Final exam for the course "Financial Computing (with C++)"

When implementing the functions below you can assume that all barrier, reset and exercise times are strictly greater than the initial time. The issue times for all derivatives equal the initial time.

Your performance on the final exam will be evaluated on the basis of the best 3 solutions.

Data curves

Volatility curve obtained through the linear interpolation of the variance curve

Input :

1. $(t_i)_{1\leq i\leq m}$: the vector of times with known volatilities, $t_1 > t_0$;
2. $(V(t_i))_{1\leq i\leq m}$: the vector of known volatilities;
3. t_0: the initial time.

Output :

1. Volatility curve $V = V(t)$ on $[t_0, t_m]$. This volatility curve is computed by applying the linear interpolation to the variance curve $D = D(t)$. The relation between the volatility and variance curves is given as follows:

   $$D(t) = (t - t_0) \left(V(t)\right)^2$$

   Note that the resulting interpolation implies that the volatility is constant on the time interval $[t_0, t_1]$, that is,

   $$V(t) = V(t_1), \quad t \in [t_0, t_1].$$

Standard options in single asset model

This problem should be solved using the "standard" implementation of the Asset Model. No additional state processes are allowed!

Compound Call on American put

T : maturity of call option on American put

P : strike of call option on American put

K : strike of American put

$(t_i)_{1\leq i\leq N}$: exercise times for American put: $t_1 > T$. The intrinsic value of the American put at t_i is given by

$$f(t_i) = \max(K - S(t_i), 0),$$

where $S(t_i)$ is the price of the stock at t_i.

At maturity T a holder of the compound call option can buy the American put option (with strike K and exercise times $(t_i)_{1\leq i\leq N}$) at the price P.

Standard options in interest rate model

This problem should be solved using the “standard” implementation of the Interest Rate Model. No additional state processes are allowed!

Callable capped floater

N : notional

C : cap rate

δt : interval of time between the payments given as year fraction.

m : total number of payments

δL : spread over Libor.

Assume that today is the issue time of the capped floater. Denote this time by t_0. Denote by $L(s, t)$ the LIBOR rate computed at s for maturity t. The payment times of the derivative security are given by

$$t_i = t_0 + i\delta t, \quad 1 \leq i \leq m.$$

At payment time t_i

1. the holder *receives* the coupon

$$N\delta t \times \min(L(t_{i-1}, t_i) + \delta L, C)$$

2. the seller of the option *has the right to cancel* the contract. In this case, in addition to the above coupon he pays the notional. No payments will be made in the future. Note that the option can not be terminated at issue time.

If the contract has not been terminated before, then at maturity t_m the holder receives the above coupon plus notional.

Path dependent options in interest rate model

You should solve this problem by adding only one additional state process.

Index amortizing swap

N : notional

R : fixed rate

δt : interval of time between the payments given as year fraction.

m : total number of payments

side : this parameter defines the side of the swap contract, i.e. whether one pays "fixed" and receives "float" or otherwise.

$f = f(r)$**:** amortizing function for the notional (decreasing and takes values in (0,1))

q**:** threshold for the cleanup call

Brief description: as in the standard swap contract the sides make interest payments to each other. However, the notional amount is reset at payment times by float (LIBOR) rate according to amortizing function $f = f(r)$. In addition, the swap is canceled as soon as the remaining notional amount will reach the level below q percents of the original notional amount.

Denote by $(t_i)_{1\leq i\leq m}$ the payment times of the swap:

$$t_i = t_0 + i\delta t, \quad 1 \leq i \leq m.$$

Denote also by $N(t_i)$ and $r(t_i)$ the notional amount and float (LIBOR) rate computed at t_i, respectively, where $0 \leq i \leq m$ and

$$N(t_0) = N.$$

In the case of index amortizing swap the evolution of notional amount over time has the form:

$$N(t_{i+1}) = f(r_{t+1})N(t_i), \quad 0 \leq i < m.$$

The swap is canceled at the first payment time τ such that $N_\tau \leq qN$ or, if this does not happen, then at maturity.

At payment time $t_{i+1} \leq \tau$

1. One side pays "float" interest, which equals

$$N(t_i)r(t_i)\delta t,$$

2. Other side pays "fixed" interest, which equals

$$N(t_i)R\delta t.$$

Hint: to implement this transaction you need to use the member function `void apply(double (*f)(double)` of the class Slice. Check the documentation!