STRING THEORY I

Lecture 16
(final lecture)

17 Epilogue: D-branes

Last lecture: we defined a Dp-brane as a $(p+1)$-dimensional subspace of target space where the ends of open strings can end

We saw how D-branes appear from T-duality

strings with

Neumann boundary conditions $\longleftrightarrow$ Dirichlet boundary conditions

Today: a number of observations about Dbranes

Last lecture

T-duality
$\longleftrightarrow$

open string with Neumann boundary conditions compactified on S^1_R

D25 space-filling brane
↳ open string ends are free to move on spacetime

$p^{25} = \frac{m}{R}$ quantized

no winding

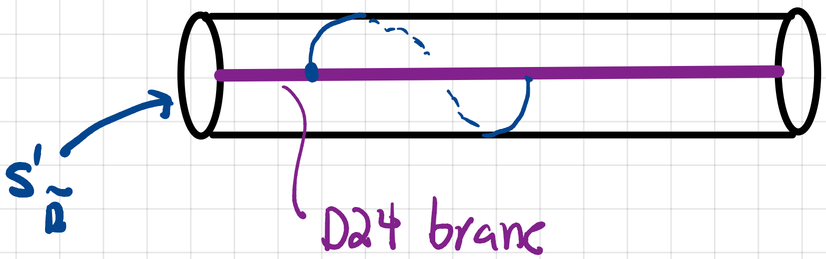
massless sector: (both sides)
25 dimensional $U(1)$ gauge field

dual open string with Dirichlet boundary conditions compactified on $S^1_{\tilde{R}}$, $\tilde{R} = \alpha'/R$

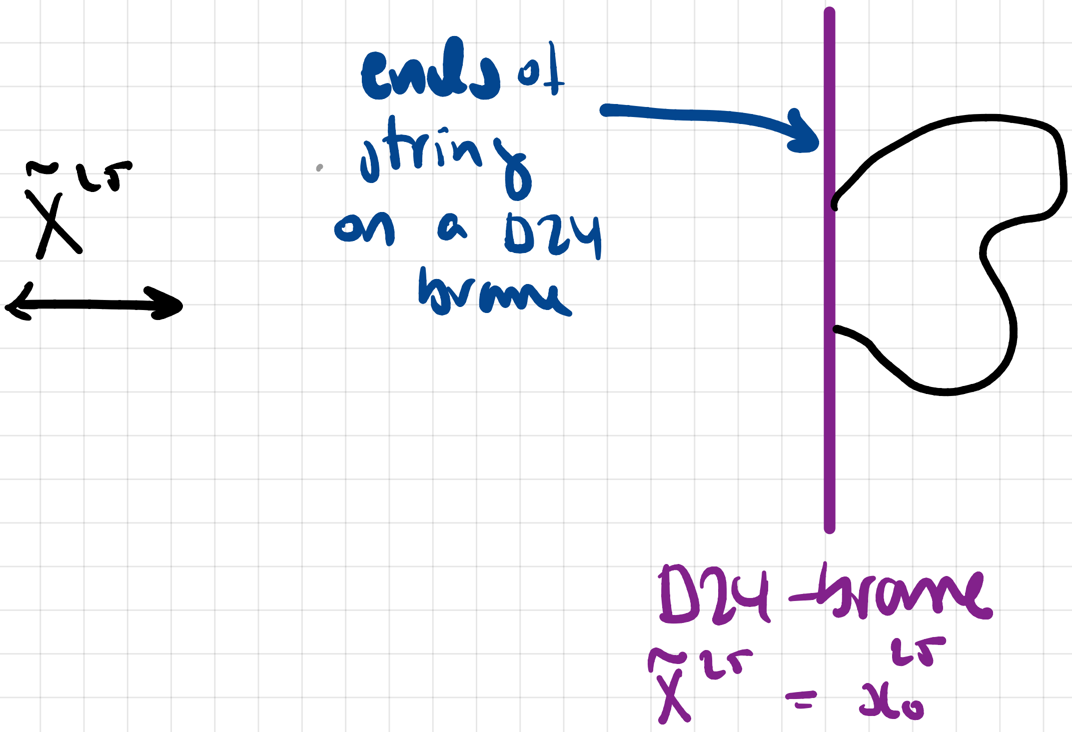
endpoints of the string live on a D24 brane

no translational symmetry along $S^1_{\tilde{R}}$

string can wind around $S^1_{\tilde{R}}$



Consider an open string on $\mathbb{R}^{1,25}$ with Dirichlet boundary conditions (no compactification) in one direction (X^{25}) and Neumann boundary in all other directions (X^i $i=0,\dots,24$)



more generally can consider an open string with $26-(p+1)$ D bcs on $(p+1)$ Neumann bcs
In this case: end of the string end on Dp brane
$SO(1,25)$
$\longrightarrow SO(1,p) \times SO(25-p)$

$\rightarrow$ no translational symmetry along $\tilde{X}^{25}$
symmetry $SO(1,24)$

Mode expansion for $\tilde{X}^\mu(\tau, \sigma)$:

Nbc $\tilde{X}^i(\tau, \sigma) = x^i + \tau p^i + i \sum_{n \neq 0} \frac{1}{n} \alpha_n^i \cos(n\sigma)$ $i = 0, \ldots, 24$

Dbc $\tilde{X}^{25}(\tau, \sigma) = x_0^{25} + i \sum_{n \neq 0} \frac{1}{n} \alpha_n^{25} e^{-in\tau} \sin(n\sigma)$

$\boxed{\text{no } \alpha_0^{25} \text{ mode}}$!

Otherwise, Virasoro operators as before

Mass-shell condition: $L_0 - 1 = \left(\frac{1}{2} p^2 + N\right) - 1$

becomes $M_{25}^2 = -|p|^2 = 2(N-1)$

$|p|^2 = p \bullet p$

(1+24)-dim inner product

Massless spectrum: must be level $N=1$

$$|\xi, \eta; K\rangle = (\xi \cdot \alpha_{-1} + \eta \alpha_{-1}^{25}) \overbrace{|0; K\rangle}$$

ground state $(N=0)$ $M_{25}^2 = -2$

25 dim momentum

25 dim momentum

(1+24)-dim polarization vector

spacetime scalar

Imposing $L_1 |\xi, \eta; K\rangle = 0$

$\langle m | \cdots \rangle = 0$
$m > 0$

we find that $|\xi, \eta; K\rangle$ is physical if

$\xi \cdot K = 0$ and η unconstrained.

(1+24)-dim inner prod

$$L_1 |\xi, \eta; K\rangle = \left(\xi_i \left([L_1, \alpha_{-1}^i] + \alpha_{-1}^i L_1 \right) + \eta \left([L_1, \alpha_{-1}^{25}] + \alpha_{-1}^{25} L_1 \right) \right) |0; K\rangle$$

$$= \left(\xi \cdot \alpha_0 + \cancel{\eta \alpha_0^{25}} + (\xi \cdot \alpha_{-1} + \eta \alpha_{-1}^{25}) L_1 \right) |0; K\rangle$$

$$= (\xi \cdot K + \cancel{(\xi \cdot \alpha_{-1} + \eta \alpha_{-1}^{25}) (\alpha_1 \cdot \alpha_0)}) |0; K\rangle = (\xi \cdot K) |0; K\rangle$$

null states at level one of the form $L_{-1}|0;K\rangle$

$$L_{-1}|0;K\rangle = K\cdot\alpha_{-1}|0;K\rangle \quad \text{with} \quad K\cdot K = 0$$

$$L_{-1}|0;K\rangle = \frac{1}{2}\sum_n \alpha_{-1-n}\cdot\alpha_n|0;K\rangle = \frac{1}{2}\sum_{n\leq 0}\alpha_{-1-n}\,\alpha_n|0;K\rangle$$

$$= \frac{1}{2}\Big(2\alpha_{-1}\cdot\alpha_0 + \cancel{\sum_{n\geq 2}\alpha_{-1+n}\,\alpha_{-n}}\Big)|0;K\rangle = \alpha_{-1}\cdot K\,|0;K\rangle$$

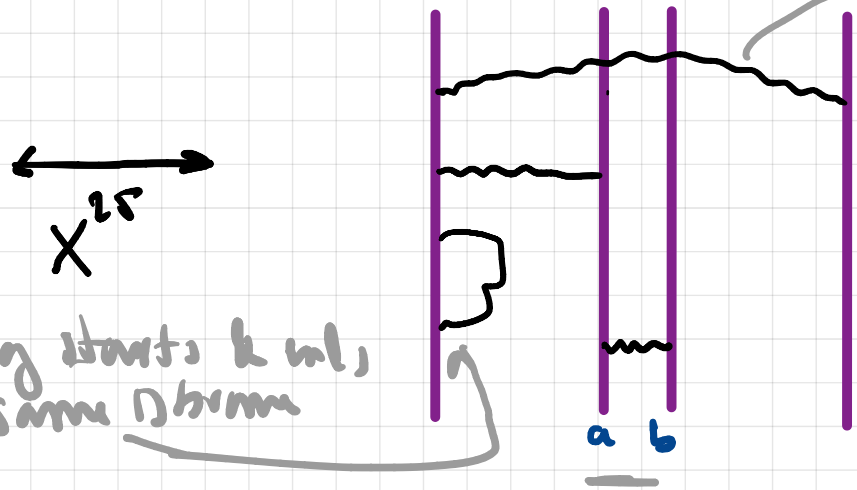
Then we have the physical states

- 25-dimensional photon $\quad \zeta\cdot\alpha_{-1}|0;K\rangle$ $\quad$ u(1) gauge field
  $\zeta\cdot K = 0$
- scalar field $\quad \phi = \eta\,\alpha_{-1}^{25}|0;K\rangle$

can be identified with fluctuations in the position of the D-brane along the transverse X^{25} direction

(no proof here)

One can also have systems of D-branes



For stretched strings between two branes

$$X^{25}_{ab} = x_a^{25} + \frac{1}{\pi}\underbrace{(X_b^{25} - X_a^{25})}_{\Delta X_{ab}}\sigma + \sum_{n\neq 0}\frac{1}{n}\alpha_n^{25} e^{-in\tau}\sin(n\sigma)$$

$X^{25}_{ab}(\sigma=0) = x_a$
$X^{25}_{ab}(\sigma=\pi) = x_b$

$$\alpha_0^{25} = \frac{1}{\pi}(X_b^{25} - X_a^{25})$$

mass-shell condition: $$M^2_{ab} = -p\cdot p = \underbrace{\left(\frac{X_b^{25} - X_a^{25}}{2\pi\alpha'}\right)^2}_{(T_s \Delta x)^2} + \frac{1}{\alpha'}(N-1)$$

$= $ mass² of a classical string stretched between the branes

shift of mass levels!

spectrum of stretched string:

info re which Dbrane string ends live

Chan Paton labels

- $\underline{N=0}$ $\quad |K, ab\rangle$

$$M_{ab}^2 = -\frac{1}{\alpha'} + \left(\frac{\Delta X_{ab}}{2\pi\alpha'}\right)^2$$

tachyon if $\quad |\Delta X_{ab}| < 2\pi\sqrt{\alpha'}$

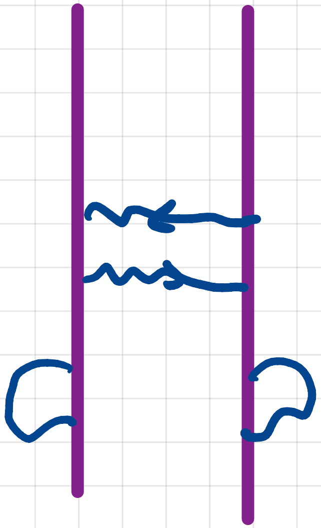
- $\underline{N=1}$ $\qquad M_{ab}^2 = \left(\frac{\Delta X^{25}}{2\pi\alpha'}\right)^2$

$$\left.\begin{array}{l} S\cdot\alpha_{-1}|K, ab\rangle \\ \eta\,\alpha_{-1}^{25}|K, ab\rangle \end{array}\right\}$$ massive vector on D24-brane

null states

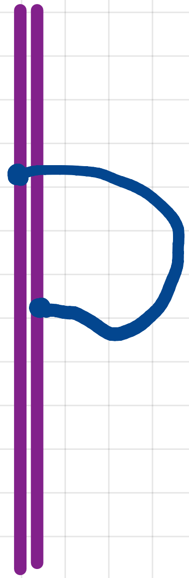
$$L_{-1}|0;K;ab\rangle = K\cdot\alpha_{-1}|K;ab\rangle + \frac{\Delta X_{ab}}{\pi}\eta\,\alpha_{-1}^{25}|K;ab\rangle$$

Coincident limit: suppose we have N D-branes



a $\qquad$ b

U(1) $\qquad$ U(1)



stretched string state between Dbrane at x_a & D-brane at x_b $\rightsquigarrow$ massless gauge fields

massive vector fields $\quad |K^i, ab\rangle$ $\qquad \xrightarrow{\Delta x_{ab} \to 0}$ $\qquad \alpha^i_{-1} |k; ab\rangle$ + scalars

$a, b = 1, \dots, N$

Chan-Paton labels

One can show that the spectrum has a $U(N)$ symmetry and that these states are indeed gauge fields in the adjoint representation of $U(N)$

One can choose a basis for these states (N^2 of them)

$$|\rho, k; A\rangle = \sum_{a,b} (t^A)^a{}_b \, |\rho, k; ab\rangle$$

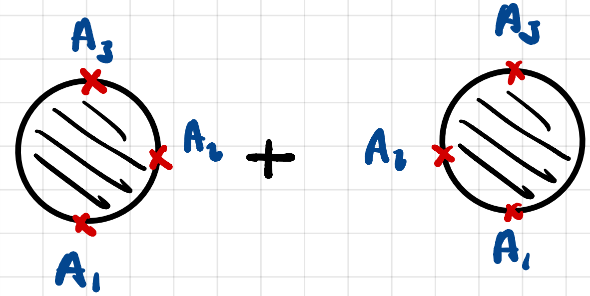
$A = 1, \ldots, N^2$

basis of $u(N)$ hermitian

$\mathrm{tr}\,(t^A t^B) = \delta^{AB}$

Chan-Paton factors

3-point coupling of massless vectors



$$\mathcal{A}(\zeta_1, k_1, A_1;\ \zeta_2, k_2, A_2;\ \zeta_3, k_3, A_3)$$

$$\sim g_o\, \delta(k_1+k_2+k_3)\Big\{ \zeta_1\cdot k_{23}\ \zeta_2\cdot\zeta_3 + \zeta_2\cdot k_{31}\ \zeta_1\cdot\zeta_3 + \zeta_3\cdot k_{12}\ \zeta_1\cdot\zeta_2$$

$$+ \frac{\alpha'}{2}\zeta_1\cdot k_{23}\ \zeta_2\cdot k_{31}\ \zeta_3\cdot\zeta_{12} \Big\} \times \mathrm{tr}\left(t^{a_1}[t^{a_2}, t^{a_3}]\right)$$

gives the 3-point vertex for the $U(N)$ non-Abelian gauge theory

$$\mathcal{L} = -\frac{1}{4}\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu}) - \frac{2i}{3}\alpha'\,\mathrm{Tr}(F_\mu{}^\nu F_\nu{}^\omega F_\omega{}^\mu) + \text{scalars}$$

EFT action on D24 brane

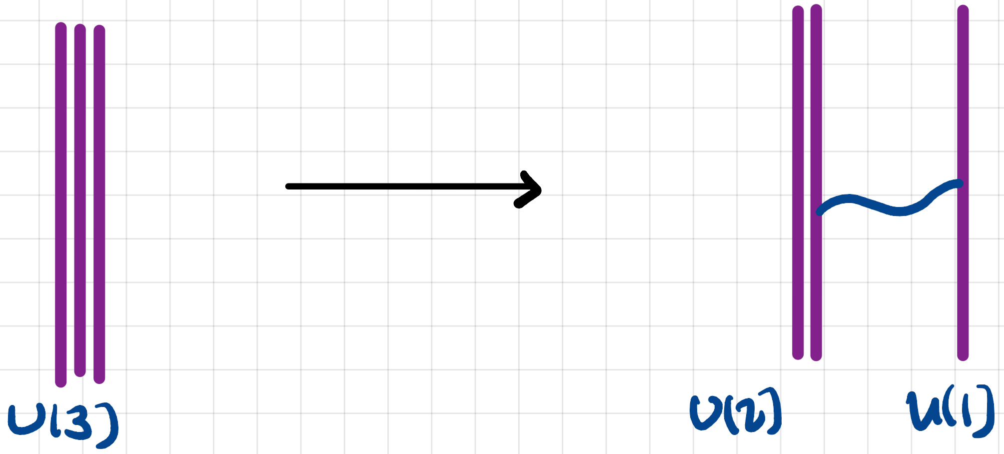
Yang-Mills

α' corrections

One can also obtain this from the β-function

(needs boundary couplings and boundary renormalization flow)

A picture of the Higgs mechanism



stack of 3 Dbranes

$$M_{ab}^2 = \left(\frac{|\Delta x|}{2\pi\alpha'}\right)^2$$

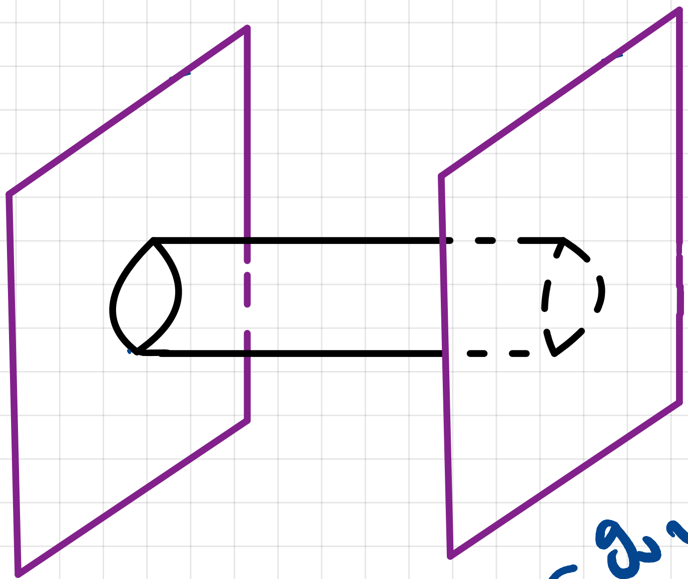
mass of string stretched between branes

D-branes as dynamical objects?

If they are, maybe they need to be included in the perturbative description of strings? how?

Estimate mass scale relevant to D-branes by computing its tension

closed string exchange between two Dbranes



gravitational coupling $K \sim g_0^2$

$$A \sim K^2 \, T_p^2 \sim (g_0)^0$$

D-brane tension   genus 0

$$T_p \sim \frac{1}{g_0^2} \Rightarrow$$ D-branes are massive "non perturbative" objects

$g_0 \sim g_0^2$

What happens then as $g_s \to \infty$?

Polchinski 1995, "duality revolution"

Final remarks

We have seen that the theory of quantized strings has a very rich structure

- dimension of spacetime fixed by consistency
- quantized gravity → at low energies Einstein's gravity
- gauge fields
- CFT
- S matrix with good UV behavior
- duality
- emergence of non-perturbative branes

String theory 2:

Superstrings (fermions in 2dim NLσM supersymmetry)

↳ tachyon removal

dimension of space time $D=10$

Beyond: strong coupling

black hole physics

phenomenology: KK approach

might be incomplete

End of String Theory I

Thanks!