Introduction to Cryptology

8.3 - Hash functions: Practical Constructions

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Constructing hash functions

Hash functions used in practice are commonly constructed in two steps:

- a fixed-length collision-resistant hash function h is constructed;
- a techniques (e.g. the Merkle-Damgård transform) is applied to extend h to arbitrary-length inputs.

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Given a block cipher with n as key length and ℓ as block- ength, h is defined as follows:

$$\begin{aligned} h : \{0,1\}^{n+\ell} &\rightarrow \{0,1\}^{\ell} \\ (k,x) &\mapsto h(k,x) = F_k(x) \oplus x. \end{aligned}$$

Hash Functions From Block Ciphers

Not known how to prove the collision resistance of h relying on the assumption that F is a strong pseudorandom permutation.

Something similar to the random oracle model is used.

- F is modelled as an ideal cipher (each key specifies a uniform permutation F_k).
- Each party has to query an oracle to compute $F(k, x)$ or $F^{-1}(k, y)$.

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MD5 has output length equal to 128. Given an input x:

- the bit 1 is appended;
- the bit 0 is appended until the length of the string is congruent to 448 modulo 512;
- the length of the original input is appended, encoded as a 64-bit string;
- the padded string is divided into blocks of length 512. Each block is divided into 16 chunks M_i of 32 bits each.

MD5

For each block, 64 operations are executed. They are grouped into 4 rounds, each of 16 operations.

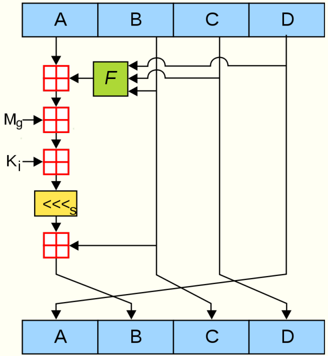
At the very beginning, four 32-bit strings, A, B, C and D, are initialised to constant values.

- The i-th operation sends B in C, C in D and D in A. The new value of B is obtained as

$$B + \lll_{s[i]} (F(B, C, D) + K[i] + M_{g(i)});$$

- $\lll_{s[i]} (\cdot)$ denotes a bit rotation to the left of $s[i]$ places, where s is a fixed list;
- the non-linear function $F(\cdot)$ and the index $g(\cdot)$ vary depending on the round;
- $K[i]$ denotes a 32-bit constant, different for each operation.
- Addition is done modulo 2^{32}.

MD5



One MD5 operation (from wikipedia)

The string obtained after processing a block is added to the value of $(A||B||C||D)$ at the beginning of the four rounds, obtaining a new initial state for the next block.

MD5

Round 1

- $F(B, C, D) = (B \wedge C) \vee (\neg B \wedge D)$;
- $g(i) = i$.

Round 2

- $F(B, C, D) = (B \wedge D) \vee (C \wedge \neg D)$;
- $g(i) = 5i + 1 \pmod{16}$.

Round 3

- $F(B, C, D) = B \oplus C \oplus D$;
- $g(i) = 3i + 5 \pmod{16}$.

Round 4

- $F(B, C, D) = C \oplus (B \vee \neg D)$;
- $g(i) = 7i \pmod{16}$.

Secure Hash Algorithms: SHA-1 and SHA-2

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The block ciphers were specifically designed for this purpose.

- SHACAL-1 - with block length equal to 160 - for SHA1.
- SHACAL-2 - with block length equal to 256 - for SHA2.
- Keys are 512-bit strings in both block ciphers.

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- SHACAL-1 - with block length equal to 160 - for SHA1.
- SHACAL-2 - with block length equal to 256 - for SHA2.
- Keys are 512-bit strings in both block ciphers.

Second, they are extended to handle arbitrary-length inputs using the Merkle-Damgård transform.

SHA-1

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- the same padding of MD5 is executed;
- after the padding, each 512-bit block is expanded into eighty 32-bit chunks W_j;
- for $17 \leq j \leq 80$, $W_j := W_{j-3} \oplus W_{j-8} \oplus W_{j-14} \oplus W_{j-16}$.

SHA-1

For each block, 80 operations are executed. They are grouped into 4 rounds, each of 20 operations.

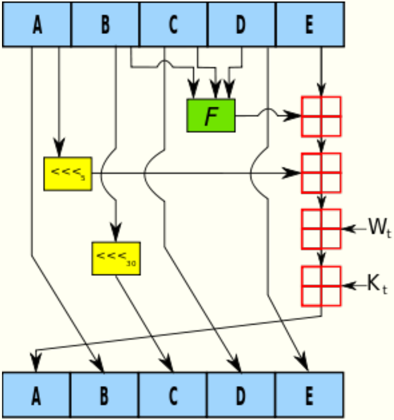
At the very beginning, five 32-bit strings, A, B, C, D and E, are initialised to constant values.

- The t-th operation sends A in B, $\lll_{30}(B)$ in C, C in D and D in E. The new value of A is obtained as

$$\lll_5 (A) + E + F(B, C, D) + K[t] + W_t;$$

- the non-linear function $F(\cdot)$ varies depending on the round;
- $K[t]$ denotes a 32-bit constant, different for each round.
- Addition is done modulo 2^{32}.

SHA-1



One SHA-1 operation (from wikipedia)

A blogpost on the functioning of SHA-1: http://www.metamorphosite.com/one-way-hash-encryption-sha1-data-software

SHA-1

Round 1

- $\mathrm{F}(B, C, D) = (B \wedge C) \vee (\neg B \wedge D)$.

Round 2

- $\mathrm{F}(B, C, D) = B \oplus C \oplus D$.

Round 3

- $\mathrm{F}(B, C, D) = (B \wedge C) \vee (B \wedge D) \vee (C \wedge D)$.

Round 4

- $\mathrm{F}(B, C, D) = B \oplus C \oplus D$.

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<u>Round 4</u>

- $\mathrm{F}(B, C, D) = B \oplus C \oplus D$.

The string obtained after processing a block is added to the value of $(A||B||C||D||E)$ at the beginning of the four rounds, obtaining a new initial state for the next block.

SHA-2

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The output length is equal to 256. Given an input x:

- the same padding of MD5 and SHA-1 is executed;
- after the padding, each 512-bit block is expanded into sixty-four 32-bit chunks W_j;
- for $17 \leq j \leq 80$:

$$
\begin{aligned}
s_0 &:= (\ggg_7 (W_{j-15}) \oplus (\ggg_{18} (W_{j-15}) \oplus (\ggg_3 (W_{j-15}) \\
s_1 &:= (\ggg_{17} (W_{j-2}) \oplus (\ggg_{19} (W_{j-2}) \oplus (\ggg_{10} (W_{j-2}) \\
W_j &:= W_{j-16} + s_0 + W_{j-7} + s_1
\end{aligned}
$$

SHA-2

For each block, 64 operations are executed.

At the very beginning, eight 32-bit strings, A, B, C, D, E, F, G and H are initialised to constant values.

- The t-th operation sends A in B, B in C, C in D, E in F, F in G and G in H. The new value of A is obtained as

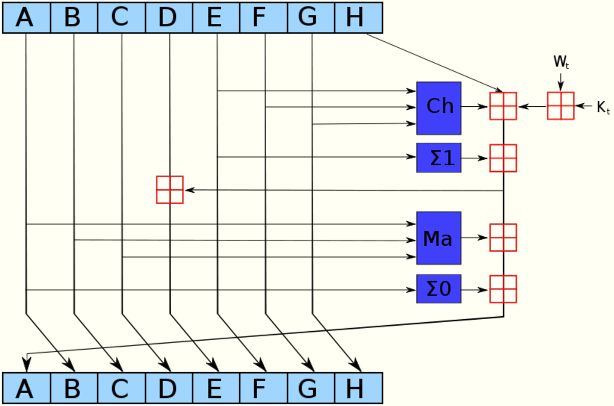
  $$W_t + K[t] + H + Ch(E, F, G) + \Sigma_1(E) + Ma(A, B, C) + \Sigma_0(A).$$

  The new value of E is obtained as

  $$W_t + K[t] + H + Ch(E, F, G) + \Sigma_1(E) + D.$$

- $K[t]$ denotes a 32-bit constant, different for each round.
- Addition is done modulo 2^{32}.

SHA-2



One SHA-2 operation (from wikipedia)

A detailed description of SHA-2:
http://www.iwar.org.uk/comsec/resources/cipher/sha256-384-512.pdf

SHA-256

The logical functions are as follows:

- $Ch(E,F,G) = (E \wedge F) \oplus (\neg E \wedge G)$
- $Ma(A,B,C) = (A \wedge B) \oplus (A \wedge C) \oplus (B \wedge C)$
- $\Sigma_0(A) = (\ggg_2 (A)) \oplus (\ggg_{13} (A)) \oplus (\ggg_{22} (A))$
- $\Sigma_1(E) = (\ggg_6 (E)) \oplus (\ggg_{11} (E)) \oplus (\ggg_{25} (E))$

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The constant words, $K[1], \cdots, K[64]$ are the first 32 bits of the fractional parts of the cube roots of the first sixty-four primes.

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The constant words, $K[1], \cdots, K[64]$ are the first 32 bits of the fractional parts of the cube roots of the first sixty-four primes.

The string obtained after processing a block is added to the value of $(A||B||C||D||E||F||G||H)$ before the 64 operations, obtaining a new initial state for the next block.

SHA-3 (Keccak)

In 2012, Keccak was announced as the winner of the NIST competition (called SHA-3) to design a new (family of) hash function(s).

It uses an unkeyed permutation f with block length equal to 1600.

Keccak uses a new construction, named sponge construction for the domain extension.

SHA-3 (Keccak)

The permutation f operates on blocks of length 1600. The output length d lies in $\{224, 256, 384, 512\}$.

A rate r is fixed. The capacity is $c = 1600 - r$. The bigger c, the bigger the number of bits of security and the slower the execution.

The digest is obtained after two phases: the absorbing phase, and the squeezing phase.

SHA-3 - Absorbing

The initial input is padded:

- the bit 1 is appended;
- the bit 0 is appended until the length of the string is congruent to $r-1$ modulo r;
- the bit 1 is appended.

The string is divided into n blocks $M_0, \ldots, M_{n-1}$ of length r. For each block, an evaluation of f is performed.

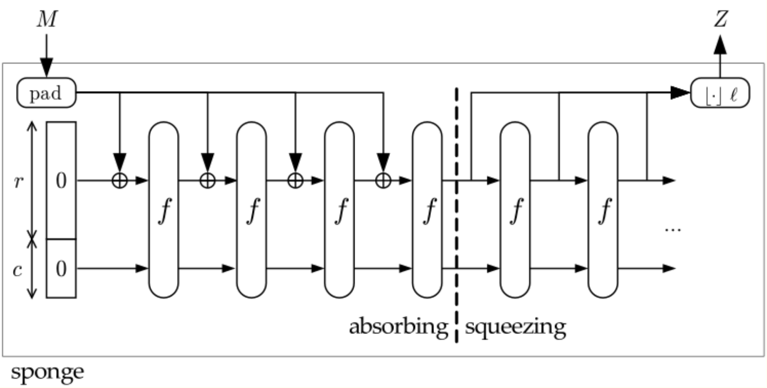
The state S is initialised as a string of 1600 zeros.

- M_i is extended appending $c = 1600 - r$ zeros, obtaining M'_i.
- The new state S is $f(S \oplus M'_i)$.

SHA-3 - Squeezing

- Z is initialised as the empty string.
- While the length of Z is less than d, the first r bits of S are appended to Z and S is updated applying f.
- Z is truncated to d bits.

Keccak- Sponge Function



Complete description: http://sponge.noekeon.org/CSF-0.1.pdf

SHA-3 (Keccak)

The permutation f works on a state S represented by a 5x5 matrix of 64-bit words.

The image is computed by repeating 24 times a round composed by 5 steps: θ, ρ, π, χ and ι.

Only χ is not linear.

Further Reading I

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