Session 3: pricing of standard and barrier options on a stock

When implementing the options below, you can assume that all the barrier and exercise times are strictly greater than the initial time. The issue time coincides with the initial time.

Down-and-out call

L : lower barrier

$(t_i)_{1\leq i\leq M}$: barrier times

K : strike

T : maturity ($T > t_M$).

At maturity T a holder of the option is given the right to buy one stock at strike K if the spot price was above lower barrier L for all barrier times $(t_i)_{1\leq i\leq M}$. Otherwise, the option expires worthless.

Call on forward price

K : strike price

T : maturity of call option

U : delivery time of forward contract

The payoff of the option at the maturity is given by

$$V(T) = \max(F(T,U) - K, 0)$$

where $F(T,U)$ is the forward price computed at T for delivery U.

American butterfly

$K:$ middle strike

$W:$ size of wing $(W < K)$

$(t_i)_{1\leq i\leq N}:$ exercise times

A holder of the option can exercise it at any time t_i. In this case, he will receive the sum of the following payments:

1. long position in call with strike $K - W$,
2. 2 short positions in call with strike K,
3. long position in call with strike $K + W$.

If the holder does not exercise the option for all exercise times, then it expires worthless.

"Corridor" option

$N:$ notional amount.

$L:$ lower barrier.

$U:$ upper barrier

$(t_i)_{1\leq i\leq m}:$ barrier times

The last barrier time is the maturity of the option. The payoff of the option at maturity equals the product of the notional on the percentage of the barrier times, when the price of the stock is less than the upper barrier and is greater than the lower barrier, that is

$$V_{t_m} = N\frac{1}{m}\sum_{1\leq i\leq m} 1_{\{L<S(t_i)<U\}}$$

Here $S(t)$ is the price of the stock at t and 1_A is the indicator of event A.