STRING THEORY I

Lecture 13

$\boxed{5}$ Strings in background fields

5.1 Background field expansion and the Weyl anomaly

5.2 Including other massless modes

5.3 Spacetime effective actions

5.4 The dilaton revisited

Last lecture:

constructed a 2 dim NLσM

$$S = S^{(G)} + S^{(B)} + S^{(\Phi)}$$

general 2 dim QFT which is reparametrization invariant (& renormalizable)

→ $S^{(G)} + S^{(B)}$ is classically Weyl invariant
but $S^{(\Phi)}$ is not (unless Φ = constant)

We analyzed the action S in terms of the background field expansion

$$X^\mu = \underbrace{X_0^\mu}_{\text{soln of classical EOM}} + x^\mu$$

x^μ ← quantum fluctuation

This gives a perturbative expansion of the NLσM in powers of the fluctuations.

Crucially we demanded that the resulting 2dim QFT be Weyl invariant at the quantum level.

This requirement leads to the computation of the β-functional. The preservation of the Weyl symmetry at the quantum level, ie

$$\beta^{(G)} = 0 \quad , \quad \beta^{(B)} = 0 \; , \quad \beta^{(\Phi)} = 0$$

then imposes constraints on the spacetime fields G, B & Φ which are interpreted as EOM for these fields (eg to first order in α' $R_{\mu\nu} = 0$, etc)

15.4 The dilaton revisited

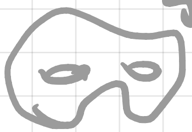
Recall

$$S^{\Phi} = \frac{1}{4\pi}\int d^2\sigma\, \sqrt{\gamma}\, R^{(2)}(\gamma)\, \Phi(X)$$

Previously we had ignored this term for $\Phi =$ constant because then the integrand is a total derivative. This however is not the right way to look at this term in an interacting theory in a background with $\Phi = \Phi_0 =$ constant.

Theorem (differential topology): Gauss-Bonnet theorem

Σ 2 dim surface



g=2

$$\frac{1}{4\pi}\int_{\Sigma_g} dA\, R^{(2)} = \underline{\chi(\Sigma_g) = 2-2g}$$

compact, no boundaries

Riemann surface no boundaries $\rightarrow \Sigma_g$ (g — genus)

geodesic curvature of $\partial\Sigma$

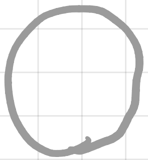
$$\frac{1}{4\pi}\int_{\Sigma_{g,h}} dA\, R^{(2)} + \frac{1}{2\pi}\int_{\partial\Sigma_{g,h}} ds\, K = \chi(\Sigma_{g,h}) = 2-2g-h$$

$\Sigma_{g,h}$ → Riemann surface, genus g and h boundaries

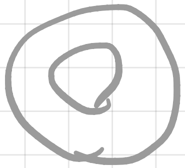
χ Euler-characteristic (<u>topological invariant</u>)

for a surface Σ with g handles and h boundaries

eg



$g=0$ $h=1$,



$g=1$, $h=2$

etc.

This means that in a background $\Phi = \Phi_0$

$$S^{\Phi} = \Phi_0 \lambda_{g,h}$$

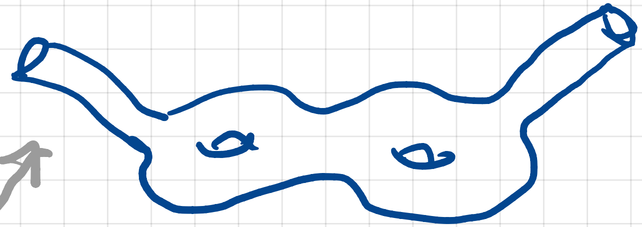
where $\lambda_{g,h}$ is determined by this theorem and depends on g and h.

Clearly this term in the total action

$$S = S^{(a)} + S^{(B)} + S^{(\Phi)}$$

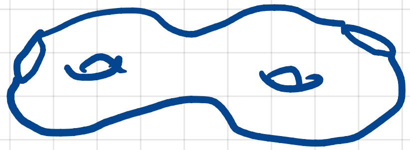
adds a factor of $e^{-\Phi_0}$ (to some power determined by λ) to the amplitudes.

$\boxed{\text{Closed}}$ string amplitudes :



$\longleftrightarrow$

$g=2, \quad h=2$



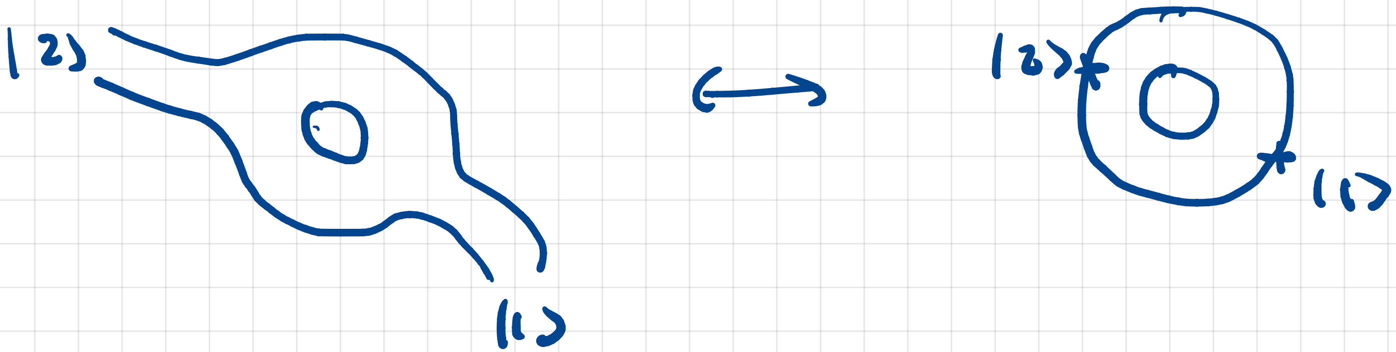
$\chi = 2-2g-h$

incomming & outgoing states introduce extra boundaries

each loop gets a factor $e^{-2\Phi_0}$

each external closed string gets a factor $e^{-\Phi_0}$

$\boxed{\text{Open}}$ string amplitudes



each open string loop gets a factor $e^{-\Phi_0}$

each external open string gets a factor $e^{-\Phi_0/2}$

how?

(gluing argument
see (Beem's lecture)

These factors are related to the factors of the open string (g_o) & closed string (g_c) coupling constants:

shifting the dilaton Φ by a zero mode

$$\Phi \to \Phi + c$$

corresponds to a rescaling of these couplings

$$g_c \longrightarrow g_c e^{-c} \qquad g_o \longrightarrow g_o e^{-c/2}$$

so the value of g_c (or g_o) can be absorbed into a shift in the expectation value of Φ

This means that the string coupling constants are **not** parameters of the theory, in fact they are dynamical and given in terms of the expectation value of Φ.

From the computation of the annulus amplitude and reinterpreting it as a (tree level) closed string amplitude one can find the precise normalization factor. We had

$$g_o^2 \sim g_c$$

The computation gives: $g_o^2 = 2^{18} \pi^{25/2} (\alpha')^6 g_c$

(Polchinski exercise 7.9)

This is related to the fact that in string theory there are no continuous parameters. Parameters appear then as expectation values of dynamical spacetime fields.

It seems that we need to have a discussion about the energy scales involved in the space-time effective action obtained by requiring it gives the Beta functions as EOM that the background spacetime fields need to satisfy.

As we have discussed this amounts to an <u>α'-expansion</u> of the effective action.

For the one-loop Beta function we obtained the effective action to leading order in α'. We found in the string frame

$$S_{26}^{(S)} = \frac{1}{2\kappa_0^2}\int d^{26}x\sqrt{-G}\,e^{-2\Phi}\left(R - \frac{1}{12}|H|^2 + 4|\partial\Phi|^2\right)$$

$H = dB$

while in the Einstein frame

$$S_{26}^{(E)} = \frac{1}{2\kappa^2}\int d^{6}x\sqrt{-\tilde{G}}\left(\tilde{R} - \frac{1}{12}e^{-\frac{1}{3}\tilde{\Phi}}|H|^2 - \frac{1}{6}|\partial\tilde{\Phi}|^2\right)$$

where $\tilde{G}_{\mu\nu} = e^{\frac{1}{6}\tilde{\Phi}}G_{\mu\nu}$, $\tilde{\Phi} = \Phi - \Phi_0$, $\kappa = \kappa_0 e^{\Phi_0}$

(and indices are raised & lowered with $\tilde{G}$)

The Einstein frame is constructed such that the Einstein-Hilbert term takes the canonical form with gravitational coupling

$$\kappa = (8\pi G_N)^{1/2}$$

The gravitational coupling

$$\kappa = \kappa_0 e^{\Phi_0} = (8\pi G_N)^{1/2} \sim (M_{Pl})^{-\frac{1}{2}(D-2)}$$

controls spacetime quantum effects

On the other hand we have the string scale

$$\alpha' \sim M_s^{2}$$

(Recall that we obtained an effective theory from the NLσM large radius expansion with cutoff M_s)
The string scale controls stringy corrections
(world sheet quantum corrections).

The gravitational coupling κ and the string scale α' are related by the expectation value of the dilaton (e^{Φ_0})

We have a dimensionless ratio

$$\frac{M_s}{M_{pl}} \sim e^{\frac{2}{D-2}\Phi_0} \qquad g_s \sim e^{\Phi_0}$$

$e^{\Phi_0} \to 0$ gives a classical limit in space time

So we have the effective action for energies $E \ll M_s$

in the limit $\frac{M_s}{M_{pl}} \to 0$

Next: compactifications

↳ illustrate · $\mathbb{R}^{1,24} \times S^1_R$

· T-duality