Final exam for the course "Financial Computing (with C++)"

When implementing the functions below you can assume that all barrier, reset and exercise times are strictly greater than the initial time. The issue times for all derivatives equal the initial time.

Your performance on the final exam will be evaluated on the basis of the best 3 solutions.

Data curves

Forward prices for a stock that pays discrete dividends

Input :

1. the spot price
2. $(t_i)_{1\leq i\leq m}$: the vector of dividend times
3. $(D_i)_{1\leq i\leq m}$: the vector of fixed dividends (in cash)
4. discount curve
5. t_0: the initial time.

Output :

1. The curve of forward prices for the stock on the interval $[t_0, t_m]$.

Standard options in single asset model

This problem should be solved using the "standard" implementation of the Asset Model. No additional state processes are allowed!

Up-range-out put

U : upper barrier

$(t_i)_{1\leq i\leq M}$: barrier times. The first barrier time is greater than initial time and the last one is less than maturity.

N : the number of barrier events after which the option is canceled, $0 < N \leq M$.

K : strike

T : maturity ($T > t_M$).

The option is canceled immediately after the stock price has been above the upper barrier U for N barrier times. Otherwise, it behaves as standard European put option with maturity T and strike K.

Standard options in interest rate model

This problem should be solved using the "standard" implementation of the Interest Rate Model. No additional state processes are allowed!

Future on cheapest bond to deliver

This problem is motivated by the existing future contract on US treasury bonds.

Input: the parameters of the future contract

- T : the maturity of the future contract
- m : the number of "future" times before the maturity
- **Bonds to deliver** with indexes $1 \leq i \leq n$. Assume that issue times for all bonds equal T (the maturity of the future contract). The parameters of the bond with index i include
  - N_i : notional
  - R_i : coupon rate
  - $(\delta t)_i$: interval of time between the payments given as year fraction.
  - m_i : the number of coupon payments.

Output: the future price $F(t_0)$ at the initial time.

Denote by

$$h = \frac{T - t_0}{m}$$

the time difference between two adjacent "future" times, where t_0 is the initial time. The set of "future" times, that is, the times, when future price is determined, is given by

$$t_i = t_0 + ih \quad 0 \leq i \leq m-1.$$

Denote by $F(t_i)$ the future price at t_i. Recall that

1. It costs nothing to enter into either a long or a short position in the future contract
2. At time t_i before maturity, when $1 \leq i < m$
   (a) the buyer (long position) pays the future price $F(t_{i-1})$ established at previous trading day
   (b) the seller (short position) pays the future price $F(t_i)$ established at the current trading day
3. At maturity $T = t_m$
   (a) the buyer (long position) pays the future price $F(t_{m-1})$ established at previous trading day
   (b) the seller (short position) delivers one of the available coupon bonds. Note that the seller has the right to choose which bond to deliver.

Path dependent options in single asset model

You should solve this problem by adding only one additional state process.

Clique option

T**:** maturity

$(t_i)_{1 \leq i \leq N}$: averaging times, $t_N < T$.

$(u_i)_{1 \leq i \leq M}$: reset times, $u_M < t_N$.

K : initial strike.

The payoff of the option at maturity is given by

$$V(T) = \frac{1}{N} \sum_{i=1}^{N} \max(S(t_i) - K(t_i), 0),$$

where $S(t_i)$ and $K(t_i)$ are, respectively, the spot price and the strike at t_i.

The value of the strike $K(t_i)$ is given by

1. the initial strike K, if there are no reset times before t_i, that is, $u_1 > t_i$.

2. the spot price $S(u(t_i))$ at the previous reset time $u(t_i)$:

$$\begin{aligned} K(t_i) &= S(u(t_i)), \\ u(t_i) &= \max\{u_j : \quad u_j \leq t_i\}. \end{aligned}$$