Post-Quantum Cryptography and Standardization

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I. Quantum Computers & Post-Quantum Cryptography

II. The NIST Standardization Process

III. Lattice-Based Cryptography

IV. Falcon

Quantum Computers & Post-Quantum Cryptography

Quantum computers

- Quantum computers exploit quantum phenomena such as superposition and entanglement
- They operate under different rules than classical computers (e.g. (qu)bits)
- Big players include Google, IBM, Honeywell, Microsoft, etc.
- Applications include genomic sequencing, finance, drug development, etc.
- What about *cryptography*?

Impact on public-key cryptography

PQSHIELD

Shor's period-finding algorithm [Sho94]

Given a finite abelian group $\mathbb{G}$ and $f : \mathbb{G} \to X$, output r so that $f(x+r) = f(x)$.
The complexity of this quantum algorithm is *polynomial*: $O((\log |\mathbb{G}|)^3)$.

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Application to discrete logarithm:

1. **Problem:** Let $g, h \in \mathbb{G}$, where $\mathbb{G}$ is a finite abelian group of order p. Find r such that $h = g^r$.
2. Let $f(x, y) = g^x \cdot h^{-y}$.
3. Periods of f are multiple of $(r, 1)$.

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Application to factoring:

1. **Problem:** Factor $N = p \cdot q$.
2. Sample $1 < a < N$, let $f_a(x) = x^a$.
3. The period r of f verifies: $a^r = 1 \bmod N$.
4. If r is even, then $(a^{r/2} \pm 1)$ are factors of N since: $(a^{r/2} + 1)(a^{r/2} - 1) = 0 \bmod N$.
5. If some steps fail, goto 2.

Impact on symmetric cryptography

Search problems:

- Given a function $f : X \to \{0, 1\}$, we want to find $x \in X$ such that $f(x) = 1$. Grover's quantum algorithm [Gro96] do that in $\Theta(\sqrt{|X|})$ calls to f (instead of $O(|X|)$ classically).
- Less dramatic impact than Shor, but much larger scope (exhaustive key search, (second) preimage, subroutines in cryptanalytic algorithms, etc.).

Collision problems:

- Given $f : W \to X$, we want to find $w_1, \neq w_2$ such that $f(w_1) = f(w_2)$. A series of works propose quantum algorithms solving this in time between $O(|X|^{2/5})$ [CNS17] and $\Theta(|X|^{1/3})$ [BHT98, Amb04, Zha15], instead of $O(\sqrt{|X|})$ classically.
- Impacts mainly hash functions.

State of affairs

	Problem	Classical Hardness	Quantum Hardness
Public-key crypto.	Factoring	$e^{\tilde{O}((\log N)^{1/3})}$	$poly(\log N)$
	Discrete Logarithm	$e^{\tilde{O}((\log p)^{1/3})}$	$poly(\log p)$
Symmetric crypto.	Exhaustive search	$O(\lvert X\rvert)$	$\Theta(\lvert X\rvert^{1/2})$
	Collision	$O(\lvert X\rvert^{1/2})$	$\Theta(\lvert X\rvert^{1/3})$

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	Collision	$O(\|X\|^{1/2})$	$\Theta(\|X\|^{1/3})$

Symmetric & keyless primitives:

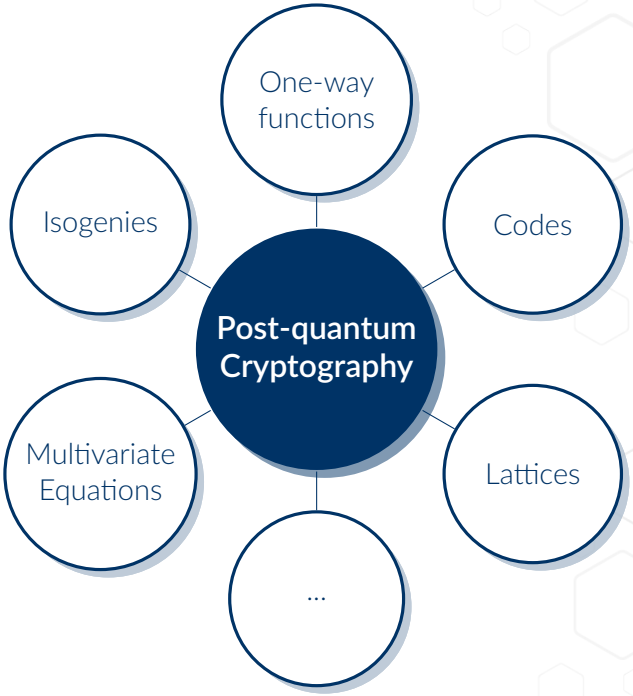
- **Impacted:**
  - Sym. Encryption (e.g. AES)
  - Hash Functions (e.g. SHA-3)
  - MACs (e.g. HMAC)
  - etc.
- **Mitigation:** Double the sizes

Public-key primitives:

- **Impacted:**
  - RSA encryption & signatures
  - (EC)DH, (EC)DSA
  - El Gamal
  - etc.
- **Mitigation:** New assumptions!

Post-quantum cryptography:

- Exploded in the last 10-20 years
- Multiple families of assumptions
- Very heterogeneous field
- Apples-to-apples comparison is hard
- Lots of work to do



Lattice-based cryptography

- Inception in 1996 [Ajt96, HPS98]
- The underlying hard problem is typically to solve a linear system $\mathbf{sA} = \mathbf{t}$ under geometric constraints (**s** short for some metric)
- Cryptanalysis is done primarily via lattice reduction
- Historically, strong connection to theoretical CS
- Balanced performances (communication cost, computational cost, etc.)
- Perhaps the most dynamic field at the moment

Code-based cryptography

- Inception in 1978 [McE78]
- Archetypal problem is to solve a linear system under sparsity constraints:

Syndrome Decoding problem

Given a matrix $\mathbf{H} \in \mathbb{F}_2^{k \times n}$ and a *syndrome* $\mathbf{s} \in \mathbb{F}_2^k$, find $\mathbf{e} \in \mathbb{F}_2^n$ of Hamming weight w such that $\mathbf{H} \times \mathbf{e} = \mathbf{s}$.

- Cryptanalysis is rather mature, but new variants are regularly broken.
- Some schemes (McElieve, Wave) have large keys.

Multivariate cryptography

- Inception in 1988 [MI88, Pat96]
- Archetypal problem:

Multivariate quadratics problem (MQ)

Given $\mathbf{y} \in \mathbb{F}^m$ and a multivariate quadratic map $\mathbf{F} : \mathbb{F}^n \to \mathbb{F}^m$, find $\mathbf{x}$ such that $\mathbf{F}(\mathbf{x}) = \mathbf{y}$.

- Cryptanalysis is done via algebraic techniques such as Gröbner bases
- Typically large keys but small signatures/ciphertexts
- Rocky security history

Isogeny-based cryptography

- Inception in 1996-2006 [Cou06, RS06]
- Archetypal problems are often generalizations of elliptic-curve problems:

Computational Supersingular Isogeny (CSSI) problem [DJP14]

Given two elliptic curves E, E' and the value of an isogeny $\varphi : E \to E'$ on the torsion subgroup $E[\ell^e]$, find φ.

- Very compact, but somewhat slow.
- Very dynamic and recent field, efficiency and cryptanalysis may evolve.

Hash-based cryptography

- Inception in 1978 [Lam79, Mer90]
- We only know how to build signature schemes
- The gold standard of security: relies only on collision/(second-)preimage resistance of generic hash functions.
- Use generic data structures (trees, tables, etc.) to improve efficiency.
- Large signatures, slow signing.
- Not to be confused with signatures that prove in ZK the knowledge of x such that $F(x) = y$ for a one-way function F (e.g. Picnic [ZCD+19])

The NIST Standardization Process

Standardizing PQC

Why standardise now if quantum computers are not practical yet?

- **T1:** Duration of standardization process
- **T2:** Time to deploy standards
- **T3:** Duration a given data must remain confidential
- **T4:** Time before quantum computers become practical

For authentication (e.g. signatures):

$$T1 + T2 > T4$$

For confidentiality (e.g. encryption and key exchange):

$$T1 + T2 > T4 - T3$$

The NIST Standardization Process

Objective:

- Standardize PQC through an open process

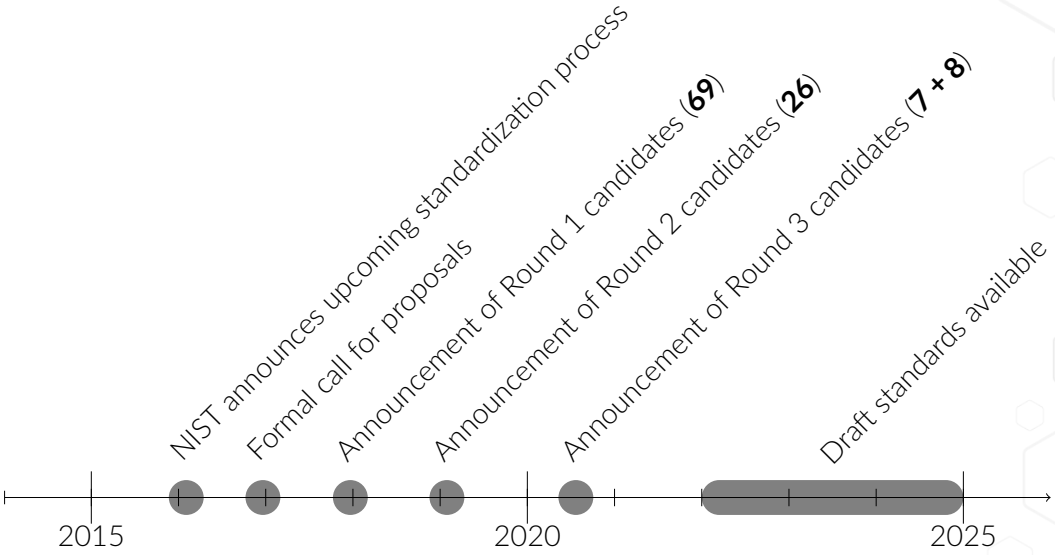
Scope:

- Signatures
- Key exchange / Key Encapsulation Mechanisms (KEM) / Public Key Encryption (PKE)

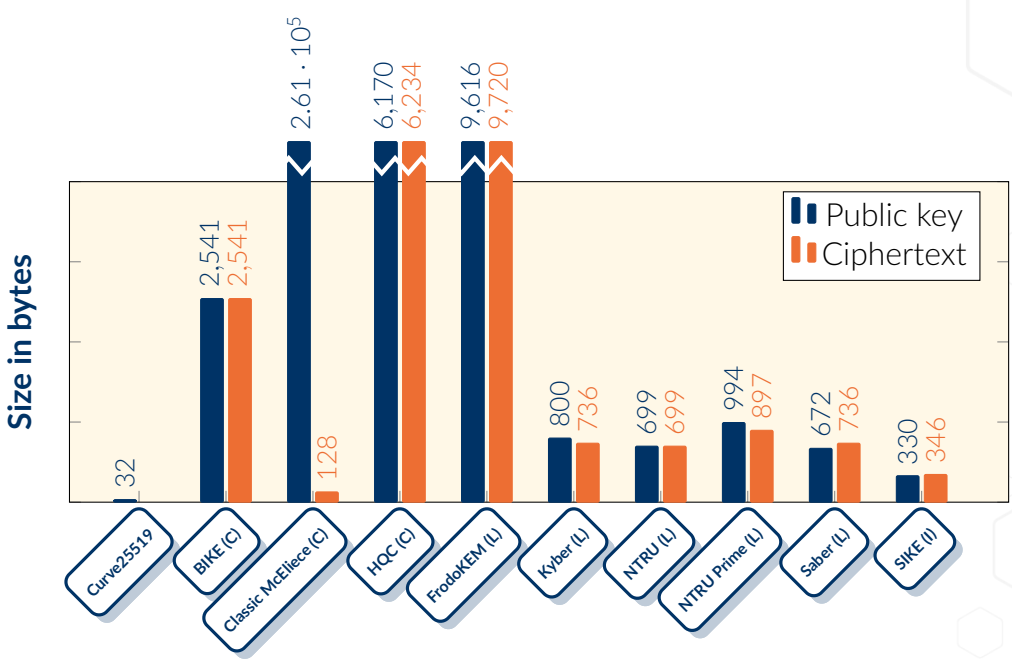
Criteria:

- Security
- Performances
  - Communication cost
  - Computational efficiency
  - Portability on embedded devices
- Suitability to real-world usecases

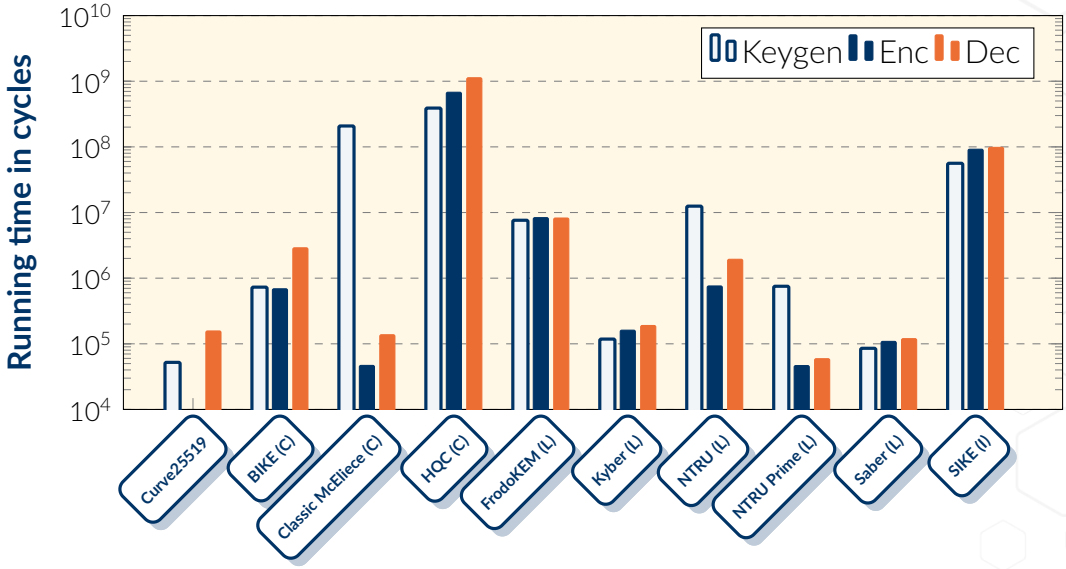
NIST Standardization Timeline



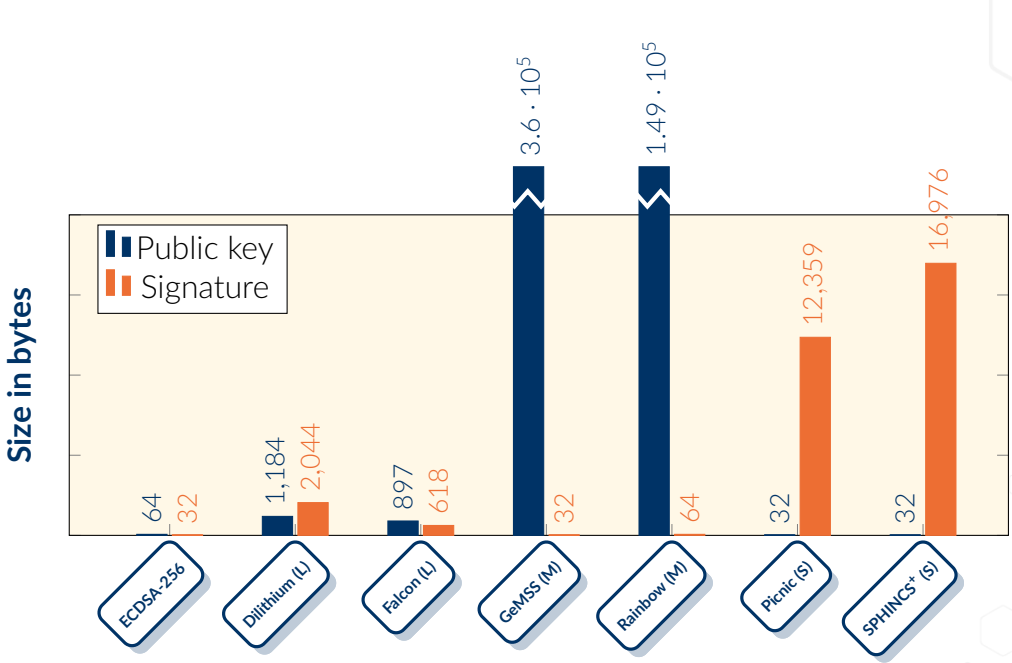
Bandwidth cost of Level 1 KEMs



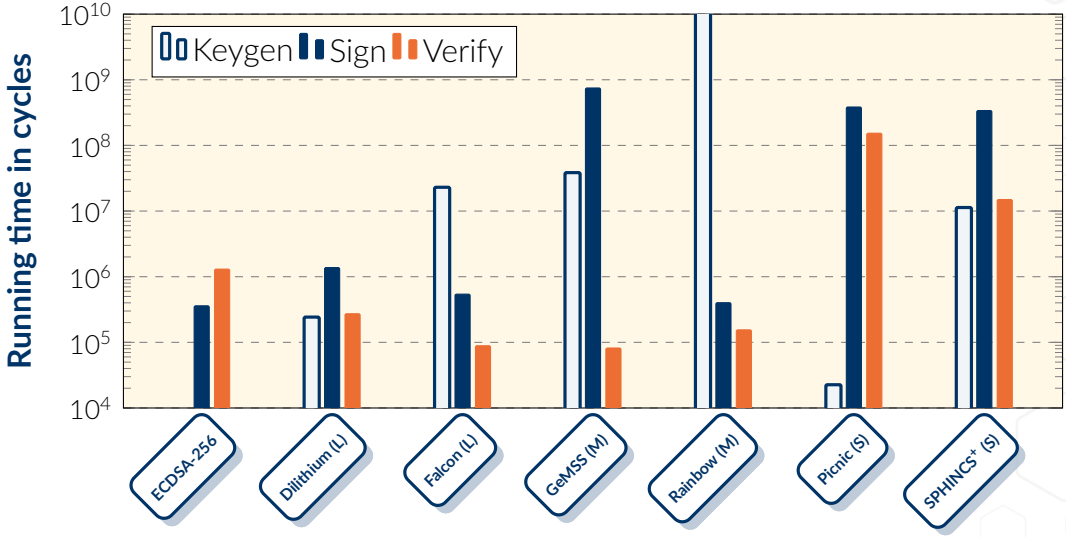
Computation Cost of Level 1 KEMs



Bandwidth cost of Level 1 Signatures



Computation Cost of Level 1 Signatures



Additional literature

Quantum computing:

- Lecture notes by Ron de Wolf: https://homepages.cwi.nl/~rdewolf/qcnotes.pdf
- Series of workshops by the Simons institute: https://simons.berkeley.edu/programs/quantum2020

Code-based cryptography:

- MOOC by INRIA: https://www.canal-u.tv/producteurs/inria/cours_en_ligne/code_based_cryptography

Isogeny-based cryptography:

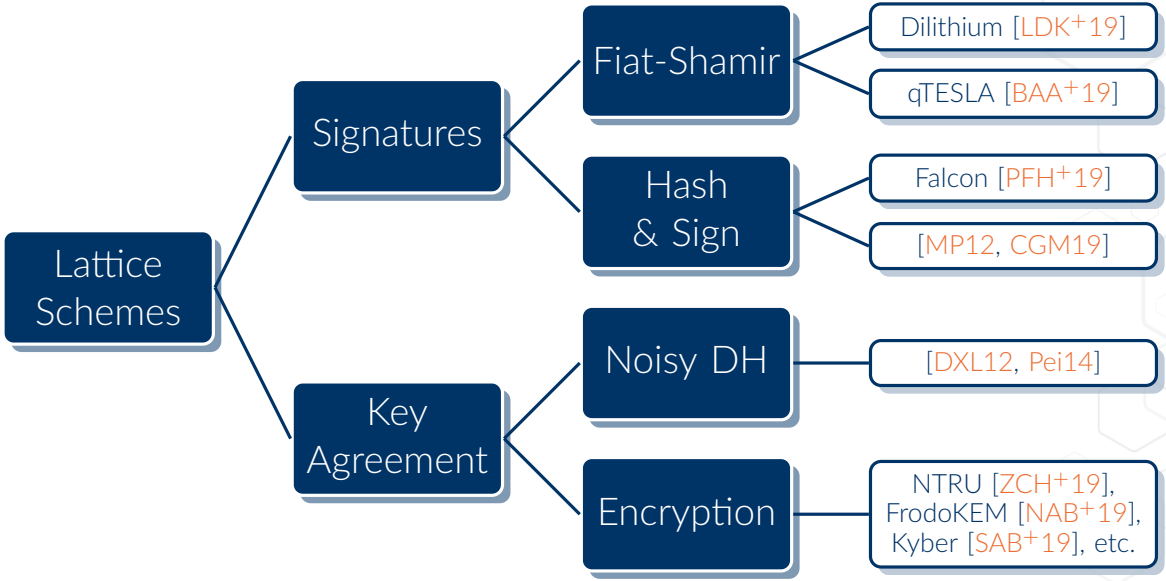
- Introduction by Luca de Feo: https://arxiv.org/pdf/1711.04062.pdf

The NIST standardization process:

- Everything is available online: specification of the candidates, reference implementations, slides, reports by NIST, mailing list, etc. https://csrc.nist.gov/projects/post-quantum-cryptography/

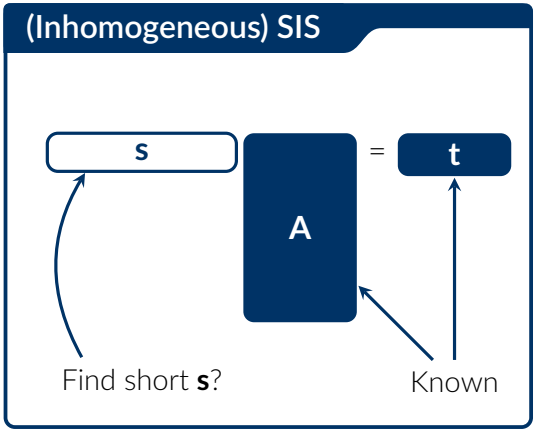
Lattices: see end of next talk

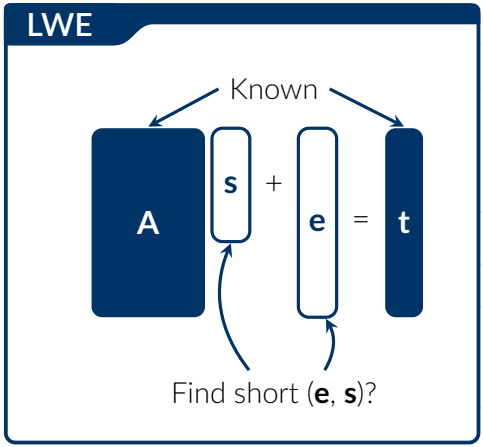
Lattice-Based Cryptography


Lattice Schemes
Signatures
Fiat-Shamir
Dilithium [LDK+19]
qTESLA [BAA+19]
Hash & Sign
Falcon [PFH+19]
[MP12, CGM19]
Key Agreement
Noisy DH
[DXL12, Pei14]
Encryption
NTRU [ZCH+19], FrodoKEM [NAB+19], Kyber [SAB+19], etc.

Main problems: SIS and LWE

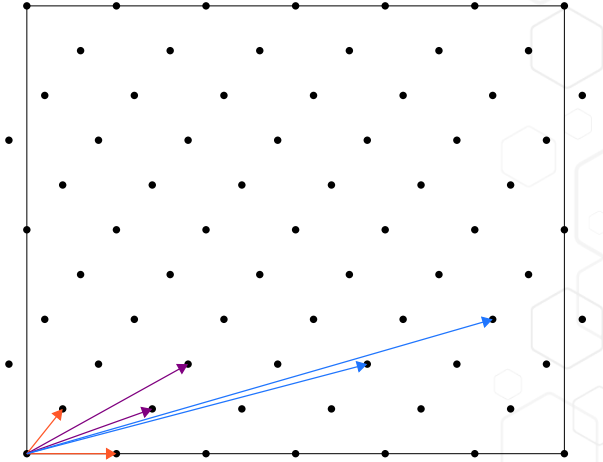
SIS stands for *Short Integer Solution*, and LWE for *Learning With Errors*.
SIS only comes in a search variant, LWE has a search and a decision variant.
The base ring may vary: $\mathbb{Z}_q$, $\mathbb{Z}_q[x]/(x^n + 1)$, etc.





How is any of that related to lattices?

- Solving $\mathbf{s} \cdot \mathbf{A} = 0 \bmod q$ is equivalent to finding a short vector of the lattice $\mathcal{L} = \{\mathbf{s} | \mathbf{s} \cdot \mathbf{A} = 0 \bmod q\}$
- Computing a (large) basis of $\mathcal{L}$ is trivial, but obtaining a short basis requires lattice reduction [LLL82, SE94]
- Similarly, (inhomogeneous) SIS and LWE can be expressed as lattice problems.



LWE as an analog to DLOG

At a very, very abstract level, one can build schemes based on LWE by adapting schemes based on the discrete logarithm:

- **DLOG:** Given (g, g^x), find x.
- **LWE:** Given $(\mathbf{A}, \mathbf{AS} + \mathbf{E})$, find $\mathbf{S}$.

Example with El Gamal in the next slide.

Keygen$(g \in \mathbb{G})$

1. Sample $x \leftarrow \mathbb{Z}_{|\mathbb{G}|}$
2. $h \leftarrow g^x$
3. $\mathtt{sk} := x, \mathtt{pk} := h$

Enc$(\mathtt{M}, \mathtt{pk})$

1. Sample $r \leftarrow \mathbb{Z}_{|\mathbb{G}|}$
2. $u \leftarrow g^r$
3. $v \leftarrow h^r \cdot \mathtt{M}$
4. $\mathtt{ct} := (u, v)$

Dec$(\mathtt{M}, \mathtt{sk})$

1. $\mathtt{M} \leftarrow v \cdot u^{-x}$

Keygen$(\mathbf{A} \in \mathcal{R}_q^{m \times m})$

1. Sample short $\mathbf{S}, \mathbf{E}$
2. $\mathbf{B} \leftarrow \mathbf{AS} + \mathbf{E}$
3. $\mathtt{sk} := (\mathbf{S}, \mathbf{E}), \mathtt{pk} := \mathbf{B}$

Enc$(\mathtt{M}, \mathtt{pk})$

1. Sample short $\mathbf{R}, \mathbf{E}', \mathbf{E}''$
2. $\mathbf{U} \leftarrow \mathbf{RA} + \mathbf{E}'$
3. $\mathbf{V} \leftarrow \mathbf{RB} + \mathbf{E}'' + \mathrm{Encode}(\mathtt{M})$
4. $\mathtt{ct} := (\mathbf{U}, \mathbf{V})$

Dec$(\mathtt{M}, \mathtt{sk})$

1. $\mathtt{M} \leftarrow \mathrm{Decode}(\mathbf{V} - \mathbf{US})$

That was easy, wasn't it?

Not so fast. Many new elements to factor in:

- More parameters: dimensions, ring $\mathcal{R}_q$, sampling distributions, etc.
- Decryption failures can be exploited [HGS99, DGJ+19, DVV19, GJY19, DRV20]
- Tricks like bit dropping and error-correcting codes [ADPS16, Ham19] may improve efficiency but complexify the security analysis!
- Transforms to achieve active security (e.g. IND-CCA) need to be studied against quantum attackers as well [HHK17].

Signatures also come out with their fair share of challenges!

Falcon

- One of the 3 finalists for NIST standardization (signature track).
- Falcon is a lattice-based signature of type *hash-then-sign*.
- At a very very high level, think RSA signatures but with lattices.

NTRU

Let $\mathcal{R}_q = \mathbb{Z}_q[x]/(x^n + 1)$. Given $h \in \mathcal{R}_q$, find short $f, g \in \mathcal{R}_q$ such that

$$g \cdot f^{-1} = h \tag{1}$$

- The NTRU problem can be seen as a special case of SIS. Indeed, given $\mathbf{A} = \begin{bmatrix} 1 \\ h \end{bmatrix}$, we seek a short $\mathbf{s} = \left[\begin{array}{c|c} g & -f \end{array}\right]$ such that $\mathbf{s} \cdot \mathbf{A} = 0 \bmod q$.
- Given f, g, one can compute a short matrix $\mathbf{B} = \left[\begin{array}{c|c} g & -f \\ \hline G & -F \end{array}\right]$ such that $\mathbf{B} \cdot \mathbf{A} = 0 \bmod q$. See [PFH+19].

GPV signatures [GPV08]

Falcon instantiates this blueprint with NTRU lattices (see previous slide).

Keygen(1^λ)

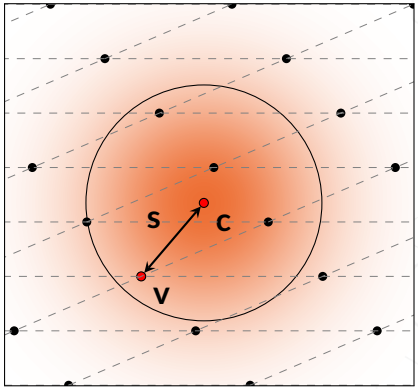
1. Gen. matrices $\mathbf{A}, \mathbf{B}$ s.t.:
   - $\mathbf{B} \cdot \mathbf{A} = 0$
   - $\mathbf{B}$ has small coefficients
2. $\mathtt{pk} := \mathbf{A}, \mathtt{sk} := \mathbf{B}$

Sign$(\mathtt{M}, \mathtt{sk} = \mathbf{B})$

1. Compute $\mathbf{c}$ such that $\mathbf{c} \cdot \mathbf{A} = H(\mathtt{M})$
2. $\mathbf{v} \leftarrow$ vector in $\mathcal{L}(\mathbf{B})$, close to $\mathbf{c}$
3. $\mathtt{sig} := \mathbf{s} = (\mathbf{c} - \mathbf{v})$

Verify$(\mathtt{M}, \mathtt{pk} = \mathbf{A}, \mathtt{sig} = \mathbf{s})$

Check ($\mathbf{s}$ short) & ($\mathbf{s} \cdot \mathbf{A} = H(\mathtt{M})$)



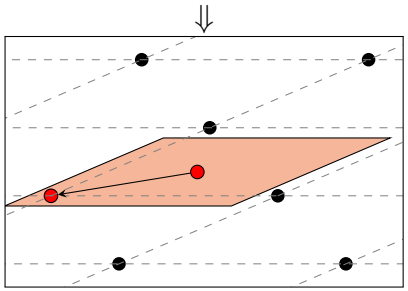
How to compute efficiently a close vector (the second algorithm assumes we precomputed the Gram-Schmidt orthogonalization $\mathbf{B} = \mathbf{L} \cdot \tilde{\mathbf{B}}$).

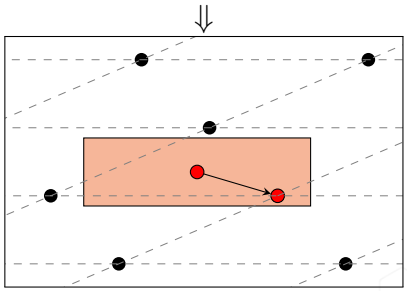
RoundOff$(\mathbf{B}, \mathbf{c})$

1. $\mathbf{t} \leftarrow \mathbf{c} \cdot \mathbf{B}^{-1}$
2. For $j \in \{n, \ldots, 1\}$:
   1. $z_j \leftarrow \lceil t_j \rfloor$
3. Return $\mathbf{v} := \mathbf{z} \cdot \mathbf{B}$

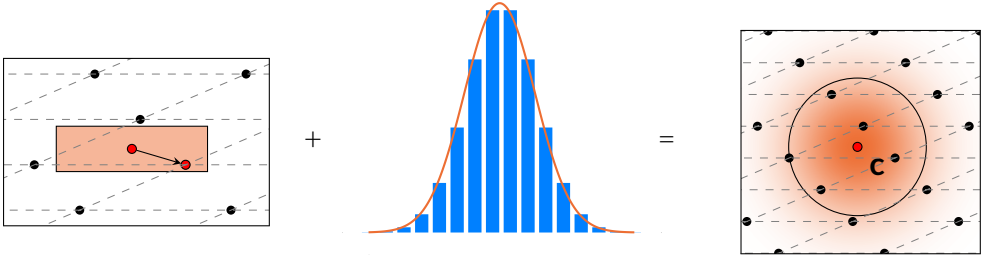
NearestPlane$(\mathbf{B}, \mathbf{L}, \mathbf{c})$

1. $\mathbf{t} \leftarrow \mathbf{c} \cdot \mathbf{B}^{-1}$
2. For $j \in \{n, \ldots, 1\}$:
   1. $z_j \leftarrow \lceil t_j + \sum_{i>j}(t_1 - z_i)L_{i,j} \rfloor$
3. Return $\mathbf{v} := \mathbf{z} \cdot \mathbf{B}$





- **Problem:** When used for signing, the algorithms RoundOff and NearestPlane leak the shape of the private key **B**, leading to attacks [NR06, DN12].
- **Solution [GPV08]:** Replace rounding with (Gaussian) randomized rounding.



Falcon applies a few optimizations not described in this talk:

- Exploiting the algebraic structure of $\mathbb{Z}[x]/(x^n + 1)$ to speed up the key generation [PP19] and signing [DP16] procedures.
- Use the Rényi divergence to optimize parameter selection [Pre17, HPRR20].

Additional literature

Lattices:

- A few courses:
  - https://homepages.cwi.nl/~dadush/teaching/lattices-2018/
  - https://cims.nyu.edu/~regev/teaching/lattices_fall_2009/index.html
  - https://cseweb.ucsd.edu/classes/fa17/cse206A-a/
  - https://web.eecs.umich.edu/~cpeikert/lic15/index.html
  - https://people.csail.mit.edu/vinodv/6876-Fall2015/index.html
- Series of workshops by the Simons institute:
  https://simons.berkeley.edu/programs/lattices2020

Falcon:

- Official website: https://falcon-sign.info/

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