Examples for the course “Financial Computing with C++”

Data curves for financial models

While implementing the functions below, you need to account for the singularities of the type 0/0.

Yield shape curve 1

Input:

$\lambda \geq 0:$ the mean-reversion rate.

$t_0:$ the initial time given as year fraction.

Output: the function

$$\Gamma(t) = \frac{1 - e^{-\lambda(t-t_0)}}{\lambda(t-t_0)}, \quad t \geq t_0,$$

appearing in the descriptions of yield curves for various financial models (Hull-White, Nelson-Siegel, Svensson, Vasicek).

Yield shape curve 2

Input:

$\lambda \geq 0:$ the mean-reversion rate.

$t_0:$ the initial time given as year fraction.

Output: the function

$$\Gamma(t) = \frac{1 - \exp(-\lambda(t-t_0))}{\lambda(t-t_0)} - \exp(-\lambda(t-t_0)), \quad t \geq t_0,$$

appearing in the descriptions of yield curves for various financial models (Nelson-Siegel, Svensson).

Forward curve for exchange rate

Input:

$S(t_0)$: the spot exchange rate (the number of units of domestic currency per one unit of foreign currency).

$D^d = (D^d(t))_{t \geq t_0}$: the discount curve in domestic currency.

$D^f = (D^f(t))_{t \geq t_0}$: the discount curve in foreign currency.

Output:

$F = (F(t))_{t \geq t_0}$: the forward FX rates.

Forward price curve for an annuity

Input:

q : the coupon rate.

δt : the time interval between coupon payments.

T : the maturity.

$D = (D(t))_{t \geq t_0}$: the discount curve.

t_0 : the initial time.

bClean : the boolean parameter specifying the type of the prices: "clean" or "dirty". The dirty price is the actual amount paid in a transaction. The clean price is the difference between the dirty price and the accrued interest. If t_i is the previous coupon time (or the initial time if no coupons have been paid so far) and t is the settlement time, then the accrued interest is given by

$$A(t) = q(t - t_i).$$

Output:

$F = (F(t))_{t \in [t_0, T]}$: the forward prices for the annuity.

The annuity pays coupons $q\delta t$ at times $(t_i)_{i=1,\dots,M}$ such that

$$t_0 < t_1 \leq t_0 + \delta t, \quad t_{i+1} - t_i = \delta t, \quad t_M = T.$$

The buyer pays forward price $F(t)$ at delivery time t and then receives coupons $q\delta t$ at payments times $t_i > t$.

Interpolation of data curves

Discount curve obtained by log linear interpolation

Input:

$$
\begin{aligned}
&(t_i)_{i=1,\dots,M} : \text{ the maturities, } t_i < t_{i+1},\\
&(d_i)_{i=1,\dots,M} : \text{ the discount factors,}\\
&t_0 : \text{ the initial time, } t_0 < t_1.
\end{aligned}
$$

Output: the discount curve

$$d(t) = \exp(l(t)), \quad t \in [t_0, t_M],$$

where function $l = l(t)$ is the linear interpolation of the logs of the market discount factors:

$$l(t) = LinInterp\,((t_i)_{i=0,1,\dots,M}, (\log d_i)_{i=0,\dots,M}), \quad d_0 = 1.$$

Forward curve obtained by interpolation of cost-of-carry rates

Input:

$$
\begin{aligned}
&S_0 : \text{ the spot price,}\\
&(t_i)_{i=1,\dots,M} : \text{ the maturities, } t_i < t_{i+1},\\
&(F_i)_{i=1,\dots,M} : \text{ the forward prices,}\\
&c_0 : \text{ the initial short-term cost-of-carry rate.}\\
&t_0 : \text{ the initial time, } t_0 < t_1,\\
&\mathcal{I} : \text{ an interpolation method.}
\end{aligned}
$$

Output: the forward curve

$$F(t) = S_0 \exp(q(t)(t - t_0)), \quad t \in [t_0, t_M],$$

where the cost-of-carry function $q = q(t)$ is the $\mathcal{I}$-interpolation of the market cost-of-carry rates:

$$
\begin{aligned}
q(t) &= \mathcal{I}((t_i)_{i=1,\dots,M}, (q_i)_{i=0,1,\dots,M}),\\
q_0 = c_0, &\quad q_i = \frac{\log(F_i/S_0)}{t_i - t_0}, \quad i = 1, \dots, M.
\end{aligned}
$$

Least-squares fitting of data curves

Discount curve by fitting of yields

Input:

$(t_i)_{i=1,\ldots,M}:$ the maturities, $t_i < t_{i+1}$.

$(d_i)_{i=1,\ldots,M}:$ the discount factors.

$t_0:$ the initial time, $t_0 < t_1$.

$\mathcal{L}:$ a fitting method for yields.

Output:

$d = d(t):$ the fitted discount curve.

$\epsilon = \epsilon(t):$ the error function of the fit for the discount curve.

The discount curve has the form:

$$d(t) = \exp(-\gamma(t)(t - t_0)), \quad t \geq t_0,$$

where yield curve $\gamma = \gamma(t)$ is the result of $\mathcal{L}$-fit of the market yields:

$$\gamma(t) = \mathcal{L}((t_i)_{i=1,\ldots,M}, (\gamma_i)_{i=1,\ldots,M}),$$
$$\gamma_i = -\frac{\log d_i}{t_i - t_0}, \quad i = 1, \ldots, M.$$

Discount curve by the Nelson-Siegel fit of yields

Input:

$(t_i)_{i=1,\ldots,M}:$ the maturities, $t_i < t_{i+1}$.

$(d_i)_{i=1,\ldots,M}:$ the discount factors.

$\lambda \geq 0:$ the mean-reversion rate.

$t_0:$ the initial time, $t_0 < t_1$.

Output:

$d = d(t):$ the fitted discount curve.

$\epsilon = \epsilon(t)$: the error function of the fit for the discount curve.

$((c_i), \Gamma, \chi^2)$: the fitted constants, their covariance matrix, and the total fitting error.

The discount curve is given by

$$d(t) = \exp(-\gamma(t)(t - t_0)), \quad t \geq t_0,$$

where yield curve $\gamma = \gamma(t)$ has the Nelson-Siegel form:

$$\begin{aligned}\gamma(t; c_0, c_1, c_2) = c_0 + c_1 \frac{1 - e^{-\lambda(t-t_0)}}{\lambda(t - t_0)} \\ + c_2 \left(\frac{1 - e^{-\lambda(t-t_0)}}{\lambda(t - t_0)} - e^{-\lambda(t-t_0)} \right), \quad t \geq t_0,\end{aligned}$$

and constants c_0, c_1, c_2 are the result of the least-squares fit of the market yields:

$$\chi^2 = \min_{c_0, c_1, c_2} \sum_{i=1}^{M} (\gamma(t_i; c_0, c_1, c_2) - \gamma_i)^2,$$

$$\gamma_i = -\frac{\log d_i}{t_i - t_0}, \quad i = 1, \ldots, M.$$

Options on a single stock

Standard put

K : the strike.

T : the maturity.

The payoff of the option at the maturity is given by

$$V(T) = \max(K - S(T), 0),$$

where $S(T)$ is the price of the stock at T.

American put

K : the strike.

$(t_i)_{1 \le i \le N}$: the exercise times.

A holder of the option can exercise it at any time t_i. In this case he receives the payment:

$$f(t_i) = \max(K - S(t_i), 0),$$

where $S(t)$ is the price of the stock at time t.

Barrier up-or-down-and-out option

U : the upper barrier.

L : the lower barrier.

$(t_i)_{1 \le i \le M}$: the barrier times.

N : the notional.

The payoff of the option at maturity (last barrier time t_M) is given by notional amount N if the stock price stays between the lower and upper barriers for all barrier times. Otherwise, the option expires worthless.

Down-and-out american call

L : the lower barrier.

$(t_i)_{1 \le i \le M}$: the barrier times.

K : the strike.

$(u_i)_{1 \le i \le N}$: the exercise times, $u_N > t_M$.

The option behaves as the american call option with strike K and exercise times $(u_i)_{1 \le i \le N}$ until the first barrier time when the stock price hits lower barrier L. At this exit time the option is canceled.

Swing option

K : the strike.

$(t_i)_{1\le i\le N}$: the exercise times.

M : the maximal number of exercises, $M \le N$.

A holder of the option is given the right to purchase M stocks at price K per share. The transactions take place at exercise times. Only *one* stock can be bought at a particular exercise time, that is, to get n stocks the holder should use n *different* exercise times. Such options are actively traded on energy markets.

Options on interest rates

Interest rate cap

N : notional

C : cap rate

δt : interval of time between the payments given as year fraction.

m : total number of payments

We assume that today is the issue time of the cap and denote this time by t_0. The payment times of the cap are given by

$$t_i = t_0 + i\delta t, \quad 1 \le i \le m.$$

At payment time t_i the holder *receives*

$$N \max(L(t_{i-1}, t_i)\delta t - C\delta t, 0),$$

where $L(s,t)$ is the LIBOR computed at time s for maturity t.

Interest rate swap

N : the notional.

R : the fixed rate.

δt : the interval of time between the payments given as year fraction.

m : the total number of payments.

side: this parameter defines the side of the swap contract, i.e. whether one pays "fixed" and receives "float" or otherwise.

We assume that today is the issue date of the swap and denote this time by t_0. The payment times of the swap are given by

$$t_i = t_0 + i\delta t, \quad 1 \leq i \leq m.$$

At time t_{i+1},

1. One side pays "float" interest

$$NL(t_i, t_{i+1})\delta t,$$

   where $L(s, t)$ is the float (LIBOR) rate computed at s for maturity t.

2. Another side pays "fixed" interest

$$NR\delta t$$

We need to compute the present value of the swap.

Swaption

T : the maturity.

Parameters of underlying swap:

N : the notional.

R : the fixed rate.

δt : the interval of time between the payments given as year fraction.

m : the total number of payments.

side : this parameter defines the side of the swap contract, i.e. whether one pays "fixed" and receives "float" or otherwise.

At maturity T, the holder has the right to enter into the swap contract with the parameters defined above and issue time T.

Cancellable interest rate collar

N : the notional.

C : the cap rate.

F : the floor rate ($F < C$).

δt : the interval of time between the payments given as year fraction.

m : the total number of payments.

We assume that today is the issue time of the collar and denote this time by t_0. The payment times of the collar are given by

$$t_i = t_0 + i\delta t, \quad 1 \le i \le m.$$

At payment time t_i:

1. If LIBOR rate $L(t_{i-1}, t_i)$ is greater than cap rate C, then the holder *receives*
$$N\delta t(L(t_{i-1}, t_i) - C).$$
2. If LIBOR rate $L(t_{i-1}, t_i)$ is less than floor rate F, then the holder *pays*
$$N\delta t(F - L(t_{i-1}, t_i)).$$
3. After the payment is either received or paid, the holder *has the right to cancel* the contract. No payments will be made after that.

Down-and-out cap

Underlying cap :

- $N :$ the notional.
- $R :$ the cap rate.
- $\delta t :$ the interval of time between the payments given as year fraction.
- $m :$ the total number of payments.

$L :$ the lower bound for float (LIBOR) rate.

We assume that today is the issue time of the cap and denote this time by t_0. The payment times of the cap are given by

$$t_i = t_0 + i\delta t, \quad 1 \leq i \leq m.$$

The down-and-out cap generates the same cash flow as the interest rate cap up to (and including) the payment time, when the float rate drops below L. After that the option is terminated. In other words, if we denote by τ the first payment time t_i, when float rate $r(t_i, t_i + \delta t)$ between t_i and $t_i + \delta t$ is less then L, then for a payment time t_j:

1. If $t_j \leq \tau$, then the holder gets standard cap payment

$$N \max(r(t_{j-1}, t_j)\delta t - R\delta t, 0).$$

2. If $t_j > \tau$, then the holder gets nothing.

Futures on LIBOR

The futures contracts of these types are traded, for example, on EUREX, where the underlying is 3 month EURO LIBOR.

Input: the parameters of futures contract.

- $\Delta :$ the period for LIBOR given as year fraction ($\Delta = 0.25$ for 3 month LIBOR).
- $T :$ the maturity of futures contract.
- $m :$ the number of settlement times between today and the maturity.

Output: futures price $F(t_0)$ computed at initial time t_0.

We assume that the settlement times are given by

$$t_i = t_0 + i\delta t, \quad 0 < i \leq m,$$

where t_0 is the initial time and

$$\delta t = \frac{T - t_0}{m}.$$

Notice that the settlement times include T, but do not contain t_0.

The futures contract on LIBOR involves the following transactions:

1. It costs nothing to enter into either a long or a short position in the futures contract
2. At time t_i before maturity, $1 \leq i < m$,
   (a) the buyer (long position) pays futures price $F(t_{i-1})$ established at the previous trading day,
   (b) the seller (short position) pays futures price $F(t_i)$ established at the current trading day.
3. At maturity $T = t_m$
   (a) the buyer (long position) pays futures price $F(t_{m-1})$ established at previous trading day,
   (b) the seller (short position) pays

   $$F(t_m) = F(T) = 1 - L(T, T + \Delta),$$

   and $L(T, T + \Delta)$ is the LIBOR computed at time T for maturity $T + \Delta$.