Introduction to Cryptology

7.2 - Hash functions: Constructions and Applications

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How to Design a Hash Function?

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Merkle-Damgård transform is a very famous approach for domain extension.

- Used for MD5 and the SHA family.
- Theoretical implication: if you can compress by a single bit, then you can compress by an arbitrary amount of bits!

The Merkle-Damgård Transform

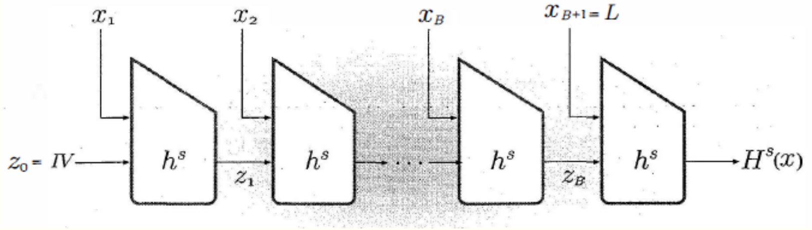
Let (KeyGen, h) be a fixed-length hash function, with $\ell'(n) = 2n$ and $\ell(n) = n$. Define an arbitrary-length hash function

$$(\text{KeyGen}, H)$$

as follows:

- $s \leftarrow \text{KeyGen}(n)$: KeyGen is the same for the two hash functions.
- $H^s(x) \leftarrow H(s, x)$: on input a key s and a string $x \in \{0, 1\}^*$ of length $L < 2^n$, it proceeds as follows:
  - x is padded with zeros to get a string of length $B \cdot n$;
  - $x = (x_1, \ldots, x_B)$ and $x_{B+1} := L$;
  - z_0 (also called IV) is set to 0^n;
  - $z_i := h^s(z_{i-1} || x_i)$, for $i = 1, \cdots, B+1$;
  - $H^s(x) := z_{B+1}$.

The Merkle-Damgård Transform



From [Katz-Lindell].

Theorem

If (KeyGen, h) *is collision-resistant, then so is* (KeyGen, H)*.*

The Merkle-Damgård Transform

Proof.

Let x and x' be two distinct strings s.t. $H^s(x) = H^s(x')$.

Assume $|x| = L$ and $|x'| = L'$. After the padding, $x = x_1, \cdots \cdots, x_{B+1}$ and $x' = x'_1, \cdots, x'_{B+1}$, with $x_{B+1} = L$ and $x'_{B'+1} = L'$.

- $\underline{L \neq L'}$: then $H^s(x) = z_{B+1} = h^s(z_B||L) = h^s(z'_{B'}||L') = z'_{B'+1} = H^s(x')$. Hence $z_B||L \neq z'_{B'}||L'$ is a collision for h^s.
- $\underline{L = L'}$: in this case $B = B'$. Consider $J_i = z_{i-1}||x_i$ and $J'_i = z'_{i-1}||x'_i$ for $i = 1, \ldots, B+2$, where

$$J_{B+2} = z_{B+1} = z'_{B+1} = J'_{B+2}.$$

  Let N be the largest integer s.t. $I_N \neq I'_N$ (which exists since $x \neq x'$). Since $N \leq B+1$, then $h^s(I_N) = z_N = z'_N = h^s(I'_N)$.

$\square$

MACs using Hash Functions

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The general idea is simple and widely used.

- Step 1: a collision-resistant hash function (KeyGen, H) is used to hash a message m into a fixed-length string $H^s(m)$.
- Step 2: a fixed-length MAC is applied to $H^s(m)$.

Hash-and-MAC

Let $S_{MAC} = (\text{KeyGen}_M, \text{Mac}, \text{Verify})$ be a fixed-length MAC for messages of length $\ell(n)$, and (KeyGen_H, H) a hash function with output length $\ell(n)$.

A MAC for arbitrary length messages

$$S'_{MAC} = (\text{KeyGen}', \text{Mac}', \text{Verify}')$$

can be defined as follows.

- $(k, s) \leftarrow \text{KeyGen}'(1^n)$: given a security parameter n, it runs KeyGen_M and KeyGen_H on input n, obtaining two keys, k and s. The output is (k, s).
- $t \leftarrow \text{Mac}'((k, s), m \in \{0, 1\}^*)$: $t := \text{Mac}_k(H^s(m))$.
- $1/0 \leftarrow \text{Verify}'((k, s), m, t)$: it outputs 1 if $\text{Verify}_k(H^s(m), t)$ is equal to 1, 0 otherwise.

Hash-and-MAC

Theorem

If S_{MAC} is a secure MAC for messages of length $\ell(n)$ and (KeyGen_H, H) is a collision-resistant hash function, then S'_{MAC} is a secure MAC for arbitrary-length messages.

HMAC

HMAC is a standardised secure message authentication code that uses two layers of hashing.

It can be viewed as an instantiation of the hash-and-MAC technique.

HMAC is very efficient and widely used in practice.

HMAC

Let (KeyGen_H, H) be a hash function obtained from a fixed-length hash function (KeyGen_h, h), with $\ell'(n) = 2n$ and $\ell(n) = n$, applying the Merkle-Damgård transform. Let opad and ipad be two fixed strings of length n.

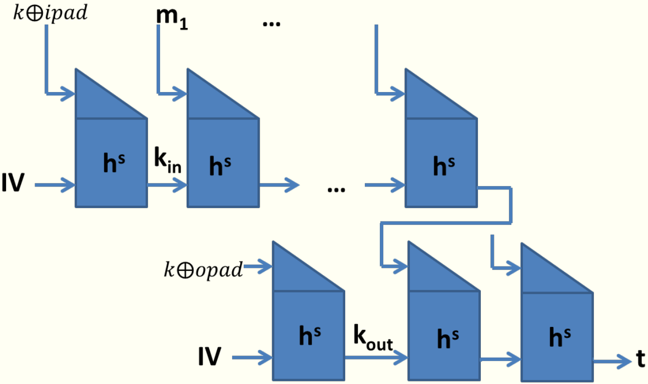
A MAC $S = (\text{KeyGen}, \text{Mac}, \text{Verify})$ for arbitrary-length messages can be defined as follows:

- $(s, k) \leftarrow \text{KeyGen}(n)$: given a security parameter n, it samples a uniform $k \in \{0,1\}^n$ and runs KeyGen_H on input n, obtaining s. It outputs the key (s, k).
- $t \leftarrow \text{Mac}((s,k), m \in \{0,1\}^*)$: it returns the output

$$H^s((k \oplus \text{opad}) || H^s((k \oplus \text{ipad}) || m)).$$

- $1/0 \leftarrow \text{Verify}((s,k), m, t)$: it is the canonical verification.

HMAC



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