Homework 1: data curves for financial models

While implementing the functions below, you need to account for the singularities of the type 0/0.

Volatility curve computed from variance curve

Input:

$V = (V(t))_{t\geq t_0}$: the variance curve.

t_0 : the initial time given as year fraction.

Output: continuously compounded volatility curve $\Sigma = (\Sigma(t))_{t\geq t_0}$.

We recall that

$$V(t) = \Sigma^2(t)(t - t_0), \quad t \geq t_0.$$

Cost-of-carry rate curve for the Black model

Input:

θ : the constant drift term (θ/λ is the mean-reversion level).

$\lambda \geq 0$: the mean-reversion rate.

$\sigma \geq 0$: the volatility.

t_0 : the initial time given as year fraction.

Output:

$q = (q(t))_{t\geq t_0}$: the continuously compounded curve of *cost-of-carry* rates.

In the Black model, the log of spot price evolves as

$$\log S_t = \log S(t_0) + X_t, \quad t \geq t_0,$$

where $X = (X_t)$ is an OU (Ornstein-Uhlenbeck) process driven by Brownian motion $B = (B_t)$:

$$dX_t = (\theta - \lambda X_t)dt + \sigma dB_t, \quad X(t_0) = 0.$$

We recall that the forward price curve has the form:

$$F(t) = S(t_0)\exp(q(t)(t-t_0)) = \mathbb{E}\left(S_t\right), \quad t \geq t_0.$$

Computations show (check them!) that cost-of-carry rate

$$q(t) = \theta\frac{1-e^{-\lambda(t-t_0)}}{\lambda(t-t_0)} + \frac{\sigma^2}{2}\frac{1-e^{-2\lambda(t-t_0)}}{2\lambda(t-t_0)}, \quad t \geq t_0.$$

The Svensson yield curve

Input:

$(c_i)_{i=0,1,2,3}$: the constant coefficients of the Svensson model of interest rates.

$(\lambda_i)_{i=1,2}$: the strictly positive mean-reversion rates, $\lambda_1 \neq \lambda_2$.

t_0 : the initial time given as year fraction.

Output: the Svensson yield curve $\gamma = (\gamma(t))_{t\geq t_0}$. It has the form:

$$\begin{aligned}\gamma(t) = c_0 + c_1\frac{1-e^{-\lambda_1(t-t_0)}}{\lambda_1(t-t_0)} + c_2\left(\frac{1-e^{-\lambda_1(t-t_0)}}{\lambda_1(t-t_0)} - e^{-\lambda_1(t-t_0)}\right) \\ + c_3\left(\frac{1-e^{-\lambda_2(t-t_0)}}{\lambda_2(t-t_0)} - e^{-\lambda_2(t-t_0)}\right), \quad t \geq t_0.\end{aligned}$$

Forward price curve for a coupon bond

Input:

q : the coupon rate.

δt : the time interval between coupon payments.

T : the maturity.

$D = (D(t))_{t\geq t_0}$: the discount curve.

t_0 : the initial time.

bClean : the boolean parameter specifying the type of the prices: “clean” or “dirty”. The dirty price is the actual amount paid in a transaction. The clean price is the difference between the dirty price and the accrued interest. If t_i is the previous coupon time (or the initial time if no coupons have been paid so far) and t is the settlement time, then the accrued interest is given by

$$A(t) = q(t - t_i).$$

Output:

$F = (F(t))_{t \in [t_0, T]}$: the forward prices for the bond.

The bond pays coupons $q\delta t$ at times $(t_i)_{i=1,\ldots,M}$ such that

$$t_0 < t_1 \leq t_0 + \delta t, \quad t_{i+1} - t_i = \delta t, \quad t_M = T.$$

The bond also pays notional $N = 1$ at maturity T. The buyer of the forward contract pays forward price $F(t)$ at delivery time t and then receives coupons $q\delta t$ at payments times $t_i > t$ and notional $N = 1$ at maturity T.