Introduction to Cryptology

4.3 - AES

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The Advanced Encryption Standard (AES)

- In 1997, the US agency NIST[1] opened a competition for a new block cipher, to be called AES.
- In 2000, Rijndael, a new block cipher designed by Vincent Rijmen and Joan Daemen, won the competition.
- The are three standardised versions of AES. For each of them, the block length ℓ is 128.
- The key length n can be either 128 (AES-128), 192 (AES-192) or 256 (AES-256).

[1] National Institute of Standards and Technology

The Advanced Encryption Standard (AES)

- Given an input $x = (x_0, \ldots, x_{15})$, where each x_i is a byte, a 4x4 matrix s is initialised as:

$$s = \begin{pmatrix} x_0 & x_4 & x_8 & x_{12} \\ x_1 & x_5 & x_9 & x_{13} \\ x_2 & x_6 & x_{10} & x_{14} \\ x_3 & x_7 & x_{11} & x_{15} \end{pmatrix}.$$

- The number of rounds Nr depends on the key length: Nr is 10 for AES-128, 12 for AES-192 and 14 for AES-256.

- The key schedule takes the key k and constructs $Nr + 1$ 4x4 matrices, $sk_0, \ldots, sk_{Nr}$, where each entry is a byte.

The Advanced Encryption Standard (AES)

Input : x, Nr, $(sk_0, \ldots, sk_{Nr})$.

Output : y.

$s = (x_0, x_4, x_8, x_{12}; x_1, x_5, x_9, x_{13}; x_6, x_8, x_{10}, x_{14}; x_3, x_7, x_{11}, x_{15})$

$s = s \oplus sk_0$

for $j = 1, \ldots, Nr - 1$;

$s = SubBytes(s)$

$s = ShiftRows(s)$

$s = MixColumns(s)$

$s = s \oplus sk_j$

$s = SubBytes(s)$

$s = ShiftRows(s)$

$s = s \oplus sk_{Nr}$

return s

AES - SubBytes

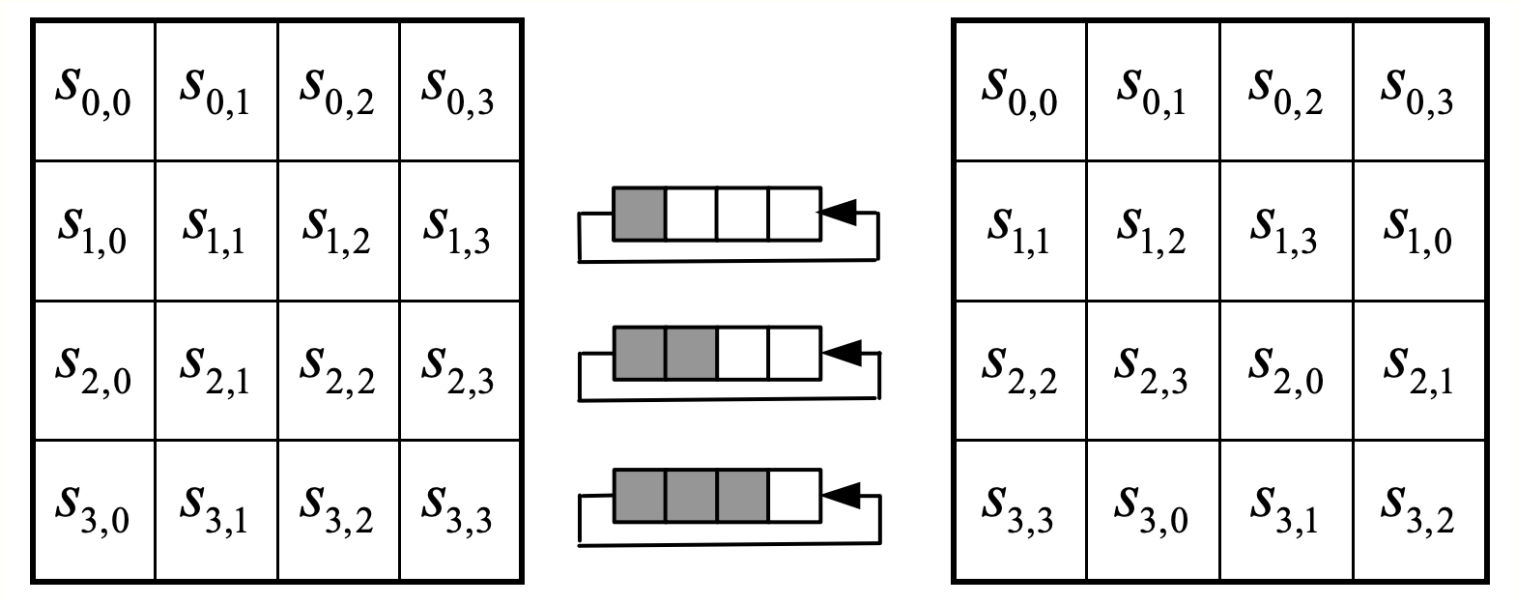
It operates independently on each byte of s.

A byte, represented by two hexadecimal digits UV, is sent into the byte of the cell in row U and column V.

		v														
		0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
u	0	63	7c	77	7b	f2	6b	6f	c5	30	01	67	2b	fe	d7	ab	76
	1	ca	82	c9	7d	fa	59	47	f0	ad	d4	a2	af	9c	a4	72	c0
	2	b7	fd	93	26	36	3f	f7	cc	34	a5	e5	f1	71	d8	31	15
	3	04	c7	23	c3	18	96	05	9a	07	12	80	e2	eb	27	b2	75
	4	09	83	2c	1a	1b	6e	5a	a0	52	3b	d6	b3	29	e3	2f	84
	5	53	d1	00	ed	20	fc	b1	5b	6a	cb	be	39	4a	4c	58	cf
	6	d0	ef	aa	fb	43	4d	33	85	45	f9	02	7f	50	3c	9f	a8
	7	51	a3	40	8f	92	9d	38	f5	bc	b6	da	21	10	ff	f3	d2
	8	cd	0c	13	ec	5f	97	44	17	c4	a7	7e	3d	64	5d	19	73
	9	60	81	4f	dc	22	2a	90	88	46	ee	b8	14	de	5e	0b	db
	a	e0	32	3a	0a	49	06	24	5c	c2	d3	ac	62	91	95	e4	79
	b	e7	c8	37	6d	8d	d5	4e	a9	6c	56	f4	ea	65	7a	ae	08
	c	ba	78	25	2e	1c	a6	b4	c6	e8	dd	74	1f	4b	bd	8b	8a
	d	70	3e	b5	66	48	03	f6	0e	61	35	57	b9	86	c1	1d	9e
	e	e1	f8	98	11	69	d9	8e	94	9b	1e	87	e9	ce	55	28	df
	f	8c	a1	89	0d	bf	e6	42	68	41	99	2d	0f	b0	54	bb	16

AES - ShiftRows

The bytes in the last three rows of the state s are cyclically shifted, while the first row remains unchanged:



AES - MixColumns

It operates on the state s column-by-column, transforming each column by means of the matrix transformation:

$$\begin{bmatrix} s'_{0,c} \\ s'_{1,c} \\ s'_{2,c} \\ s'_{3,c} \end{bmatrix} = \begin{bmatrix} 02 & 03 & 01 & 01 \\ 01 & 02 & 03 & 01 \\ 01 & 01 & 02 & 03 \\ 03 & 01 & 01 & 02 \end{bmatrix} \begin{bmatrix} s_{0,c} \\ s_{1,c} \\ s_{2,c} \\ s_{3,c} \end{bmatrix}$$

Given a byte $b = (b_0, b_1, \ldots, b_7)$, we have that:

- $(01) \cdot b = b$;
- $(02) \cdot b = (b_7, b_0 + b_7, b_1, b_2 + b_7, b_3 + b_7, b_4, b_5, b_6)$;
- $(03) \cdot b = (b_7 + b_0, b_0 + b_7 + b_1, b_1 + b_2, b_2 + b_7 + b_3, b_3 + b_7 + b_4, b_4 + b_5, b_5 + b_6, b_6 + b_7)$.

AES - Key schedule

The key k is divided into 4-byte words $k_1, \ldots, k_{Nk}$.

The functions/conventions used are:

- SubWord: takes a 4-byte input word and applies the S-box to each of the four bytes.
- RotWord: takes a 4-byte word (a_0, a_1, a_2, a_3) as input and returns (a_1, a_2, a_3, a_0).
- Rcon[i] is the 4-byte word $(x^{i-1}, 00, 00, 00)$, where x^{i-1} is the byte corresponding to the reduction of x^{i-1} modulo $x^8 + x^4 + x^3 + x + 1$ (the reduction is a polynomial of degree smaller than 8 in $\mathbb{F}_2[x]$).

AES - Key schedule

Input : $k = (k_1, \ldots, k_{Nk})$, Nr.

Output : $sk_0, \ldots, sk_{Nr}$.

$w = [], \text{temp} = [], i = 0$

while $i < Nk$

  $w[i] = k_{i+1}$

  $i = i + 1$

$i = Nk$

while $i < 4(Nr + 1)$

  $\text{temp} = w[i - 1]$

  if $i \equiv 0 \pmod{Nk}$

    $\text{temp} = \text{SubWord}(\text{RotWord}(\text{temp})) \oplus \text{Rcon[i/Nk]}$

  elseif $Nk > 6 \wedge i \equiv 4 \pmod{Nk}$

    $\text{temp} = \text{SubWord}(\text{temp})$

  $w[i] = w[i - Nk] \oplus \text{temp}$

  $i = i + 1$

return w

Security of AES

- No practical attacks that are notably better than brute-force search for the key.
- A practical instantiation of a (strong) pseudo-random permutation.
- Free, standardised, and efficient.

Further Reading I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Further Reading III

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