MATHEMATICAL INSTITUTE
UNIVERSITY OF OXFORD

SAMPLE EXAM 1
FINANCIAL COMPUTING WITH C++
HILARY TERM, 2021

- Your grade will be based on the best 3 solutions.
- Implement every function in a separate *.cpp file. Thus, you will submit 4 *.cpp files and output text file SampleExam1.txt. *You have to submit the output file to avoid a failing grade.*
- While implementing the functions below, you need to account for the singularities of the type 0/0.
- The issue time for all options coincides with the initial time. The maturities, barrier, and exercise times are strictly greater than the initial time.

Data curves for financial models

Yield curve for the Vasicek model of interest rates

Input:

θ : the constant drift term (θ/λ is the mean-reversion level).

$\lambda \geq 0$: the mean-reversion rate.

$\sigma > 0$: the volatility.

$r(t_0)$: the initial short-term interest rate.

t_0 : the initial time given as year fraction.

Output:

$\gamma = (\gamma(t))_{t \geq t_0}$: the continuously compounded yield curve in the Vasicek model of interest rates.

In the Vasicek model, short-term interest rate $r = (r_t)$ is an OU (Ornstein-Uhlenbeck) process driven by Brownian motion $B = (B_t)$:

$$dr_t = (\theta - \lambda r_t)dt + \sigma dB_t.$$

We recall that the discount curve has the form:

$$D(t) = e^{-\gamma(t)(t-t_0)} = \mathbb{E}\left(e^{-\int_{t_0}^t r_s ds}\right), \quad t \geq t_0.$$

Computations show (please, check) that

$$\gamma(t) = r(t_0)A(t) + \frac{\theta}{\lambda}(1 - A(t)) - \frac{\sigma^2}{2\lambda^2}(1 - 2A(t) + B(t)), \quad t \geq t_0.$$

where

$$A(t) = \frac{1 - e^{-\lambda(t-t_0)}}{\lambda(t - t_0)}, \quad B(t) = \frac{1 - e^{-2\lambda(t-t_0)}}{2\lambda(t - t_0)}.$$

Interpolation of data curves

Forward exchange curve obtained by the log-linear interpolation

Input:

S_0 : the current spot exchange rate.

$(t_i)_{i=1,\ldots,M}$: the maturities of discount factors, $t_0 < t_1$.

$(D_i^d)_{i=1,\ldots,M}$: the discount factors in domestic currency.

$(D_i^f)_{i=1,\ldots,M}$: the discount factors in foreign currency.

t_0 : the initial time given as year fraction.

Output: the forward curve on $[t_0, t_M]$ obtained by the log-linear interpolation of the market spot and forward FX rates.

Options on a single stock

Up-range-out put

U : the upper barrier.

$(t_i)_{1 \le i \le M}$: the barrier times. The first barrier time is greater than the initial time and the last one is less than the maturity.

N : the number of barrier events after which the option is canceled, $0 < N \le M$.

K : the strike.

T : the maturity ($T > t_M$).

The option is canceled immediately after the stock price has been above the upper barrier U for N barrier times. Otherwise, it behaves as standard European put option with maturity T and strike K.

Options on interest rates

Futures on cheapest bond to deliver

This problem is motivated by the existing futures contract on US treasury bonds.

The parameters of the futures contract:

T : the maturity.

m : the number of "future" times before the maturity.

Bonds to deliver with indexes $1 \le i \le n$. We assume that all the bonds are issued at T (the maturity of the futures contract). The parameters of the bond with index i have the form:

N_i : the notional.

R_i : the coupon rate.

$(\delta t)_i$: the interval of time between the payments given as year fraction.

m_i : the number of coupon payments.

Output: the futures price $F(t_0)$ at the initial time.

We denote by $F(t_i)$ the futures price at

$$t_i = t_0 + ih \quad 0 \le i \le m-1,$$

where

$$h = \frac{T - t_0}{m}.$$

We recall that

1. It costs nothing to enter the futures contract.
2. At a futures time t_i before the maturity, $1 \le i < m$,
   (a) the buyer (long position) pays previous futures price $F(t_{i-1})$.
   (b) the seller (short position) pays current futures price $F(t_i)$.
3. At maturity $T = t_m$,
   (a) the buyer (long position) pays previous futures price $F(t_{m-1})$.
   (b) the seller (short position) delivers one of the available coupon bonds. Note that the seller has the right to choose which bond to deliver.