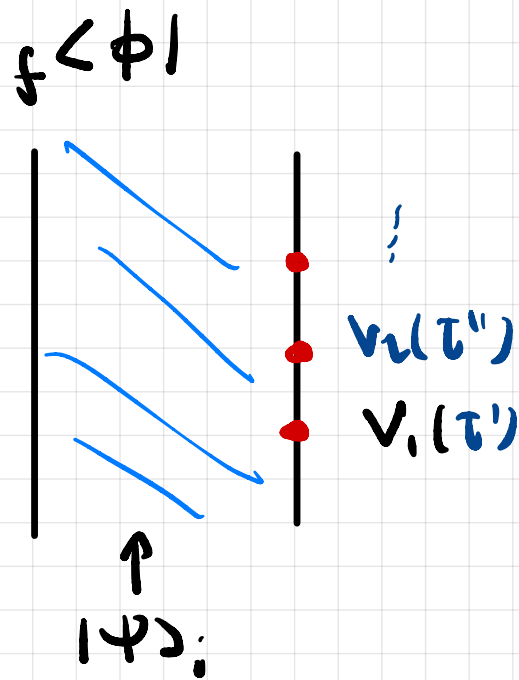
and incoming/outgoing states are represented by

$$|\psi\rangle = \lim_{z \rightarrow 0} z^{-1} V_\psi(z) |0;0\rangle$$

(action of V_ψ on zero momentum vacuum state in the infinite Euclidean past)

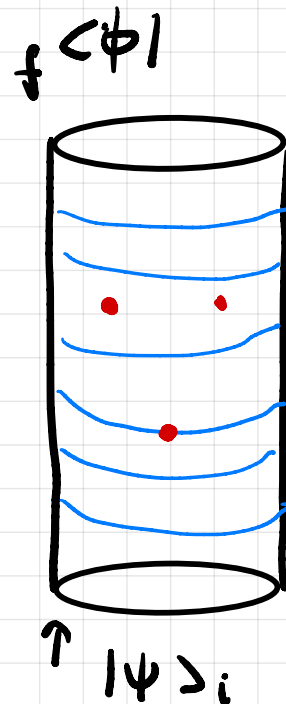
$$\langle\phi| = \lim_{z \rightarrow \infty} z \langle 0;0| V_\psi(z)$$

open strings
(tree level)



Vertex operators
inserted on the
boundary of the
world sheet

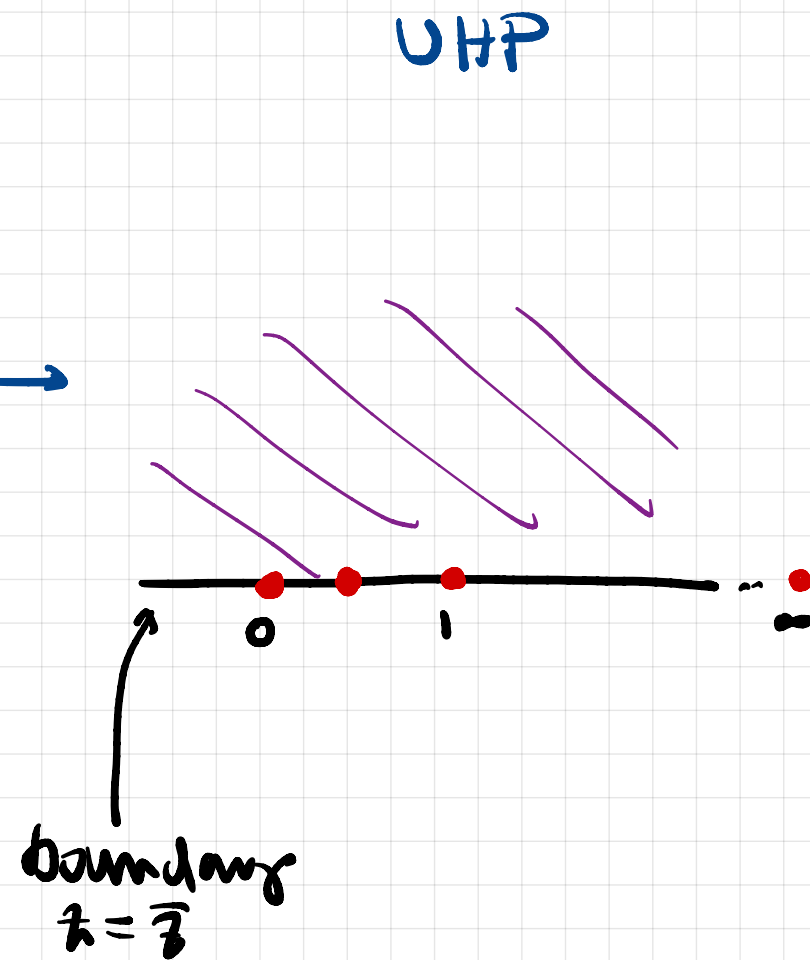
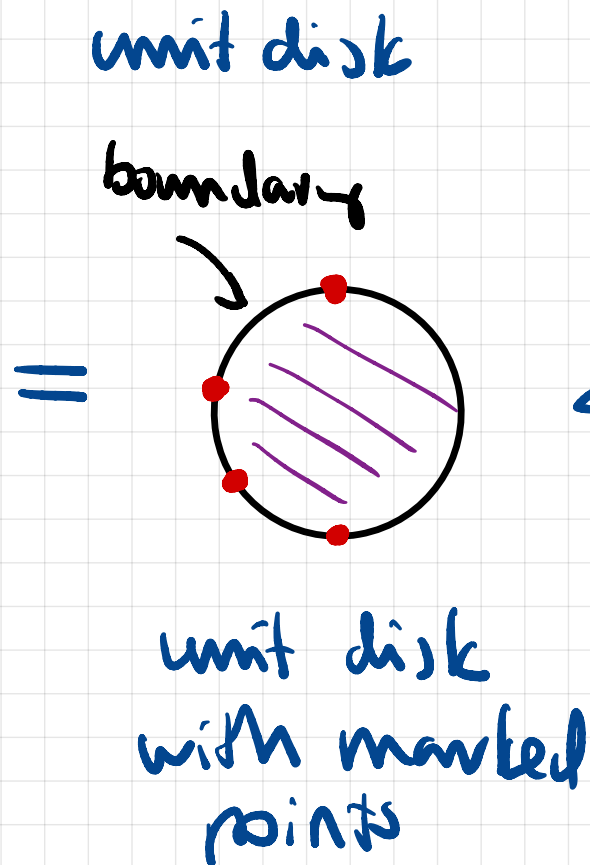
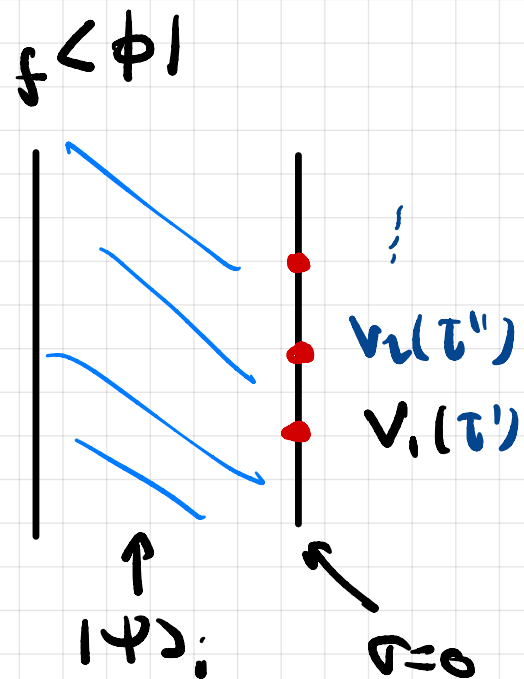
closed strings
(tree level)



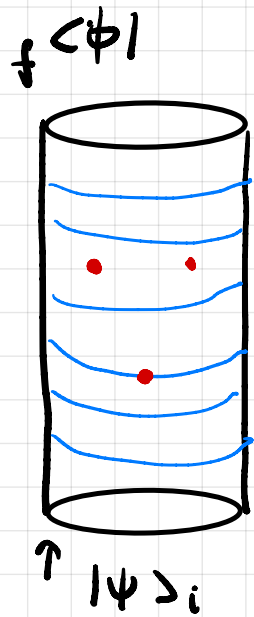
Vertex operators
inserted in the
interior of the
world sheet

Moreover, by a Wick rotation together with wise coordinate changes we map the Lorentzian world sheet into Euclidean world sheet and the amplitudes have now an interpretation on this Euclidean world sheet.

open strings
(tree level)

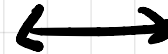
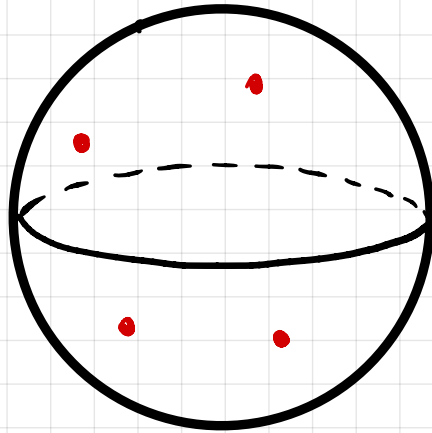


closed strings
(tree level)

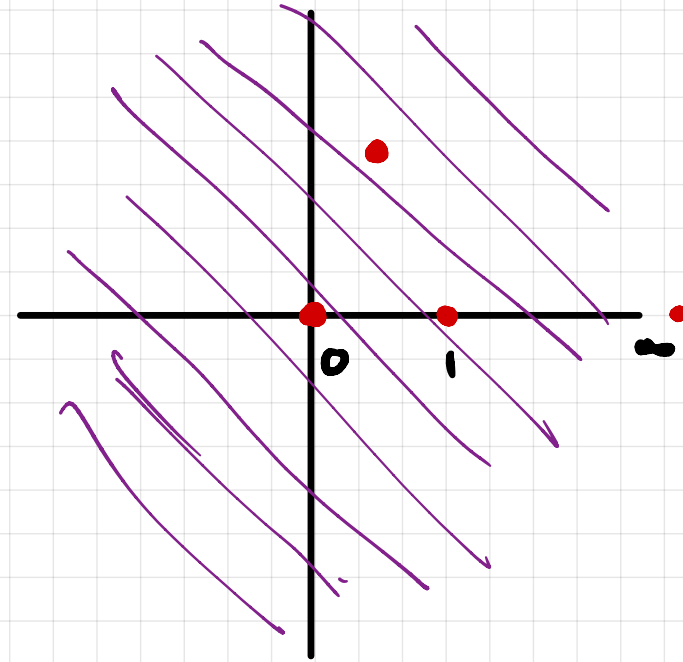


=

sphere with
marked points

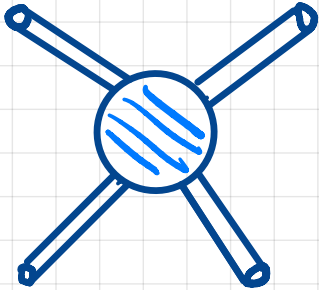


complex plane $\cup \{\infty\}$





The string perturbation series is a genus expansion
 For example, for the closed string

tree 1-loop 2-loops ...

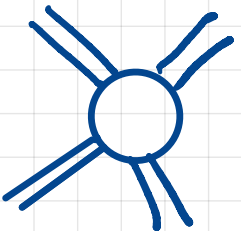


$$= g_c^2 \text{ (sphere with 4 red dots and blue lines) } + g_c^4 \text{ (torus with 4 red dots) } + g_c^6 \text{ (genus 2 surface with 4 red dots) } + \dots$$






without boundaries sum over all topologies of Riemann surfaces

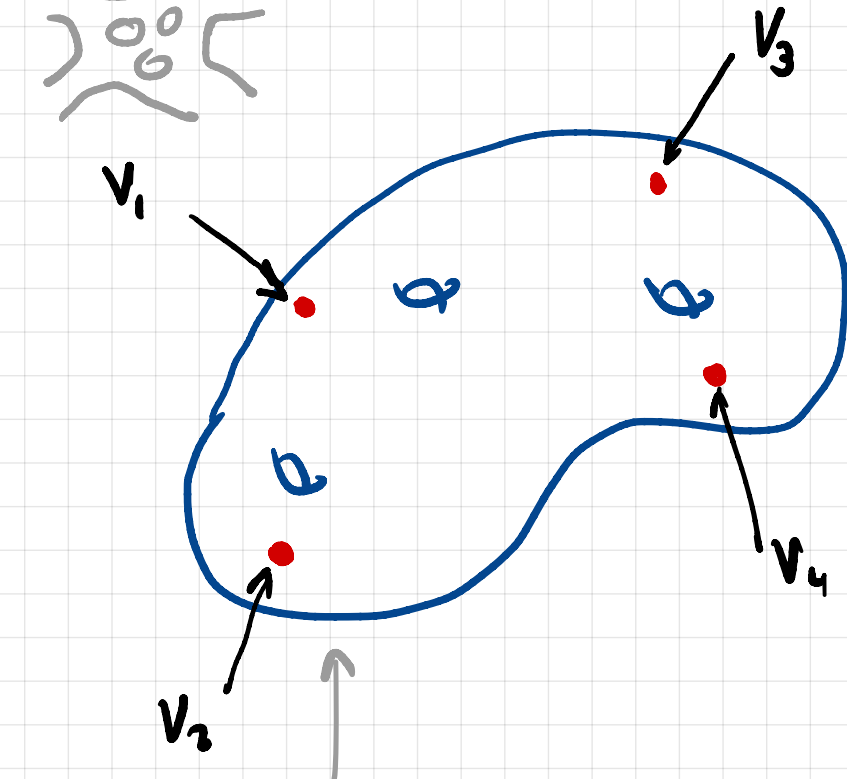
with boundaries



$$= g_o^2 \text{ (disk with 4 red dots and blue lines) } + g_o^4 \text{ (annulus with 4 red dots) } + \dots$$

At each order in perturbation level: one diagram



genus g Riemann
surfaces
(\mathcal{C} -surfaces)

Moduli space of
Riemann surfaces with marked points

$$\mathcal{A}_g(\psi_1, \dots, \psi_n)$$

$$= \int [d\mu] \langle \psi_1 \psi_2 \psi_3 \psi_4 \rangle$$

parametrizes
Weyl equivalent
classes of metrics
i.e. \mathcal{C} -structures

$$\Sigma_{g,4}(\mu)$$

$$\mathcal{M}_{g,4}$$

• $\mathcal{M}_g \ni [d\mu] \rightarrow$ complicated

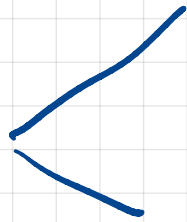
however low genus isn't so bad (we did tree level examples)
(1-loop amplitude calculations are rather interesting)

Remark 1:

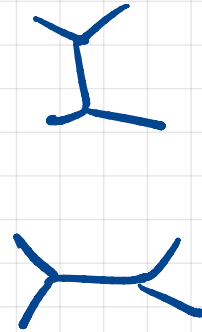
① One diagrams per order in perturbation theory

② Degeneration limits look like many Feynman diagrams

eg



or



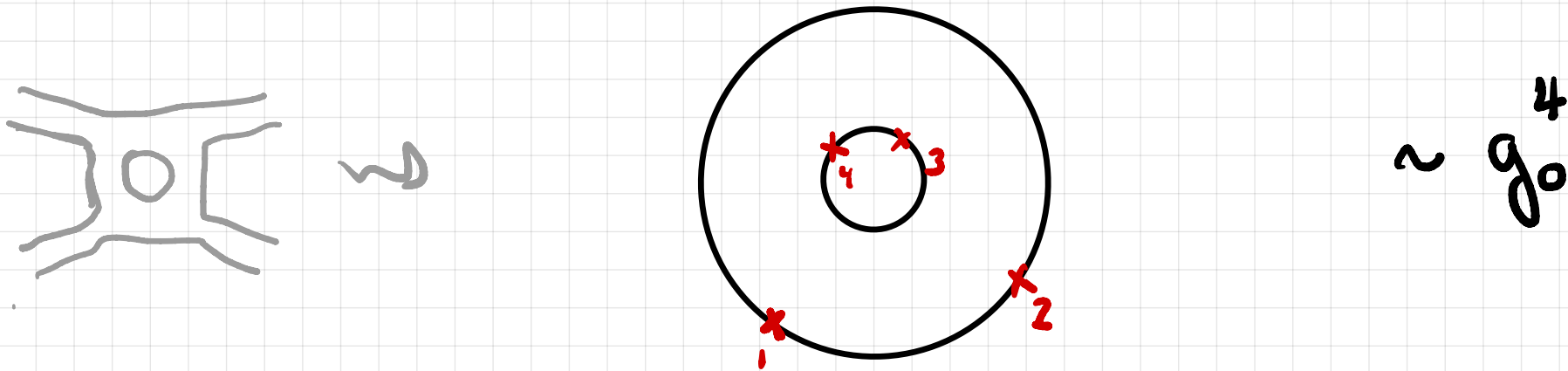
etc

③ Generalization of DHS duality

[4] An interesting generalization of DTS duality is the open/closed duality

Consider the following 1-loop open string amplitude: the annulus amplitude

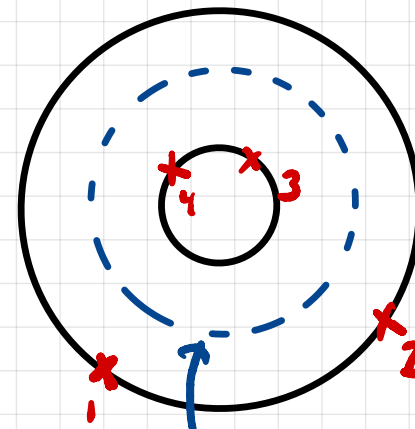
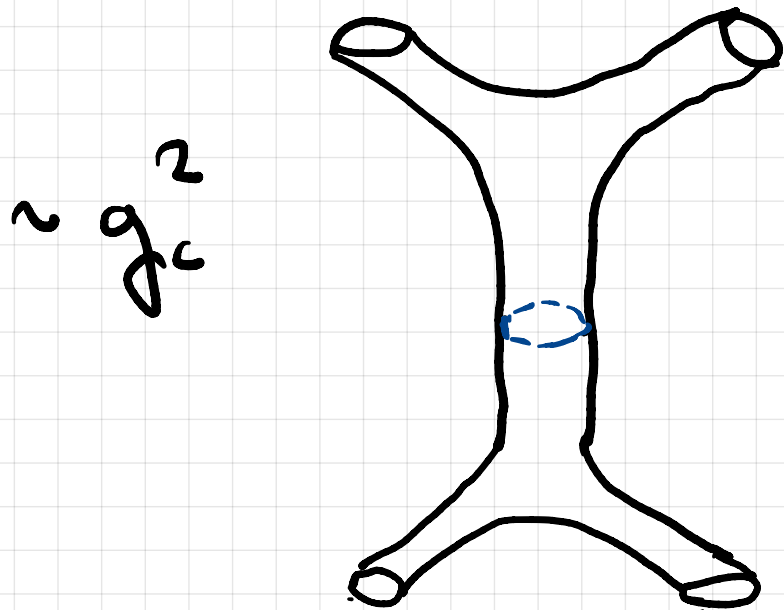
say for 2 incoming string states $|1\rangle, |2\rangle$ and two outgoing string states $\langle 3, \langle 4|$



open string in a loop

But topologically this is the same as a cylinder so instead we can reinterpret this one loop open string amplitude as a tree level closed string amplitude

C. Been
magical amplitude



a single
geometry
with two
interpretations

closed string state

Consistency : $g_c = g_o^2$

$g_c \sim$ grav. coupling
 $g_o \sim U(1)$ gauge coupling.

[5] Final remark: UV behavior (see D Tong's lecture notes)

- UV-finiteness holds for all loop diagrams up to 2 loops.

↳ 2-loop finite theory of gravity interacting with matter in higher dims

However so far there is no proof for all loops...

Next: strings in background fields