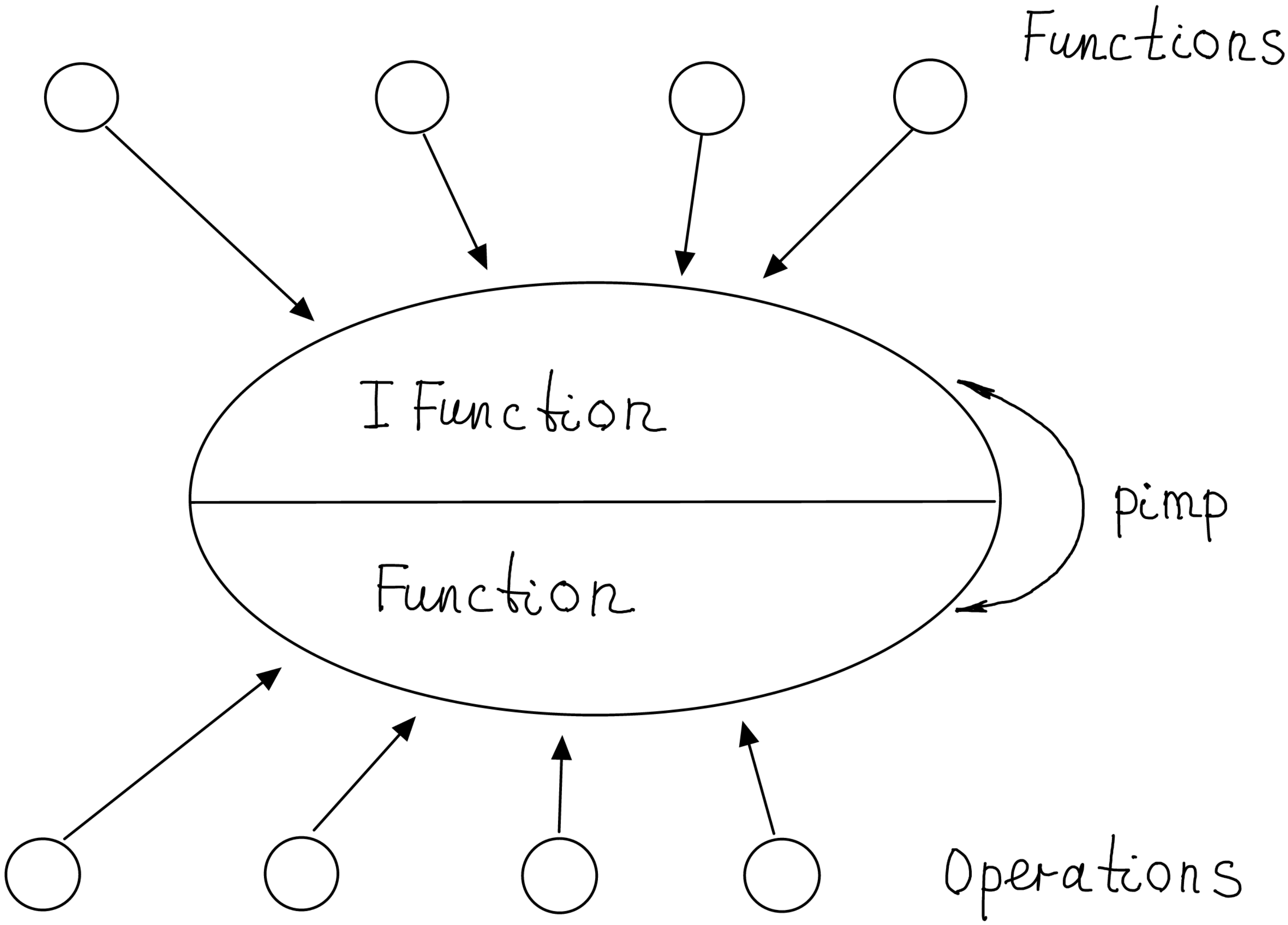
I: Construction of data curves for financial models.

Design :



Methods :

(a) Interpolation

(b) Least square fitting.

Data: Shape function for changes in yield curve in Hull-White model.

Inputs

$\lambda \geq 0$: the mean-reversion rate

t_0: the initial time

Output: the function

$$\Gamma(t) = \frac{1 - \exp(-\lambda(t-t_0))}{\lambda(t-t_0)}, \quad t \geq t_0.$$

Domain: $[t_0, +\infty)$

Algorithm: given $t \geq t_0$ we define

$$F = \lambda(t-t_0)$$

if ($F < \varepsilon$ ($\approx 10^{-8} \approx$ 1cek))

{

return $\lim_{x \to 0} \frac{1-\exp(-x)}{x} = 1$

}

// otherwise

$$\text{return} \quad \frac{1 - \exp(-F)}{F}$$

$\blacksquare$

Data: log-linear interpolation of discount curve.

Inputs:

$(t_i)_{i=1,\dots,M}$: maturities, $t_{i+1} > t_i$.

$(d_i)_{i=1,\dots,M}$: discount factors.

t_0 : initial time, $t_0 < t_1$.

Output: discount curve $D = D(t)$ on $[t_0, t_M]$ obtained by log-linear interpolation.

Domain: $[t_0, t_M]$

Algorithm:

Step 1: Compute the vectors of arguments: $\overline{x} = (t_0, \dots, t_M)$

& values: $\overline{y} = (\underset{\overset{\|}{\log 1}}{0}, \log d_1, \dots, \log d_M)$

for linear interpolation.

<u>Step 2</u>: Linear interpolation of

$$(\bar{x}, \bar{y}) \implies L(t), \; t \in [t_0, t_M].$$

<u>Step 3</u>:

return $D(t) = \exp(L(t)), \; t \in [t_0, t_M]$.

$\blacksquare$

Data: Least square fit in Hull-White model.

Inputs:

$(t_i)_{i=1,\dots,M}$: maturities, $t_i < t_{i+1}$.

$(d_i)_{i=1,\dots,M}$: discount factors.

$\lambda \geq 0$: mean-reversion rate.

t_0 : initial time.

Output: the discount curve

$$D(t) = \exp\left(-\hat{A}\,\Gamma(t)\,(t-t_0)\right),$$

$$t \in [t_0, +\infty),$$

where

$$\Gamma(t) = \frac{1-\exp(-\lambda(t-t_0))}{\lambda(t-t_0)}$$

is the shape of changes in yield curve in Hull-White model and

$$\hat{A} = \arg\min_{A} \sum_{i=1}^{M} \left(y_i - A\Gamma(t_i) \right)^2$$

with the market yields

$$y_i = -\frac{\log d_i}{t_i - t_0}, \quad i = 1, \ldots, M.$$

<u>Domain:</u> $[t_0, +\infty)$

<u>Algorithm:</u>

<u>Step 1:</u> Compute the market yields:

$$y_i = -\frac{\log d_i}{t_i - t_0}, \quad i = 1, \ldots, M.$$

<u>Step 2:</u> Compute the values

$$\Gamma_i = \Gamma(t_i), \quad i = 1, \ldots, M.$$

<u>Step 3:</u> Compute the constant $\hat{A}$ from the least square fit:

$$\hat{A} = \arg\min_{A} \sum_{i=1}^{M} \left(y_i - A\Gamma_i \right)^2$$

We deduce that

$$\hat{A} = \frac{\mathrm{Cov}(\gamma, \Gamma)}{\mathrm{Var}(\Gamma)},$$

where

$$\mathrm{Cov}(\gamma, \Gamma) = \sum_{i=1}^{M} \gamma_i \Gamma_i,$$

$$\mathrm{Var}(\Gamma) = \sum_{i=1}^{M} \Gamma_i^2.$$

Step 4

return $D(t) = \exp\left(-\underbrace{\hat{A}\,\Gamma(t)}_{\text{yield}}(t - t_0)\right)$

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