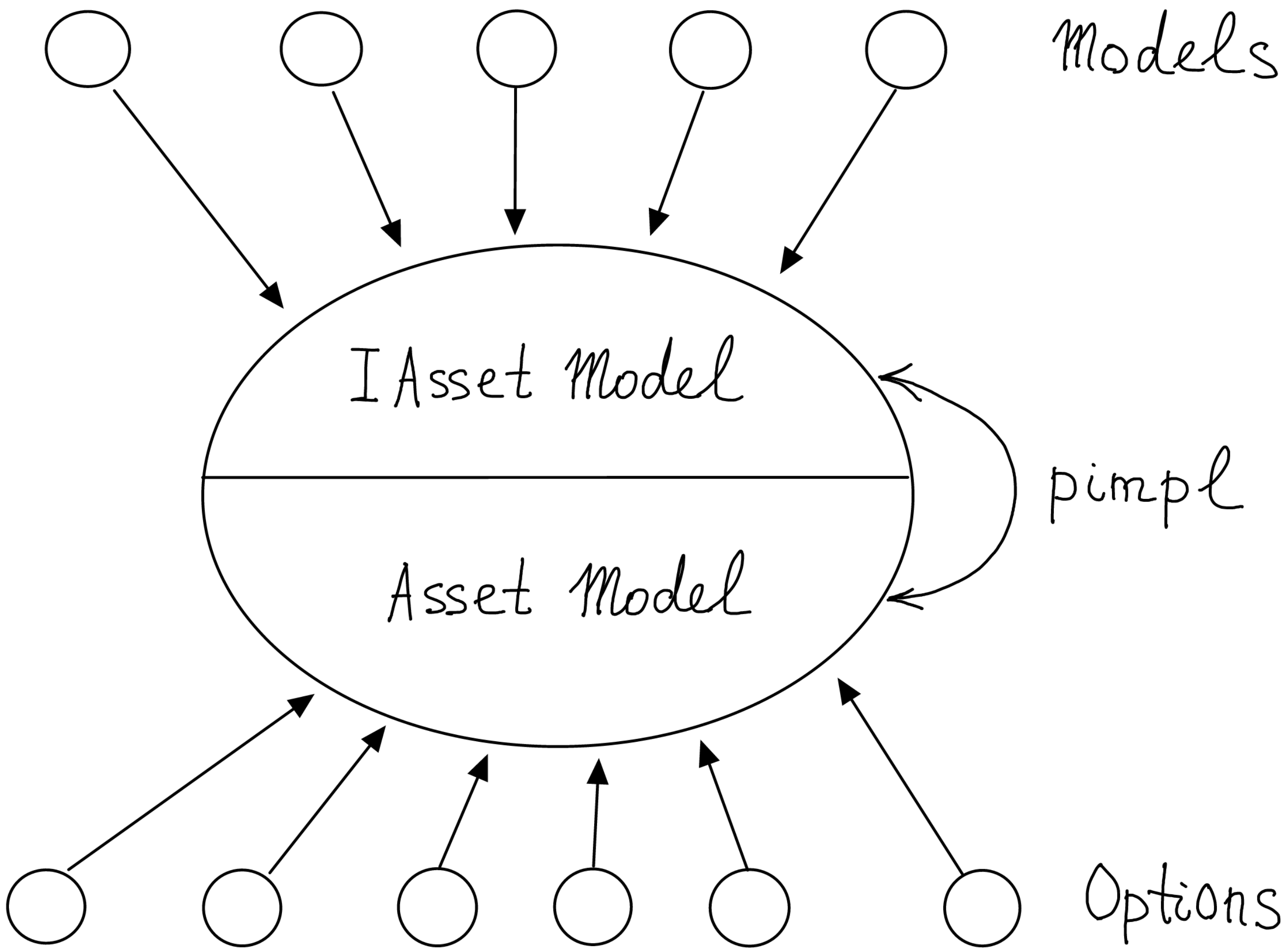
II : Standard and barrier options on a stock.

Design :



Work with Asset Model :

Step 1 : Event times

$$t_0 \quad \underbrace{t_1 \; t_2 \; \cdots \; t_M}_{\text{depend on option}}$$

initial time

<u>Goal</u>: make the vector of event times as small as possible.

<u>Step 2</u>:

(a) Basic payoffs (Slices):

(i) cash

(ii) spot prices

(iii) discount factors

(iv) forward prices

(b) Operations on Slices:

(i) At given event time: everything

(ii) Between event times: rollback.

Asset Standard: put.

Inputs

K : strike

T : maturity

t_0 : initial time, $t_0 < T$.

Event times: $\{ t_0, T \}$

Algorithm

$$X_T = \max(K - \overset{\text{spot}}{S_T}, 0)$$

$$X_{t_0} = \mathcal{R}_{t_0}(X_T) \qquad \blacksquare$$

Asset Standard : American Put

Inputs :

K : strike

$(t_i)_{i=1,\dots,M}$: exercise times

t_0 : initial time

Event times : $\{t_0, t_1, \dots, t_M\}$

① Boundary condition.

$$X_{t_M} = 0$$

② Loop $\quad \underset{\text{end}}{t_0} \longleftarrow \underset{\text{begin}}{t_M}$

$$X_{t_m} (?) \longleftarrow X_{t_{m+1}} \text{ (known)}$$

// $X_{t_{m+1}}$: value to continue (no exercise $\leq t_{m+1}$)

$$\underset{(\leq t_m)}{X_{t_{m+1}}} = \max\Big(\underset{(\leq t_{m+1})}{X_{t_{m+1}}},\ K - \underset{\text{spot}}{S_{t_{m+1}}} \Big)$$

$$\underset{(\leq t_m)}{X_{t_m}} = \mathcal{R}_{t_m}\Big(\underset{(\leq t_m)}{X_{t_{m+1}}}\Big)$$

(3) After loop.

return X_{t_0}. $\blacksquare$

Asset Standard: up-down-out barrier.

Inputs:

L : lower barrier

U : upper barrier

N : notional

$(t_i)_{i=1,\dots,M}$: barrier times

t_0 : initial time

Event times: $\{t_0, t_1, \dots, t_M\}$

Algorithm

1. Boundary condition:

$$X_{t_M} = N$$

2. Loop $\overset{\text{end}}{t_0} \longleftarrow \overset{\text{begin}}{t_M}$

$$X_{t_m} (?) \longleftarrow X_{t_{m+1}} \text{ (known)}$$

// $X_{t_{m+1}}$: value to continue (no barriers $\leq t_{m+1}$)

$$\underset{(\le t_m)}{X_{t_{m+1}}} \mathrel{*}= 1\{\underbrace{S_{t_{m+1}}}_{\text{spot}} \geq L\} * 1\{U \geq S_{t_{m+1}}\}$$

$$\underset{(\le t_m)}{X_{t_m}} = \mathcal{R}_{t_m}\Big(\underset{(\le t_m)}{X_{t_{m+1}}}\Big)$$

(3) After loop

return X_{t_0}. $\blacksquare$

Asset Standard: down-out American Call.

Inputs:

L : lower barrier

$(s_i)_{i=1,\dots,I}$: barrier times

K : strike

$(\tau_j)_{j=1,\dots,J}$: exercise times

t_0 : initial time

Event times: $\{t_0, \underbrace{t_1, \dots, t_M}_{\text{sorted union of barrier \& exercise times}}\}$

Algorithm

1. Boundary condition

$$X_{t_M} = 0$$

2. Loop $\overset{\text{end}}{t_0} \longleftarrow \overset{\text{begin}}{t_M}$

$$X_{t_m}\,(?) \longleftarrow X_{t_{m+1}}\,(\text{known})$$

// $X_{t_{m+1}}$: value to continue (no barriers & exercises $\leq t_{m+1}$)

Assume that barrier events take precedence.

if $\{ t_{m+1} \in (\text{exercise times}) \}$

{

$$\underbrace{X_{t_{m+1}}}_{(\leq t_m,\, \leq t_{m+1})} = \max \Big(\underbrace{X_{t_{m+1}}, S_{t_{m+1}} - K}_{(\leq t_{m+1},\, \leq t_{m+1})} \Big)$$

call — $\leq t_m$; barrier — $\leq t_{m+1}$

}

if $\{ t_{m+1} \in (\text{barrier times}) \}$

{

$$\underbrace{X_{t_{m+1}}}_{(\leq t_m,\, \leq t_m)} *= 1\{ S_{t_{m+1}} \geq L \}$$

}

$$X_{t_m} = R_{t_m}\left(X_{t_{m+1}} \right)$$

3 After loop.

return X_{t_0}. $\blacksquare$

Asset Standard: swing

Inputs:

K : strike

$(t_i)_{i=1,\dots,N}$: exercise times

M : # of exercises

t_0 : initial time

Event times: $\{t_0, \underbrace{t_1, \dots, t_N}_{\text{exercise times}}\}$

① Boundary condition.

$$X_{t_N}[m] = 0, \quad m = 0, \dots, M-1$$

② Loop $\overset{\text{end}}{t_0} \longleftarrow \overset{\text{begin}}{t_N}$

$$X_{t_n} (?) \longleftarrow X_{t_{n+1}} \text{ (known)}$$

// $X_{t_{n+1}}[m]$: value to continue after m exercises $\leq t_{n+1}$, $m = 0, \dots, M-1$.

for $(m=0;\ m<M-1;\ m{+}{+})$

{

$$\underset{(\leq t_n)}{X_{t_{n+1}}[m]} = \max\Big(\underset{(\leq t_{n+1})}{X_{t_{n+1}}[m]},\ S_{t_{n+1}} - K + \underset{(\leq t_{n+1})}{X_{t_{n+1}}[m+1]}\Big)$$

}

$$\underset{(\leq t_n)}{X_{t_{n+1}}[M-1]} = \max\Big(\underset{(\leq t_{n+1})}{X_{t_{n+1}}[M-1]},\ S_{t_{n+1}} - K\Big)$$

for $(m=0;\ m<M;\ m{+}{+})$

{

$$\underset{(\leq t_n)}{X_{t_n}[m]} = \mathcal{R}_{t_n}\Big(\underset{(\leq t_n)}{X_{t_{n+1}}[m]}\Big)$$

}

③ After loop

return $X_{t_0}[0]$. $\blacksquare$