STRING THEORY I

Lecture 15

$\boxed{6}$ Compactifications (continued..)

S^1- compactifications of the bosonic string

$$\mathbb{R}^{1,25} \longrightarrow \mathbb{R}^{1,24} \times S^1_R$$

We are discussing this from our two perspectives

1 From the spacetime EFT:

Kaluza-Klein mechanism on the $\mathbb{R}^{1,25}$ EFT

We obtained a massless sector

$$G_{\mu\nu}(X) \longrightarrow G_{\mu\nu}(X^i) : \{ G_{ij}(X^i),\ G_{i,25}(X^i),\ G_{25,25}(X^i) \}$$

25 dim graviton

graviphoton $e^{2\sigma}A$

radion $e^{2\sigma}$

$$B_{\mu\nu}(X) \longrightarrow B_{\mu\nu}(X^i) : \{ B_{ij}(X^i),\ B_{i,25}(X^i) \}$$

25 dim KR field

KR-photon $\tilde{A}$

2 $U(1)$ gauge symmetries

$$\Phi(X) \longrightarrow \Phi(X^i) \quad \text{25 dim dilaton}$$

$\boxed{\text{Plus}}$ a discrete infinite tower of massive states (KK-modes)

For example: the 26 dimensional dilaton gives rise to a discrete infinite tower of scalar fields (KK modes)

$$\phi_n \quad \text{with mass} \quad M_n^2 = \frac{n^2}{R^2} \qquad \forall n \in \mathbb{Z}$$

These are charged $(n \neq 0)$ under the graviphoton

$$\text{charge} \quad \frac{n}{R} \qquad \text{(KK-momentum charge)}$$

There are no modes charged under the KR photon.

Of course this introduces a new scale

$$M_{KK} \sim \frac{1}{R}$$

In fact one can show that $\langle \sigma \rangle = R$
(Blumenhagen + Lüst + Theisen)

We should not trust the EFT analysis for $M_{KK} \sim M_s$: however in this case one can perform an exact analysis of worldsheet CFT.

② The <u>world sheet</u> perspective

2dim World sheet NLσM with target space with a nontrivial topology

$$X^\mu \longrightarrow X^i \qquad i = 0, \ldots, 24$$

$$X^{25} \sim X^{25} + 2\pi R \qquad (X^{25} \text{ parametrizes a circle } S^1_R)$$

states in the string Hilbert space are similar to those of $\mathbb{R}^{1,25}$ however we have now quantized KK-modes on S^1_R <u>and</u> quantized winding modes

The winding modes come from requiring

$$X^{25}(\tau, \sigma+\pi) = X^{25}(\tau,\sigma) + 2\pi R\, \omega \qquad \omega \in \mathbb{Z}$$

($X^{a}(\tau,\sigma)$ & $X^{25}(\tau,\sigma+\pi)$ periodic up to $2\pi R\,\omega$)

after quantization gives rise to new states (winding states) with $\omega \to$ conserved charge

The expansion of $X^i(\tau,\sigma)$ $i=0,\dots,24$

is as before, but the expansion for X^{25} changes

$$X^{25}(\tau,\sigma) = x^{25} + 2\alpha' p^{25}\tau + 2Rw\sigma + \text{oscillator modes}$$

where the p^{25} momentum eigenvalue is quantized

$$p^{25} = \frac{m}{R} \qquad m \in \mathbb{Z}$$

Separating this into left & right movers:

$$X^{25}(\tau,\sigma) = X_L^{25}(\sigma^+) + X_R^{25}(\sigma^-)$$

$$X_R^{25}(\sigma^-) = x_R^{25} + \left(\frac{\alpha' m}{R} - wR\right)\sigma^- + \text{osc}$$

$$X_L^{25}(\sigma^+) = x_L^{25} + \left(\frac{\alpha' m}{R} + wR\right)\sigma^+ + \widetilde{\text{osc}}$$

$$p_L^{25} = p^{25} + \frac{1}{\alpha'}Rw \qquad p_R^{25} = p^{25} - \frac{1}{\alpha'}Rw$$

construct $T_{\pm\pm}$

The Virasoro operators are as before except

$$\tilde{\alpha}_0^{25} = \sqrt{\frac{\alpha'}{2}}\left(p^{25} + w\frac{R}{\alpha'}\right) \qquad \alpha_0^{25} = \sqrt{\frac{\alpha'}{2}}\left(p^{25} - w\frac{R}{\alpha'}\right)$$

The states are of the form: mass-shell condition

$$\prod \alpha^{\mu}_{-n} \prod \tilde{\alpha}^{\nu}_{-m} \, | K ; m ; w \rangle$$

25-dim momentum evalue

$p^{25} = \frac{m}{R}$

$X^{25}(\tau, \sigma + \pi) = X^{25}(\tau, \sigma) + 2\pi R w$

and must satisfy the conditions:

(apart from $L_m|\phi\rangle = 0 \ \forall m>0$)

$$M^2_{(25)} = \frac{m^2}{R^2} + \underbrace{\frac{R^2 w^2}{\alpha'^2}} + \frac{2}{\alpha'}(N + \tilde{N} - 2)$$

mass shell condition

contribution to the mass from momentum along the compact direction

energy to wrap string around S^1_R w times

($M_{(25)}$ depends on R!)

$$N - \tilde{N} = m w$$

level-(mis)matching condition

tachyon with $m = w = 0$

$M^2_{(25)} = -8$

Massless spectrum: (For any R)

- 25 dim graviton $\gamma_{ij}\ \alpha^i_{-1}\tilde{\alpha}^j_{-1}\,|0,K\rangle \otimes |0,0\rangle$
- 25 dim B-field $B_{ij}\ \alpha^i_{-1}\tilde{\alpha}^j_{-1}\,|0,K\rangle \otimes |0,0\rangle$
- scalar from the trace part of γ: $\Phi_{(25)}$
- 2x 25 dim $U(1)\times U(1)$ gauge fields $\varsigma\cdot(\alpha_{-1}\tilde{\alpha}^{25}_{-1} \pm \tilde{\alpha}_{-1}\alpha^{25}_{-1})\,|0,K\rangle \otimes |0,0\rangle$

(graviphoton from the 26 dim metric + another photon from the 26 dim KR field)

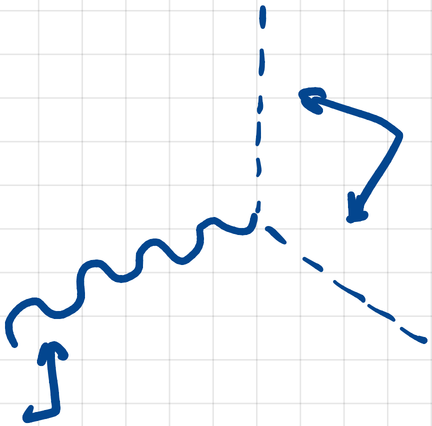
- scalar ("radion") $\alpha^{25}_{-1}\tilde{\alpha}^{25}_{-1}\,|0,K\rangle \otimes |0,0\rangle$

identified with the scalar σ (associated to R)

massless string spectrum $\longleftrightarrow$ massless spectrum from KK reduction of EFT

(except when $\alpha' = \sqrt{R}$!)

Gauge fields: by considering the 3-amplitude



winding tachyon

$|0,K\rangle \otimes |0,0\rangle$

$M_{25}^2 = -8 + 4R^2 w^2$

$V_{m,w}(p) \sim \int d\bar{\tau}\, d\sigma\; e^{ip\cdot X}\; e^{ip_L X^{25} + i p_R X^{25}}$

Kalb-Ramond photon

$V_-(K) \sim \int d\bar{\tau}\, d\sigma\; \zeta \cdot (\partial_+ X\, \partial_- X^{25} - \partial_- X\, \partial_+ X^{25})\, e^{iK\cdot X}$

$A = \underbrace{g \cdot K_3\, (2Rw)}\, \delta(K_1 + K_2 + K_3)$

which reproduces the vertex from a term $A_\mu \phi\, \partial^\mu \phi$ in the spacetime Lagrangian

$\therefore$ winding number is the KR-photon charge

Remark: we have introduced a new scale R
In fact, we have a one parameter family
of compactifications with $R \in (0, \infty)$

R is called a modulus

(More general compactification give rise to a
moduli space)

However $(0, \infty]$ containts points R & $\tilde{R}$
which give rise to indistinguishable physical
theories.

T- duality

Returning to the mass formula

$$M^2_{(25)} = \frac{m^2}{R^2} + \frac{1}{(\alpha')^2} w^2 R^2 + \frac{2}{\alpha'} (N + \tilde{N} - 2)$$

$$N - \tilde{N} = mw$$

Limiting cases :

- $R \to \infty$: continuum of KK modes $\to$ sign of 25th dimension

- $R \to 0$: continuum of winding modes ?

$\boxed{\text{Symmetry}}$ of the spectrum:

observe that the formulas

$$M^2_{(2\pi)} = \frac{m^2}{R^2} + \frac{1}{(\alpha')^2} w^2 R^2 + \frac{2}{\alpha'}(N+\tilde{N}-2), \qquad N-\tilde{N} = mw$$

are invariant under: $m \leftrightarrow w$ & $R \leftrightarrow \frac{\alpha'}{R} = \tilde{R}$

$\Rightarrow$ compactifications on S_R & $S_{\alpha'/R}$ have the $\boxed{\text{same}}$ spectrum.

Note that $R = \sqrt{\alpha'}$ is a fixed point of this transformation: something special happens at this point.

This is in fact an **exact** symmetry of the CFT

$\boxed{\text{T-duality}}$

so compactifications on S_R & $S_{\tilde{R}}$ with $\tilde{R} = \frac{\alpha'}{R}$ are indistinguishable as physical theories.

The interchange $m \leftrightarrow w$ means that

momentum excitations $\longleftrightarrow$ winding mode excitations

so continuum of KK modes for $R \to \infty$ $\longleftrightarrow$ continuum of winding modes for $\tilde{R} \to 0$

T-duality as an exact symmetry of the CFT

BUT, we have only shown that the spectrum is the same for two theories where

$$R \longleftrightarrow \tilde{R} = \frac{\alpha'}{R}$$

and simultaneously

$$(m, w) \longleftrightarrow (w, m)$$

We need to consider the full CFT to prove this is an $\boxed{\text{exact}}$ symmetry of the CFT

Noting that

$$\left\{ \begin{array}{ll} \alpha_0^{25} = \sqrt{\frac{\alpha'}{2}}\left(\frac{m}{R} - \frac{Rw}{\alpha'}\right) \longrightarrow & -\alpha_0^{25} \\ \tilde{\alpha}_0^{25} = \sqrt{\frac{\alpha'}{2}}\left(\frac{n}{R} + \frac{Rw}{\alpha'}\right) \longrightarrow & \tilde{\alpha}_0^{25} \end{array} \right.$$

need to extend action of the transformation to the oscillator modes

$$X_R^{25}(\sigma^-) \longleftrightarrow -X_R^{25}(\sigma^-)$$

$$X_L^{25}(\sigma^+) \longleftrightarrow X_L^{25}(\sigma^+)$$

equivalently

$$X^{25}(\tau,\sigma) = X_L^{25}(\sigma^+) + X_R^{25}(\sigma^-) \longleftrightarrow \tilde{X}^{25}(\tau,\sigma) = X_L^{25}(\sigma^+) - X_R^{25}(\sigma^-)$$

$$= x^{25} + 2\alpha' \frac{m}{R}\tau + 2Rw\sigma + \dots \qquad = \tilde{x}^{25} + 2\alpha' \frac{m}{R}\sigma + 2Rw\tau + \dots$$

circle radius R
conj momentum $p^{25} = \frac{m}{R}$

circle radius $\frac{\alpha'}{R}$
conjugate momentum $\tilde{p}^{25} = \frac{w\alpha'}{R}$

X & $\tilde{X}$ have the same energy momentum tensor

$$T_{\pm\pm} = \partial_\pm X \cdot \partial_\pm X = \partial_\pm \tilde{X}\, \partial_\pm \tilde{X}$$

One can recover L_m & $\tilde{L}_m$ as Fourier modes

$\Longrightarrow$ CFT's of X & $\tilde{X}$ are the $\boxed{\text{same}}$

As a consequence of this duality the moduli space of circle compactifications of the bosonic string is not $(0, \infty]$ but instead

$R \in (0, \sqrt{\alpha'}]$ or equivalently $R \in [\sqrt{\alpha'}, \infty)$

Fixed point of the duality transformation:

$$R \longleftrightarrow \tilde{R} = \frac{\alpha'}{R} \quad \text{when} \quad \boxed{R = \sqrt{\alpha'}}$$

$R = \sqrt{\alpha'}$ is special $\rightsquigarrow$ more massless states and enhanced gauge symmetry

$$M^2_{(25)} = \frac{m^2}{R^2} + \frac{1}{(\alpha')^2} w^2 R^2 + \frac{2}{\alpha'}(N + \tilde{N} - 2) = \frac{1}{R^2}\left(m^2 + w^2 + 2(N+\tilde{N}-2)\right)$$

so $M^2_{25} = 0$ when $m^2 + w^2 + 2(N+\tilde{N}) = 4$

also $mw = N - \tilde{N}$

Interestingly there are 4 extra massless vectors which enhance the $U(1) \times U(1)$ symmetry to

$$SU(2) \times SU(2)$$

and 8 additional scalar fields in a $(\underline{3}, \underline{3})$ representation of $SU(2) \times SU(2)$

Open strings

What happens to T-duality?

Recall: open string boundary conditions compatible with Poincaré invariance in 26 dimensions

$$\frac{\partial}{\partial\sigma} X^\mu(\tau,\sigma) = 0 \quad \text{at} \quad \sigma = 0, \pi$$

Neumann boundary conditions

Consider now compactifying on a circle

$\rightarrow$ no winding modes !

while KK-momentum momentum modes still make sense

Recall the mode expansion of the coordinates X^μ

$$X^\mu(\tau,\sigma) = x + 2\alpha' p\tau + i\sqrt{2\alpha'}\sum_{n\neq 0}\frac{1}{n}\alpha_n e^{-in\tau}\cos(n\sigma)$$

$$= X_R^\mu(\sigma^-) + X_L^\mu(\sigma^+)$$

$$X_R^\mu(\sigma^-) = x_R + \alpha' p^\mu \sigma^- + i\sqrt{\frac{\alpha'}{2}}\sum_{n\neq 0}\frac{1}{n}\alpha_n^\mu e^{-in\sigma^-}$$

$$X_L^\mu(\sigma^+) = x_L + \alpha' p^\mu \sigma^+ + i\sqrt{\frac{\alpha'}{2}}\sum_{n\neq 0}\frac{1}{n}\alpha_n^\mu e^{-in\sigma^+}$$

compactify on a circle with x^{25} parametrizing the circle of radius R and consider what happens when interchanging

$$X_L^{25} \longleftrightarrow X_L^{25} \qquad X_R^{25} \longleftrightarrow -X_R^{25}$$

Should we expect a string for which there is a winding quantum number but no KK-momentum?

The proposed dual coordinate is

$$\tilde{X}^{25}(\tau,\sigma) = X_L^{25}(\sigma^+) - X_R^{25}(\sigma^-)$$

$$= \tilde{x} + 2\alpha' p^{25}\sigma + i\sqrt{2\alpha'}\sum_{n\neq 0}\frac{1}{n}\,\hat{\alpha}_n e^{-in\tau}\sin(n\sigma)$$

$$= \tilde{x} + 2\alpha'\frac{m}{R}\sigma + \ldots = \tilde{x} + 2m\tilde{R}\sigma + \ldots$$

→ no term linear in τ ie the dual string has no momentum in the circle direction (translation invariance along S^1 is broken)

→ Moreover dual string wraps around the dual circle m times

Boundary conditions of the dual string: at $\sigma = 0, \pi$

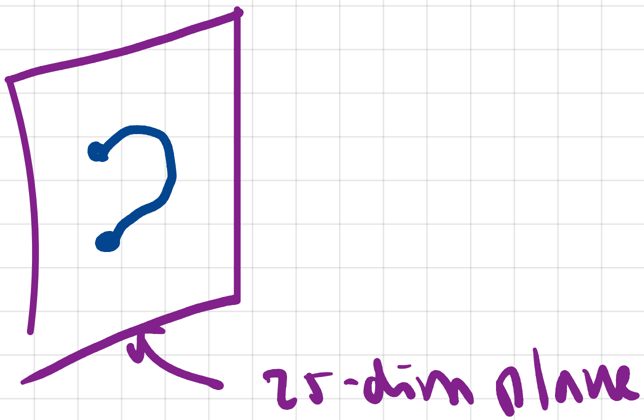
$$\left.\tilde{X}^{25}(\tau,\sigma)\right|_{\sigma=0} = \tilde{x}$$

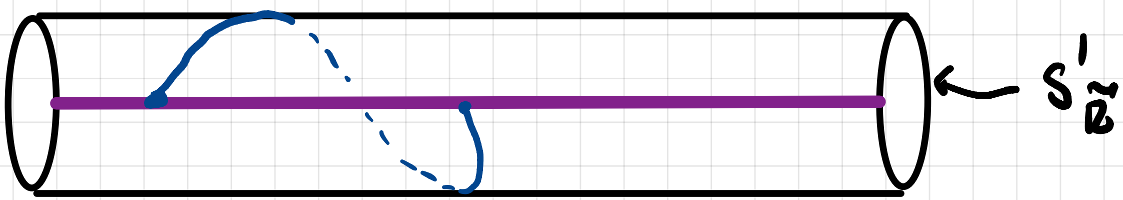
$$\left.\tilde{X}^{25}(\tau,\sigma)\right|_{\sigma=\pi} = \tilde{x} + 2\alpha' \frac{m}{R}\pi = \tilde{x} + 2\pi m \tilde{R}$$

position of the end points of dual string are fixed

→ This is a Dirichlet boundary condition!

The dual open string attached to a (1+24) dimensional hyperplane, a $\boxed{\text{D24-brane}}$





Under a T-duality transformation:

open string with Newman boundary condition on S^1_R $\longleftrightarrow$ open string with Dirichlet boundary condition on $S^1_{\tilde{R}}$

[momentum $\frac{m}{R}$ along S^1_R | no momentum along $S^1_{\tilde{R}}$

no winding around S^1_R | winding around $S^1_{\tilde{R}}$]

subspace where the string ends are attached to is called a D-brane

convention: a Dp-brane is a D-brane with p spatial dimensions (so it is $p+1$ dimensional)