Mathematical Institute
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Sample Exam 2
Financial Computing with C++
Hilary Term, 2021

- Your grade will be based on the best 3 solutions.
- Implement every function in a separate *.cpp file. Thus, you will submit 4 *.cpp files and output text file SampleExam2.txt. *You have to submit the output file to avoid a failing grade.*
- While implementing the functions below, you need to account for the singularities of the type 0/0.
- The issue time for all options coincides with the initial time. The maturities, barrier, and exercise times are strictly greater than the initial time.

Interpolation of data curves

Volatility curve obtained by the linear interpolation of the variance curve

Input:

$(t_i)_{i=1,\dots,M}$: the maturities, $t_1 > t_0$;

$(V(t_i))_{i=1,\dots,M}$: the market volatilities;

t_0 : the initial time.

Output: volatility curve $V = V(t)$ on $[t_0, t_M]$ obtained by the linear interpolation of variance curve

$$D(t) = (t - t_0)V^2(t), \quad t \in [t_0, t_M].$$

In particular, the volatility is constant on $[t_0, t_1]$:

$$V(t) = V(t_1), \quad t \in [t_0, t_1].$$

Least-squares fitting of data curves

Discount curve for Hull and White model obtained by least squire fitting of market yields

Input:

$(t_i)_{i=1,\dots,M}$: the maturities of market discount factors, $t_i < t_{i+1}$.

$(d_i)_{i=1,\dots,M}$: the market discount factors.

$\lambda \geq 0$: the mean reversion rate.

t_0 : the initial time, $t_0 < t_1$.

Output:

$d = d(t)$: the fitted discount curve with constant yield.

$\epsilon = \epsilon(t)$: the error function of the fit for the discount curve.

$((c_0, c_1), \Gamma, \chi^2)$: the fitted constants, their covariance matrix, and the total fitting error.

The discount curve is given by

$$d(t) = \exp(-\gamma(t)(t - t_0)), \quad t \geq t_0,$$

where yield curve $\gamma = \gamma(t)$ has the form

$$\gamma(t) = c_0 + c_1 \frac{1 - e^{-\lambda(t-t_0)}}{\lambda(t - t_0)}, \quad t \geq t_0. \tag{1}$$

Constants c_0 and c_1 are chosen to provide the best least-squares fit to market yields

$$\gamma_i = -\frac{\log d_i}{t_i - t_0}, \quad i = 1, \dots, M.$$

The problem is motivated by the fact that (1) represents the *minimal* family of yield curves that is generated by the Hull and White model with mean-reversion rate λ and that contains all constant yield curves.

Options on a single stock

"BOOST" (Banking On Overall Stability) option

The following option was introduced by Societe Generale.

$N:$ the notional amount.

$L:$ the lower barrier.

$U:$ the upper barrier.

$(t_i)_{1\le i\le m}:$ the barrier times.

The option terminates at the first barrier time, when the price of the stock hits either of the barriers, that is, at the barrier time t_{i^*}, whose index

$$i^* = \min\{1 \le i \le m : \quad S(t_i) > U \text{ or } S(t_i) < L\}.$$

At the exit time t_{i^*} the holder receives the payoff $N\frac{i^*-1}{m}$ (the product of the notional amount on the percentage of the barrier times the price of the stock spends inside two barriers).

If the price of the stock never exits the barriers, then at the last barrier time the holder receives notional amount N.

Options on interest rates

Putable bond with resettable coupon

The following bond is a variant of the so-called *ratchet bonds*.

$N:$ the notional.

$c:$ the initial coupon rate.

$d:$ the reset value for the coupon rate ($d < c$).

$\delta t:$ the interval of time between the payments given as year fraction.

$m:$ the total number of coupon payments.

L : the redemption price of the bond as percentage of the notional. Typically, $L < 1$.

Brief description: after a coupon payment the issuer can reset the coupon rate from original (higher) value c to (lower) reset value d. However, then at any payment time greater or equal the reset time and less than maturity the holder can sell the bond back to the issuer for redemption value LN.

Denote by t_0 the current time and by $(t_i)_{1 \leq i \leq m}$ the future coupon times:

$$t_i = t_0 + i\delta t, \quad 1 \leq i \leq m.$$

Depending on the past there are the following possibilities:

1. If $t_i < t_m$ (not the maturity) and if the coupon rate has not been reset before, then
   (a) the holder receives original coupon $Nc\delta t$
   (b) the issuer can reset original coupon rate c to lower coupon rate d and if he does so, then the holder has the right to sell the bond back to the issuer at redemption price LN.
2. If $t_i < t_m$ (not the maturity) and if the coupon rate has been already reset to d, but the bond has not been terminated, then
   (a) the holder receives reset coupon $Nd\delta t$,
   (b) the holder can sell the bond back to the issuer for redemption amount LN.
3. If $t_i = t_m$ (the maturity) and the bond has not been terminated before, the holder of the bond receives coupon payment (original or reset) as well as notional amount N.