Mathematical Institute
University of Oxford

Sample Exam 3
Financial Computing with C++
Hilary Term, 2021

- Your grade will be based on the best 3 solutions.
- Implement every function in a separate *.cpp file. Thus, you will submit 4 *.cpp files and output text file SampleExam3.txt. *You have to submit the output file to avoid a failing grade.*
- While implementing the functions below, you need to account for the singularities of the type 0/0.
- The issue time for all options coincides with the initial time. The maturities, barrier, and exercise times are strictly greater than the initial time.

Interpolation of data curves

Discount curve obtained from swap rates by log linear interpolation

Input:

δt : the time interval between payments given as year fraction.

$(R_i)_{i=1,\dots,M}$: the vector of swap rates, where R_i is the current swap rate in the contract with time interval δt and the number of payments i.

t_0 : the initial time given as year fraction.

Output: discount curve $D = D(t)$ on $[t_0, t_0 + M\delta t]$. It is obtained by the following procedure:

1. We compute discount factors d_i for maturities $t_i = t_0 + i\delta t$, $i = 1, \dots, M$.
2. We apply the log linear interpolation to (t_i, d_i), $i = 0, 1, \dots, M$, where $d_0 = 1$.

Least-squares fitting of data curves

Forward exchange curve by the Svensson fit of cost-of-carry rates

Input:

S_0 : the spot FX rate.

$(d_i^d)_{i=1,\dots,M}$: the discount factors in domestic currency.

$(d_i^f)_{i=1,\dots,M}$: the discount factors in foreign currency.

$(t_i)_{i=1,\dots,M}$: the maturities of discount factors, $t_0 < t_1$.

λ_1 : the first mean-reversion rate, $\lambda_1 > 0$.

λ_2 : the second mean-reversion rate, $\lambda_2 > 0$, $\lambda_2 \neq \lambda_1$.

t_0 : the initial time, $t_0 < t_1$.

Output:

$F = F(t)$: the fitted forward exchange curve.

$\epsilon = \epsilon(t)$: the error function of the fit for the forward exchange curve.

$((c_i), \Gamma, \chi^2)$: the fitted constants, their covariance matrix, and the total fitting error.

The forward exchange curve has the form:

$$F(t) = S_0 \exp(q(t)(t - t_0)), \quad t \geq t_0,$$

where cost-of-carry curve $q = q(t)$ has the Svensson form:

$$q(t) = c_0 + c_1 \frac{1 - e^{-\lambda_1(t-t_0)}}{\lambda_1(t - t_0)} + c_2 \left(\frac{1 - e^{-\lambda_1(t-t_0)}}{\lambda_1(t - t_0)} - e^{-\lambda_1(t-t_0)} \right) + c_3 \left(\frac{1 - e^{-\lambda_2(t-t_0)}}{\lambda_2(t - t_0)} - e^{-\lambda_2(t-t_0)} \right), \quad t \geq t_0,$$

and constants c_0, c_1, c_2, c_3 fit the market cost of carry rates.

Options on a single stock

Strike of variance swap

$T:$ the maturity,

$M:$ the number of times used in the computation of the variance.

We assume that initial time t_0 is the issue time for the swap and denote

$$t_i = t_0 + i\delta t, \quad 1 \leq i \leq M,$$

where

$$\delta t = \frac{T - t_0}{M}.$$

The payoff of the variance swap at maturity $T = t_M$ is given by

$$V(T) = \frac{1}{T - t_0} \sum_{i=1}^{M} \left(\ln \left(\frac{S(t_i)}{S(t_{i-1})} \right) \right)^2 - K^2,$$

where $S(t_i)$ is the spot price at t_i and K is the strike. Compute K using the fact, that it costs nothing to enter the variance swap. In other words, K^2 is the forward price on the annualized variance of the stock price.

Options on interest rates

Callable capped floater

$N:$ the notional.

$C:$ the cap rate.

$\delta t:$ the interval of time between the payments given as year fraction.

$m:$ the total number of payments.

$\delta L:$ the spread over Libor.

We assume that today is the issue time of the capped floater and denote this time by t_0. Let $L(s,t)$ be the LIBOR rate computed at s for maturity t. The payment times are given by

$$t_i = t_0 + i\delta t, \quad 1 \leq i \leq m.$$

At payment time t_i,

1. the holder *receives* coupon

$$N\delta t \times \min(L(t_{i-1}, t_i) + \delta L, C),$$

2. the seller of the option *has the right to cancel* the contract. In this case, in addition to the above coupon he pays the notional. No payments will be made in the future. Note that the option can not be terminated at issue time.

If the contract has not been terminated before, then at maturity t_m the holder receives the above coupon plus notional.