Session 1: data curves for financial models

While implementing the functions below, you need to account for the singularities of the type 0/0.

Yield curve computed from discount curve

Input:

$D = (D(t))_{t \geq t_0}$: the discount curve.

t_0 : the initial time given as year fraction.

Output: continuously compounded yield curve $\gamma = (\gamma(t))_{t \geq t_0}$.

We recall that

$$D(t) = e^{-\gamma(t)(t-t_0)}, \quad t \geq t_0.$$

Implied volatility curve for the Black model

Input:

$\lambda \geq 0$: the mean-reversion rate.

$\sigma > 0$: the short-term volatility.

t_0 : the initial time given as year fraction.

Output: the stationary implied volatility curve for the Black model. It has the form:

$$\Sigma(t) = \sigma \sqrt{\frac{1 - \exp(-2\lambda(t-t_0))}{2\lambda(t-t_0)}}, \quad t \geq t_0.$$

The Nelson-Siegel yield curve

Input:

$(c_i)_{i=0,1,2}$: the constant coefficients of the Nelson-Siegel model of interest rates.

$\lambda > 0$: the mean-reversion rate.

t_0 : the initial time given as year fraction.

Output: the Nelson-Siege yield curve $\gamma = (\gamma(t))_{t\geq t_0}$. It has the form:

$$\gamma(t) = c_0 + c_1 \frac{1 - e^{-\lambda(t-t_0)}}{\lambda(t - t_0)} + c_2 \left(\frac{1 - e^{-\lambda(t-t_0)}}{\lambda(t - t_0)} - e^{-\lambda(t-t_0)} \right), \quad t \geq t_0.$$

Forward prices for a stock that pays dividends

Input:

$S(t_0)$: the spot price.

$(t_i)_{i=1,\dots,M}$: the dividend times, $t_1 > t_0$.

$(Q_i)_{i=1,\dots,M}$: the dividend payments.

$D = (D(t))_{t\geq t_0}$: the discount curve.

t_0 : the initial time.

Output:

$F = (F(t))_{t\in[t_0,t_M]}$: the forward prices for the stock.

The buyer pays forward price $F(t)$ at delivery time t and then receives the stock. If t is a dividend time, then the buyer gets the dividend paid at t.