Practical Numerical Analysis: Sheet 2

1. In this problem we will approximate the values of four integrals

$$\text{(a)} \quad \int_0^1 4\pi x \sin(20\pi x)\cos(2\pi x)\mathrm{d}x = -20/99 \,,$$

$$\text{(b)} \quad \int_0^1 \sin(2\pi x)\cos(4\pi x)\mathrm{d}x = 0 \,,$$

$$\text{(c)} \quad \int_0^5 G(x)\mathrm{d}x = 7.5 \,, \qquad \text{where } G(x) = \begin{cases} x+1 & x < 1, \\ 3-x & 1 \le x \le 3, \\ 2 & x > 3. \end{cases}$$

$$\text{(d)} \quad \int_0^1 x^{3/2}\mathrm{d}x = 0.4 \,.$$

   Note that in Matlab you can implement $G(x)$ as:

```
G=@(x) (x+1).*(x<1)+(3-x).*(1<=x).*(x<=3)+2*(x>3);
```

   Compute each integral using the composite trapezium rule, the Clenshaw-Curtis rule and a Gauss-Legendre rule with `n=10:10:100`.

   Note that the Legendre polynomials are the orthogonal polynomials on $[-1, 1]$ with the unit weight function. The orthonormal Legendre polynomials are defined by $P_0(x) = 1/\sqrt{2}$, $P_1(x) = \sqrt{3/2}x$ and

$$xP_n(x) = \frac{1}{2}\frac{1}{\sqrt{1-1/(2(n+1))^2}}P_{n+1}(x) + \frac{1}{2}\frac{1}{\sqrt{1-1/(2n)^2}}P_{n-1}(x) \,,$$

   for $n = 1, 2, \ldots$.

   For each integral produce a plot showing the convergence of the error with n for each of the three different methods.

2. Use Romberg integration to compute the integral in 1(a) accurately.

3. (Optional) Use an adaptive composite trapezium rule to compute the integral in 1(a) accurately.

Further Reading

1. E. Süli and D. Mayers, *An Introduction to Numerical Analysis*, CUP, 2003.

2. L.N. Trefethen, *Is Gauss Quadrature Better than Clenshaw-Curtis?*, SIAM Review, vol 50, pages 67–87, 2008.

3. N. Hale and A. Townsend, *Fast and Accurate Computation of Gauss-Legendre and Gauss-Jacobi Quadrature Nodes and Weights*, SISC, vol 35, pages A652–A674, 2013.