Examples for the course “Financial Computing with C++”

The following functions are implemented in the project **Examples**.

Data curves

The stationary form of changes in yield shape curve for Hull and White model

Input :

1. t_0: the initial time given as year fraction.
2. λ: the mean-reversion rate in Hull and White model.

Output :

1. The shape of changes in continuously compounded yield curve on $[t_0, +\infty]$, that is, the function

$$\Gamma(t) = \frac{1 - \exp(-\lambda(t - t_0))}{\lambda(t - t_0)}.$$

Discount curve obtained by log linear interpolation

Input :

1. $(t_i)_{1 \leq i \leq m}$: the vector of times with known discount factors;
2. $(d_i)_{1 \leq i \leq m}$: the vector of known discount factors;
3. t_0: the initial time.

Output :

1. Discount curve on $[t_0, t_m]$ obtained by log linear interpolation of the given collection of discount factors.

Discount curve for Hull and White model obtained by least squire fitting of market yields

Input :

$(t_i)_{1\le i\le m}$: times for known discount factors ($t_1 > t_0$).

$(d_i)_{1\le i\le m}$: known discount factors

λ : mean reversion rate

t_0 : initial time

Output : discount curve in the form

$$d(t) = \exp\left(-A\left(\frac{1-e^{-\lambda(t-t_0)}}{\lambda}\right)\right), \quad t \ge t_0, \tag{1}$$

where the constant A minimizes the quadratic approximation error for continuously compounded yields:

$$\sum_{i=1}^{m}(r(t_i) - r_i)^2 \xrightarrow{A} \min .$$

Here $r(t_i)$ and r_i are continuously compounded yields:

$$r(t_i) = -\frac{\ln d(t_i)}{t_i - t_0}, \quad r_i = -\frac{\ln d_i}{t_i - t_0}.$$

The problem is motivated by the fact that (1) represents the *minimal* family of discount curves appearing in Hull and White model with constant mean-reversion rate λ and containing the "flat" discount curve: $d(t) \equiv 1$.

Standard options in single asset model

Standard put

K : strike price

T : maturity

The payoff of the option at the maturity is given by

$$V(T) = \max(K - S(T), 0)$$

American put

K : strike

$(t_i)_{1 \le i \le N}$: exercise times

A holder of the option can exercise it at any time t_i. In this case he receives the payment:

$$f(t_i) = \max(K - S(t_i), 0),$$

where $S(t)$ is the price of the stock at time t.

Barrier up-or-down-and-out option

U : upper barrier

L : lower barrier

$(t_i)_{1 \le i \le M}$: barrier times

N : notional

The payoff of the option at maturity (the last barrier time t_M) is given by the notional amount N if the stock price will not cross lower and upper barriers for all barrier times. Otherwise the option expires worthless. In other words, the payment of the option is given by

$$f_{t_M} = N 1_{\{\tau > t_M\}},$$

where

$$\tau = \inf\{t_i : \quad S(t_i) \geq U \text{ or } S(t_i) \leq L\}$$

is the exit time of the stock price through the barriers U and L at the barrier times $(t_i)_{1 \leq i \leq M}$.

Down-and-out american call

L : lower barrier

$(t_i)_{1 \leq i \leq M}$: barrier times

K : strike

$(u_i)_{1 \leq i \leq N}$: exercise times ($u_N > t_M$).

The option behaves as the american call option with the strike K and the exercise times $(u_i)_{1 \leq i \leq N}$ until the first barrier time when the stock price is below the lower barrier L. At this exit time the option is canceled.

Swing option

K : strike

$(t_i)_{1 \leq i \leq N}$: exercise times.

M**:** the maximal number of exercises, $M \leq N$.

A holder of the option is given the right to purchase M stocks at the price K per share. The transactions should take place at exercise times. Only *one* stock can be bought at a particular exercise time, that is, to get n stocks the holder of the option should exercise it at n *different* exercise times. The options of this type are actively traded on energy markets.

Standard options in interest rate model

Interest rate cap

N : notional

C : cap rate

δt : interval of time between the payments given as year fraction.

m : total number of payments

Assume that today is the issue time of the cap. Denote this time by t_0. The payment times of the cap are given by

$$t_i = t_0 + i\delta t, \quad 1 \leq i \leq m.$$

At payment time t_i the owner of the cap *receives* the payment

$$N \max(L(t_{i-1}, t_i)\delta t - C\delta t, 0),$$

where $L(s,t)$ is the LIBOR computed at time s for maturity t.

Swaption

T : maturity

Parameters of underlying swap:

N : notional

R : fixed rate

δt : interval of time between the payments given as year fraction.

m : total number of payments

side : this parameter defines the side of the swap contract, i.e. whether one pays “fixed” and receives “float” or otherwise.

At maturity T the owner of the option has the right to enter into the swap contract with the parameters defined above and the issue time T.

Cancellable interest rate collar

N : notional

C : cap rate

F : floor rate ($F < C$).

δt : interval of time between the payments given as year fraction.

m : total number of payments

Assume that today is the issue time of the collar. Denote this time by t_0. The payment times of the collar are given by

$$t_i = t_0 + i\delta t, \quad 1 \leq i \leq m.$$

At payment time t_i

1. if the LIBOR rate $L(t_{i-1}, t_i)$ is greater then the cap rate C, then the owner of the collar *receives* the payment

$$N\delta t(L(t_{i-1}, t_i) - C).$$

2. if the LIBOR rate $L(t_{i-1}, t_i)$ is less then the floor rate F, then the owner of the collar *pays* the payment

$$N\delta t(F - L(t_{i-1}, t_i)).$$

3. after the payment is either received or paid the owner of the collar *has the right to cancel* the contract. No payments will be made after that.

Down-and-out cap

Underlying cap :

N : notional

R : cap rate

δt : interval of time between the payments given as year fraction.

m : total number of payments

L : lower bound for float (LIBOR) rate.

Assume that today is the issue time of the cap. Denote this time by t_0. The payment times of the cap are given by

$$t_i = t_0 + i\delta t, \quad 1 \leq i \leq m.$$

The down-and-out cap generates the same cash flow as interest rate cap up to (and including) the payment time, when the float rate is less then L. After the first payment time when the float rate is less then L the cap is terminated.

In other words, if we denote by τ the first payment time t_i, when float rate $L(t_i, t_i + \delta t)$ between t_i and $t_i + \delta t$ is less then L, then for a payment time t_j

1. if $t_j \leq \tau$, then the owner of the cap receives the same payment as in the case of interest rate cap:

$$N \max(L(t_{j-1}, t_j)\delta t - R\delta t, 0)$$

2. if $t_j > \tau$, then the payment equals 0.

Future on LIBOR

The future contracts of these types are traded, for example, on EUREX, where the underlying is 3 month EURO LIBOR.

Input: the parameters of future contract.

- Δ : period for LIBOR given as year fraction (for example, $\Delta = 0.25$ for 3 month LIBOR).
- T : maturity for future contract.
- m : the number of settlement times between today and maturity

Output: the future price $F(t_0)$ computed at initial time

We assume that the settlement times are given by

$$t_i = t_0 + i\delta t, \quad 0 < i \leq m,$$

(include T, but do not contain initial time), where t_0 is the initial time and

$$\delta t = \frac{T - t_0}{m}.$$

Future contract on LIBOR involves the following transactions:

1. It costs nothing to enter into either a long or a short position in the future contract
2. At time t_i before maturity, when $1 \leq i < m$
   (a) the buyer (long position) pays the future price $F(t_{i-1})$ established at previous trading day
   (b) the seller (short position) pays the future price $F(t_i)$ established at the current trading day
3. At maturity $T = t_m$
   (a) the buyer (long position) pays the future price $F(t_{m-1})$ established at previous trading day
   (b) the seller (short position) pays the amount of money given by
   $$F(t_m) = F(T) = 1 - L(T, T + \Delta),$$
   and $L(T, T+\Delta)$ is the LIBOR computed at time T for maturity $T + \Delta$.

Path dependent options in single asset model

Asian call

T : maturity

K : strike

$(t_i)_{1 \leq i \leq N}$: reset times, $t_N < T$.

At the maturity a holder of the option receives the payment:

$$f(T) = \max\left(\frac{1}{N} \sum_{1 \leq i \leq N} S(t(i)) - K, 0\right),$$

where $S(t)$ is the price of the underlying stock at time t.

Barrier up-or-down-and-out option

U : upper barrier

L : lower barrier

$(t_i)_{1\leq i\leq M}$: barrier times

N : notional

The payoff of the option at maturity (the last barrier time t_M) is given by the notional amount N if the stock price will not cross lower and upper barriers for all barrier times. Otherwise the option expires worthless. In other words, the payment of the option is given by

$$f_{t_M} = N1_{\{\tau > t_M\}},$$

where

$$\tau = \inf\{t_i : \quad S(t_i) \geq U \text{ or } S(t_i) \leq L\}$$

is the exit time of the stock price through the barriers U and L at the barrier times $(t_i)_{1\leq i\leq M}$.

Path dependent options in interest rate model

Savings account

δt : interval of time between the payments given as year fraction.

m : total number of payments

N : notional amount.

Assume that today is the issue time for the savings account. We denote this time by t_0. We also denote by $L(s,t)$ the LIBOR computed at s for maturity t. The interest payment times are given by

$$t_i = t_0 + i\delta t, \quad 1 \leq i \leq m.$$

The values of the savings account at times t_i and t_{i+1} are related by

$$X(t_{i+1}) = X(t_i)(1 + L(t_i, t_{i+1})\delta t).$$

Put on savings account

δt : interval of time between the payments given as year fraction.

m : total number of payments

N : notional amount.

R**:** strike fixed rate.

Assume that today is the issue time for the savings account. We denote this time by t_0. We also denote by $L(s,t)$ the LIBOR computed at s for maturity t. The interest payment times are given by

$$t_i = t_0 + i\delta t, \quad 1 \leq i \leq m.$$

The values of the savings account at times t_i and t_{i+1} are related by

$$X(t_{i+1}) = X(t_i)(1 + L(t_i, t_{i+1})\delta t).$$

The values of the account that pays fixed rate R at times t_i and t_{i+1} are related by the formula:

$$Y(t_{i+1}) = Y(t_i)(1 + R\delta t).$$

The payoff of the put option at maturity is given by

$$V(T) = \max(Y(t_m) - X(t_m), 0).$$