Final exam for the course "Financial Computing (with C++)"

When implementing the functions below you can assume that all barrier, reset and exercise times are strictly greater than the initial time. The issue times for all derivatives equal the initial time.

Your performance on the final exam will be evaluated on the basis of the best 3 solutions.

Data curves

Discount curve obtained from swap rates by log linear interpolation

Input :

1. t_0: the initial time given as year fraction.
2. δt: the time interval between payments given as year fraction.
3. $(R_i)_{1\leq i\leq m}$: the vector of swap rates, where R_i is the current swap rate in the contract with time interval δt and the number of payments i.

Output :

1. Discount curve on $[t_0, t_0 + m\delta t]$ obtained by the following procedure:
   (a) first, we compute discount factors for maturities: $t_i = t_0 + i\delta t$, $0 \leq i \leq m$,
   (b) second, we apply log linear interpolation to the discount factors computed in the previous item.

Standard options in single asset model

This problem should be solved using the "standard" implementation of the Asset Model. No additional state processes are allowed!

“BOOST” (Banking On Overall Stability) option

The following option was introduced by Societe Generale in 1994.

N : notional amount.

L : lower barrier.

U : upper barrier

$(t_i)_{1\leq i\leq m}$: barrier times

The option terminates at the first barrier time, when the price of the stock hits either of the barriers, that is, at the barrier time t_{i^*}, which index is given by

$$i^* = \min\{1 \leq i \leq m : \quad S(t_i) > U \text{ or } S(t_i) < L\}.$$

At the exit time t_{i^*} the holder of the option receives the payoff $N\frac{i^*-1}{m}$ (the product of the notional amount on the percentage of the barrier times that the price of the stock spends inside two barriers).

If the price of the stock never exits the barriers, then at the last barrier time the holder of the option is paid the notional amount N.

Standard options in interest rate model

This problem should be solved using the “standard” implementation of the Interest Rate Model. No additional state processes are allowed!

Putable bond with resettable coupon

The following bond is a variant of so-called *ratchet bonds*. It should be evaluated using the “standard” implementation of interest rate model.

N : notional

c : initial coupon rate

d : reset value for the coupon rate ($d < c$).

δt : interval of time between the payments given as year fraction.

m : total number of coupon payments.

L : the redemption price of the bond as percentage of the notional. Typically, $L < 1$.

Brief description : after coupon payment the issuer can reset the coupon rate from the original (higher) value c to the (lower) reset value d. However, then at any payment time greater or equal the reset time and less than maturity the holder can sell the bond back to the issuer for the redemption value LN.

Denote by t_0 the current time and by $(t_i)_{1 \le i \le m}$ the future coupon times:

$$t_i = t_0 + i\delta t, \quad 1 \le i \le m.$$

Depending on what has happened in the past there are the following possibilities for coupon time t_i:

1. If $t_i < t_m$ (not the maturity) and if the coupon rate has not been reset before, then
   (a) the holder receives the original coupon $Nc\delta t$
   (b) the issuer can reset the original coupon rate c to a lower coupon rate d and if he does so, then the holder has the right to sell the bond back to the issuer at the redemption price LN.
2. If $t_i < t_m$ (not the maturity) and if the coupon rate has been already reset to d, but the bond has not been terminated, then
   (a) the holder receives the reset coupon $Nd\delta t$
   (b) the holder can sell the bond back to the issuer for the redemption amount LN.
3. If $t_i = t_m$ (the maturity) and the bond has not been terminated before, the holder of the bond receives coupon payment (original or reset) as well as the notional amount N.

Path dependent options in interest rate model

You should solve this problem by adding only one additional state process.

Resettable interest rate cap

N : notional amount

q : the initial cap rate.

δL : cap spread over float rate at reset times.

δt : interval of time between two payments given as year fraction.

m : total number of payments

(s_i) : reset times of the cap

The payment times of the cap are given by

$$t_i = t_0 + i\delta t, \quad 1 \leq i \leq m,$$

where t_0 is the initial time.

At any payment time t_i the owner of the cap receives the payment

$$N \max(L(t_{i-1}, t_i)\delta t - R(t_{i-1})\delta t, 0),$$

where

- $L(s, t)$ is the float (Libor) rate at s for maturity t,
- $R(t)$ is the cap rate at t given by
  1. the initial cap rate q if there are no reset times less or equal t
  2. the reset value determined at the previous reset time. If we denote by $s(t)$ the maximal reset time, which is less or equal t then the cap rate equals

$$R(t) = L(s(t), s(t) + \delta t) + \delta L.$$