Introduction to Cryptology

4.1 - Block Ciphers

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Block Ciphers

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A block cipher is a keyed map $F : \{0,1\}^n \times \{0,1\}^\ell \to \{0,1\}^\ell$ s.t.

- $F_k : \{0,1\}^\ell \to \{0,1\}^\ell$, $x \mapsto F(k,x)$ is a permutation for all $k \in \{0,1\}^n$;
- F_k and F_k^{-1} are efficiently computable for all $k \in \{0,1\}^n$.

Naming: n is the key length, ℓ is the block length.

Concrete security of Block Ciphers

Let Perm_ℓ be the set of all permutations of $\{0,1\}^\ell$, and consider a block cipher $F : \{0,1\}^k \times \{0,1\}^\ell \to \{0,1\}^\ell$.

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For a PPT distinguisher D, we define their advantage as

$$\mathrm{Adv}^{PRP}_{F,\mathrm{D}} = |\,\Pr(\mathrm{D}^{f()} = 1) - \Pr(\mathrm{D}^{F_k()} = 1)|$$

where the first probability is taken over a uniform choice of f in Perm_ℓ and the randomness of D, the second one over a uniform choice of k in $\{0,1\}^n$ and the randomness of D.

Concrete security of Block Ciphers

For any integers t and q, we define

$$\mathrm{Adv}_F^{PRP}(t,q) = \max_{\mathrm{D}}\{\mathrm{Adv}_{F,\mathrm{D}}^{PRP}\}$$

where the maximum is over all distinguishers D with time complexity at most t and making at most q queries.

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Currently, a block cipher is considered secure if the best known attack has time complexity approximately equal to a brute-force attack to recover the key.

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Representing an arbitrary permutation of $\{0,1\}^\ell$ needs $\ell \cdot 2^\ell$ bits (infeasible for $\ell > 50$; for modern block ciphers $\ell \geq 128$).

Confusion-Diffusion Paradigm

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- Example: given x $\in \{0,1\}^{128}$, split it into 16 bytes $x_1, \cdots, x_{16}$ and define

$$F_k(x) = f_1(x_1) || \cdots || f_{16}(x_{16}).$$

- $F_k(x)$ and $F_k(x')$ have only one different byte if $w(x, x') = 1$.

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Diffusion: use a mixing permutation to make a change in one bit affect the entire output!

Confusion-Diffusion Paradigm

Each function f_i is called round function.

The confusion-diffusion steps together are called round.

Substitution-permutation Networks (SPNs)

A substitution-permutation network is an implementation of the confusion-diffusion paradigm.

- Using a fixed public algorithm called key schedule, sub-keys $k_1, \ldots, k_{r+1}$ are derived from the key k.
- Different permutations $\{S_i\}$ with small block length are used to define the round functions:

$$f_i(x_i) = S_i(x_i \oplus k_{j,i})$$

  where $k_{j,i}$ denotes the i-th *chunk* of the sub-key k_j.
- S_i is called S-box

Key Schedule: a simple example

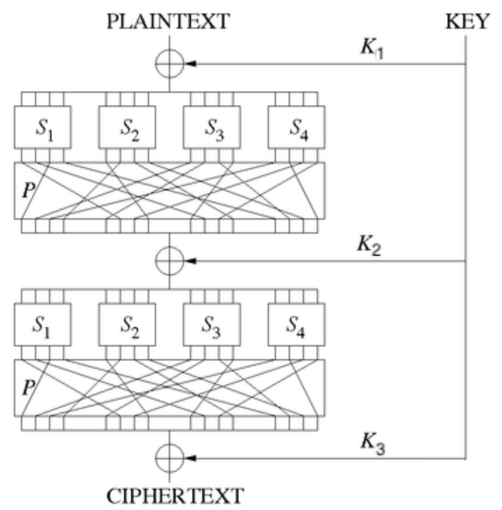
Let the key k be as follows:

$$k = 1110\ 0111\ 0110\ 0111\ 1001\ 0000\ 0011\ 1101.$$

Define k_i as the 16 consecutive bits of k starting at bit $4i - 3$:

- $k_1 = 1110\ 0111\ 0110\ 0111$
- $k_2 = 0111\ 0110\ 0111\ 1001$
- $k_3 = 0110\ 0111\ 1001\ 0000$
- $k_4 = 0111\ 1001\ 0000\ 0011$
- $k_5 = 1001\ 0000\ 0011\ 1101$

SPN - Example



Input : m, S-boxes,

mixing permutation P,

$(k_1, \ldots, k_{r+1})$.

Output : c.

state $= m$

for $j = 1, \ldots, r$;

state $=$ state $\oplus\ k_j$ (key-mixing)

apply S-boxes to the t sub-strings..

..of state (substitution)

apply P to state (permutation)

$c =$ state $\oplus\ k_{r+1}$

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mixing permutation P: the bits of the output of one S-box are fed to multiple S-boxes in the next round.

SPNs - Miscellaneous

- The Advanced Encryption Standard (AES) has a similar structure (will see it soon).
- The security of a SPN depends on the number of rounds.
  - for a SPN with a single round with no key mixing as final step, it is easy to recover the key k;
  - a one round SPN is also not secure;
  - same for a two round SPN.

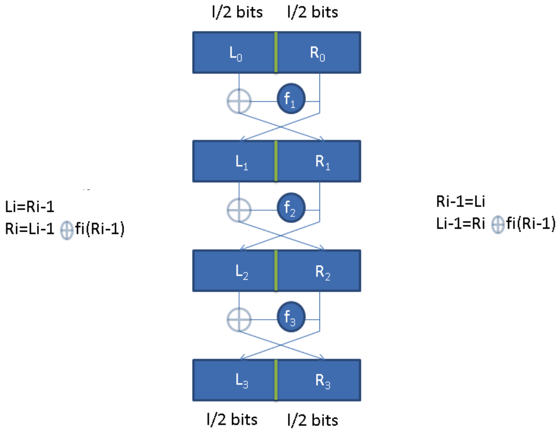
Feistel Networks

Different approach to construct block ciphers following the confusion-diffusion paradigm.

Advantage over SPNs: the round functions do not need to be permutations.

For a permutation $F_k : \{0,1\}^\ell \to \{0,1\}^\ell$, r key-dependent round functions $f_1, \cdots, f_r$, where $f_i : \{0,1\}^{\ell/2} \to \{0,1\}^{\ell/2}$, are used.

Feistel Networks - An example



Attacks on Block Ciphers

Linear Attacks

Exploit linear combinations of input, output and key bits.

- The linearity here refers to $\oplus$ (the mod 2 bit-wise sum).
- Goal: collect combinations whose probabilities of holding[1] (linear probability biases) are as close to 0 or 1 as possible.
- The relations are used in conjunction with known input-output pairs to recover the key.

[1] Over the space of all possible values of their variables.

Differential Attacks

Exploit relationship between $\Delta X = X_1 \oplus X_2$ and $\Delta Y = Y_1 \oplus Y_2$ for pairs of inputs (X_1, X_2) and corresponding outputs (Y_1, Y_2).

- Ideally, $P_{d_1,d_2} = \Pr(\Delta Y = d_2 | \Delta X = d_1) = 1/2^\ell$, for every d_1, d_2.
- Pairs (d_1, d_2) s.t. $P_{d_1,d_2} \gg 1/2^\ell$ are collected.
- It is a chosen plaintext attack, so an attacker aims at encrypting pairs (X_{i_1}, X_{i_2}) for which they know that a certain ΔY_i occurs with high probability.

Quantum attacks

Search problem: given $A \subset X$ and $f : X \to \{0, 1\}$ s.t. $f(x) = 1$ iff $x \in A$, find A having oracle accesso to f.

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Key length should be doubled
to protect against quantum attacks.

Further Reading I

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