Data: forward prices for stock paying dividends

Inputs:

S_{t_0} : spot price

$(t_m)_{m=1,\dots,M}$: dividend times

$(D_m)_{m=1,\dots,M}$: cash dividends

$B(t_0,t)$, $t \in [t_0, T]$: discount curve

t_0 : initial time

T : some time

Output: $F(t_0,t)$, $t \in [t_0, T]$, the curve of forward prices

Domain: $[t_0, T]$, same as for discount curve.

Algorithm:

Value to get stock at t

$$= B(t_0,t)\,F(t_0,t)$$

$$= S_{t_0} - \sum_{t_m<t} B(t_0,t_m)\,D_m$$

Thus we obtain that

$$F(t_0, t) = \Big(S_{t_0} - \sum_{t_m < t} B(t_0, t_m) D_m \Big) \Big/ B(t_0, t)$$

$\blacksquare$

Asset Standard : up-range-out put

Inputs

U : upper barrier

M : # of "out" times

$(t_n)_{n=1,\dots,N}$: barrier times

K : strike

T : maturity

t_0 : initial time

Event times : $\{ \underbrace{t_0}_{\text{initial time}} \underbrace{t_1, \dots, t_N}_{\text{barrier times}}, \underbrace{T}_{\text{maturity}} \}$

① Boundary condition

$$P_T = \max(K - S_T, 0)$$

$$P_{t_N} = \mathcal{R}_{t_N}(P_T)$$

$$X_{t_N}[m] = P_{t_N}, \quad m = 0, \dots, M-1.$$

② Loop $\underbrace{t_0}_{\text{end}} \longleftarrow \underbrace{t_N}_{\text{begin}}$

$$X_{t_n}(?) \longleftarrow X_{t_{n+1}} \text{ (known)}$$

// $X_{t_{n+1}}[m]$: value to continue if the upper barrier was crossed m times $\leq t_{n+1}$, $m = 0, \dots, M-1$.

$$\underset{(\leq t_n)}{X_{t_{n+1}}[m]} = \underset{(\leq t_{n+1})}{X_{t_{n+1}}[m]}\, \mathbb{1}_{\{S_{t_{n+1}} < U\}} + \underset{(\leq t_{n+1})}{X_{t_{n+1}}[m+1]}\, \mathbb{1}_{\{S_{t_{n+1}} \geq U\}},$$

$$m = 0, \dots, M-2.$$

$$\underset{(\leq t_n)}{X_{t_{n+1}}[M-1]} = \underset{(\leq t_{n+1})}{X_{t_{n+1}}[M-1]}\, \mathbb{1}_{\{S_{t_{n+1}} < U\}}$$

$$\underset{(\leq t_n)}{X_{t_n}[m]} = \mathcal{R}_{t_n}\Big(\underset{(\leq t_n)}{X_{t_{n+1}}[m]} \Big), \quad m = 0, \dots, M-1$$

3 After loop

return $X_{t_0}[0]$. $\blacksquare$

Interest Rate Standard:

futures on cheapest bond to deliver

Inputs:

T : maturity of futures

M : # of futures times

Bonds with issue time T : $i = 1, \dots, d$

N_i : notional

q_i : coupon rate

δt_i : period between coupons

L_i : # of coupon payments

Output: futures price $F(t_0)$

Event times: $\{t_0, t_1, \dots, t_M = T\}$

$$t_m = t_0 + \Delta m, \quad \Delta = \frac{T - t_0}{M}.$$

① Boundary condition

$$F(t_M) = \min_{1 \le i \le d} \big(\mathrm{bond}(i) \big)$$

$$\text{bond}(i) =$$

$$N_i \left\{ \left(\sum_{j=1}^{L_i} B(t_M, t_M + j\,\delta t_i) \right) q\,\delta t_i + B(t_M, t_M + L_i\,\delta t_i) \right\}$$

(2) Loop $\underset{\text{end}}{t_0} \longleftarrow \underset{\text{begin}}{t_M}$

$$F_{t_m}\ (?) \longleftarrow F_{t_{m+1}}\ (\text{known})$$

$$F_{t_m} = \mathcal{R}_{t_m}\left(F_{t_{m+1}}\right) / B(t_m, t_{m+1})$$

(3) After loop

return F_{t_0}. $\blacksquare$

Asset Path-Dependent: clique

Inputs:

T: maturity

$(s_n)_{n=1,\dots,N}$: averaging times ; $s_N < T$

$(\tau_m)_{m=1,\dots,M}$: reset times ; $\tau_M < s_N$

K: initial strike

t_0: initial time

Event times:

$$t_0 \quad \underbrace{t_1 \dots t_J}_{\text{sorted union of averaging and reset times}}$$

Path-dependent state process:

Y_{t_m} : current strike

(a) Origin

$$Y_{t_0} = K$$

(b) <u>Reset indexes:</u> indexes of reset times

(c) <u>One-step evolution:</u>

$$Y_{t_m} \text{ (known)} \longrightarrow Y_{t_{m+1}} \text{ (?)}$$

$$Y_{t_{m+1}} = S_{t_{m+1}}$$

stock price

<u>Algorithm</u>

① Boundary condition

$$X_{t_J} = 0$$

② Loop $t_0 \longleftarrow t_J$

end begin

$$X_{t_j} \text{ (?)} \longleftarrow X_{t_{j+1}} \text{ (known)}$$

$X_{t_{j+1}}$: value to continue (value of call payments computed $> t_{j+1}$)

if $t_{j+1} \in$ averaging times, then

$$X_{t_{j+1}} \mathrel{+}= B(t_{j+1}, T) * \max\left(S(t_{j+1}) - Y(t_{j+1}), 0 \right)$$

discount factor

$$X_{t_j} = \mathcal{R}_{t_j}\left(X_{t_{j+1}} \right)$$

③ After loop

$$X_{t_0} \mathrel{*}= \frac{1}{N}$$

# of averaging times

return X_{t_0}

$\blacksquare$