Session 2: interpolation and least-squares fitting

While implementing the functions below, you need to account for the singularities of the type 0/0.

Interpolation of data curves

Discount curve obtained by interpolation of yields

Input:

$(t_i)_{i=1,\dots,M}$: the maturities, $t_i < t_{i+1}$,

$(d_i)_{i=1,\dots,M}$: the discount factors,

r_0 : the initial short-term interest rate.

t_0 : the initial time, $t_0 < t_1$,

$\mathcal{I}$: an interpolation method.

Output: the discount curve

$$d(t) = \exp(-\gamma(t)(t-t_0)), \quad t \in [t_0, t_M],$$

where the yield curve $\gamma = \gamma(t)$ is the $\mathcal{I}$-interpolation of the market yields to maturity

$$\gamma(t) = \mathcal{I}((t_i)_{i=0,1,\dots,M}, (\gamma_i)_{i=0,1,\dots,M}), \quad t \in [t_0, t_M],$$
$$\gamma_0 = r_0, \quad \gamma_i = -\frac{\log(d_i)}{t_i - t_0}, \quad i = 1, \dots, M,$$

Forward curve obtained by log linear interpolation

Input:

S_0 : the current spot price.

$(F_i)_{i=1,\dots,M}$: the market forward prices.

$(t_i)_{i=1,\dots,M}$: the maturities of forward contracts, $t_0 < t_1$.

t_0 : the initial time given as year fraction.

Output: the forward curve on $[t_0, t_M]$ obtained by log linear interpolation from the market spot and forward prices.

Least-squares fitting of data curves

Forward curve by fitting of cost-of-carry rates

Input:

S_0 : the market spot price.

$(t_i)_{i=1,\ldots,M}$: the maturities of the forward contracts, $t_i < t_{i+1}$.

$(F_i)_{i=1,\ldots,M}$: the market forward prices.

t_0 : the initial time, $t_0 < t_1$.

$\mathcal{L}$: a fitting method for cost-of-carry rates.

Output:

$d = d(t)$: the fitted forward curve.

$\epsilon = \epsilon(t)$: the error function of the fit for the forward curve.

The forward curve has the form:

$$F(t) = S_0 \exp(q(t)(t - t_0)), \quad t \geq t_0,$$

where cost-of-carry function $q = q(t)$ is the result of $\mathcal{L}$-fit of the market cost-of-carry rates:

$$\begin{aligned} q(t) &= \mathcal{L}((t_i)_{i=1,\ldots,M}, (q_i)_{i=1,\ldots,M}), \\ q_i &= \frac{\log(F_i/S_0)}{t_i - t_0}, \quad i = 1, \ldots, M. \end{aligned}$$

Forward curve by fitting of cost-of-carry rates in the Black model for commodities

Input:

S_0 : the market spot price.

$(t_i)_{i=1,\ldots,M}$: the maturities of the forward contracts, $t_i < t_{i+1}$.

$(F_i)_{i=1,\ldots,M}$: the market forward prices.

$\lambda \geq 0$: the mean-reversion rate.

$\sigma > 0:$ the short-term volatility.

$t_0:$ the initial time, $t_0 < t_1$.

Output:

$d = d(t):$ the fitted forward curve.

$\epsilon = \epsilon(t):$ the error function of the fit for the forward curve.

$(\theta, \Gamma, \chi^2):$ the fitted constant, its variance, and the total fitting error.

Returns the forward curve in the form:

$$F(t) = S_0 \exp(q(t)(t - t_0)), \quad t \geq t_0,$$

where cost-of-carry rate $q = q(t)$ fits the market cost-of-carry rates in the Black model for commodities. We recall that under the Black model, the log of spot price has the form:

$$\log S_t = \log S_{t_0} + X_t, \quad t \geq t_0,$$

where $X = (X_t)$ is an OU process:

$$dX_t = (\theta - \lambda X_t)dt + \sigma dB_t, \quad t \geq t_0, \quad X_0 = 0.$$

Here $B = (B_t)$ is a Brownian motion and drift θ is the fitting parameter. Computations show (check them!) that

$$q(t) = \theta \frac{1 - e^{-\lambda(t-t_0)}}{\lambda(t - t_0)} + \frac{\sigma^2}{2} \frac{1 - e^{-2\lambda(t-t_0)}}{2\lambda(t - t_0)}, \quad t \geq t_0.$$