Homework 4: pricing of standard and barrier options on interest rates

When implementing the options below, you can assume that all the barrier and exercise times are strictly greater than the initial time. The issue time coincides with the initial time.

Interest rate collar

$N:$ the notional.

$C:$ the cap rate

$F:$ the floor rate, $F < C$.

$\delta t:$ the interval of time between the payments given as year fraction.

$M:$ the total number of payments.

$t_0:$ the initial time.

The payment times are given by

$$t_i = t_0 + i\delta t, \quad i = 1, \dots, M.$$

At payment time t_i, we receive cap payment

$$N \max(L(t_{i-1}, t_i)\delta t - C\delta t, 0)$$

and make floor payment

$$N \max(F\delta t - L(t_{i-1}, t_i)\delta t, 0).$$

Here, $L(t_{i-1}, t_i)$ is the market interest rate computed at t_{i-1} for maturity t_i.

American swaption

$(t_i)_{i=1,\dots,n}:$ the exercise times.

Parameters of underlying swap:

$N:$ the notional.

$R:$ the fixed rate.

$\delta t:$ the interval of time between the payments.

$m:$ the total number of payments.

side: the side of the swap contract, that is, whether one pays "fixed" and receives "float" or otherwise.

A holder of the option can enter into the underlying swap agreement at any exercise time t_i. This time then becomes the issue time of the swap.

Putable and callable bond

Coupon bond:

$N:$ notional

$R:$ coupon rate

$\delta t:$ interval of time between the payments given as year fraction.

$m:$ total number of coupon payments.

$U:$ the repurchase price of the bond as percentage of the notional. After the coupon payment the issuer of the bond can repurchase the bond from the holder by paying amount NU. Typically, this payment is greater than the notional ($U > 1$).

$L:$ the redemption price of the bond as percentage of the notional. After the coupon payment the holder of the bond can sell it back to the issuer for amount LN. Typically, this amount is less than the notional ($L < 1$).

Denote by t_0 the current time and by $(t_i)_{1\leq i\leq m}$ the future coupon times:

$$t_i = t_0 + i\delta t, \quad 1 \leq i \leq m.$$

1. At maturity $T = t_m$, if the bond has not been terminated before, the owner of the bond receives coupon $RN\delta t$ and notional N.

2. At coupon time t_i other than the maturity, if the bond has not been terminated before,

(a) the owner of the bond receives coupon $RN\delta t$.

(b) the owner of the bond has the right to redeem the bond. In this case he receives amount LN from the issuer of the bond and the bond is terminated.

(c) the issuer of the bond has the right to repurchase the bond. In this case the holder of the bond receives amount UN and the bond is terminated.

Note that the events take place in their respective order, i.e. first 2a, then 2b and finally 2c.

American put on futures price of zero-coupon bond

$N:$ notional amount of the bond.

$U:$ maturity of the bond.

$T:$ maturity of the futures contract ($T < U$)

$m:$ the number of futures times after the issue

$K:$ the strike of the put option

Denote by

$$\delta t = \frac{T - t_0}{m}$$

the time difference between two adjacent futures times, where t_0 is the initial time.

We assume that the set of the exercise times for the option has the form:

$$t_i = t_0 + i\delta t, \quad 0 < i \leq m,$$

(includes T). The holder of the American put option on the futures price can exercise it at any time t_i in which case he receives the payment

$$V_{t_i} = \max(K - F(t_i), 0),$$

where $F(t_i)$ is the futures price at t_i, and the option is canceled.

Recall that a long position in the futures contract assumes

1. payments $F(t_i) - F(t_{i-1})$ at t_i if $1 \leq i < m$,
2. the purchase of the zero-coupon bond at $t_m = T$ for $F(t_{m-1})$.