Data: discount curve from swap rates by log linear interpolation

Inputs:

$(R_m)_{m=1,\dots,M}$: swap rates

δt : period

t_0 : initial time

Output:

discount curve on $[t_0, t_0 + M\delta t]$.

Domain: $[t_0, t_M]$,

where $t_m = t_0 + m\delta t$, $m = 1, \dots, M$.

Algorithm:

Step 1:

Vectors of arguments and values:

$\bar{x} = (t_0, t_1, \dots, t_M)$

$\bar{y} = (y_0, \dots, y_M)$: logs of

discount factors.

We have that

value of float payments =

$$= 1 - B(t_0, t_m) = R_m \delta t \, S_m$$

where

$$S_m = \sum_{k=1}^{m} B(t_0, t_k)$$

We obtain the recursion:

$$S_0 = 0$$

$$B(t_0, t_{m+1}) = \frac{1 - S_m R_{m+1} \delta t}{1 + R_{m+1} \delta t}$$

$$S_{m+1} = S_m + B(t_0, t_{m+1})$$

We obtain that

$$y_0 = \log B(t_0, t_0) = \log 1 = 0$$

$$y_m = \log B(t_0, t_m), \; m = 1, \dots, M.$$

Step 2 : linear interpolation

$$(\bar{x}, \bar{y}) \implies b = b(t), \; t \in [t_0, t_M]$$

Step 3 :

$$\text{return} \quad B(t_0, t) = \exp\left(b(t) \right)$$

$\blacksquare$

Asset Standard : boost

Inputs :

N : notional

L : lower barrier

U : upper barrier

$(t_m)_{m=1,\ldots,M}$: barrier times

t_0 : initial time

Event times : $\{t_0, t_1, \ldots, t_M\}$

Algorithm

① Boundary condition

$$X_{t_M} = N$$

② Loop $\underbrace{t_0}_{\text{end}} \longleftarrow \underbrace{t_M}_{\text{begin}}$

$$X_{t_m}\ (?) \longleftarrow X_{t_{m+1}}\ (\text{known})$$

$X_{t_{m+1}}$: value to continue (if the barriers were not crossed $\leq t_{m+1}$)

$$\underset{(\leq t_m)}{X_{t_{m+1}}} \mathrel{+}= \left(\frac{\sqrt{m}}{M} - \underset{(\leq t_{m+1})}{X_{t_{m+1}}}\right) *$$

$$* \left(1_{\{S_{t_{m+1}} \geq U\}} + 1_{\{L \geq S_{t_{m+1}}\}}\right)$$

$$\underset{(\leq t_m)}{X_{t_m}} = \mathcal{R}_{t_m}\left(\underset{(\leq t_m)}{X_{t_{m+1}}}\right)$$

(3) After loop.

return X_{t_0}

∎

Interest Rate Standard:

putable bond with reset coupon

Inputs:

N : notional

c : initial coupon rate

d : reset value for coupon rate $(d < c)$

δt : time interval between coupons

M : # of coupons

b : redemption percentage ; $b < 1$.

Event times: $\{t_0, t_1, \dots, t_{M-1}\}$

$$t_m = t_0 + m\,\delta t \,, \quad m = 1, \dots, M-1.$$

① Boundary condition

$$X_{t_{M-1}} = B(t_{M-1}, t_{M-1} + \delta t)(1 + c\,\delta t) N$$

$$Y_{t_{M-1}} = B(t_{M-1}, t_{M-1} + \delta t)(1 + d\,\delta t) N$$

(2) Loop $t_0 \longleftarrow t_{M-1}$

end begin

$(X_{t_m}, Y_{t_m})(?) \longleftarrow (X_{t_{m+1}}, Y_{t_{m+1}})$ (known)

// $X_{t_{m+1}}$: value to continue before reset

// $Y_{t_{m+1}}$: value to continue after reset & before redemption

$$Y_{t_{m+1}} = \max(LN, Y_{t_{m+1}})$$

$$X_{t_{m+1}} = \max(Y_{t_{m+1}}, X_{t_{m+1}})$$

$$Y_{t_{m+1}} += d * \delta t * N$$

$$X_{t_{m+1}} += c * \delta t * N$$

$$Y_{t_m} = \mathcal{R}_{t_m}(Y_{t_{m+1}})$$

$$X_{t_m} = \mathcal{R}_{t_m}(X_{t_{m+1}})$$

③ After loop

return X_{t_0} $\blacksquare$

Interest Rate Path Dependent:

resettable cap

Inputs:

N: notional amount

q: initial cap rate

δL: cap spread over LIBOR

δt: period between payments

M: # of payments

$(u_j)_{j=1,\dots,J}$: reset times for cap

t_0: initial time

Event times: $\underbrace{t_0, t_1, \dots, t_N}_{\text{union of reset and cap times}}$

cap times $= t_0, s_1, \dots, s_{M-1}$:
times when caplets are computed

$$s_m = t_0 + m\,\delta t, \quad m = 0, \ldots, M-1$$

Path-dependent state process:

Y_{t_n} : historical float rate

(a) Origin:

$$Y_{t_0} = q$$

(b) Reset indexes: indexes of reset times.

(c) One-step evolution:

$$Y_{t_n} \text{ (known)} \longrightarrow Y_{t_{n+1}} \text{ (?)}$$

$$Y_{t_{n+1}} = \left(\frac{1}{B(t_{n+1}, t_{n+1} + \delta t)} - 1 \right) \frac{1}{\delta t}$$

Algorithm

(a) Boundary condition:

$$X_{t_N} = \underbrace{\text{NextCaplet}}_{\text{see below}}(t_N)$$

(b) loop $t_0 \longleftarrow t_N$

end t_0, begin t_N

X_{t_n} (?) $\longleftarrow$ $X_{t_{n+1}}$ (known)

// $X_{t_{n+1}}$: value to continue (value of caplets $> t_{n+1}$)

// We multiply on notional at the end

$$X_{t_n} = \mathcal{R}_{t_n}\left(X_{t_{n+1}} \right)$$

if $t_n \in$ cap times, then

$$X_{t_n} \mathrel{+}= \text{NextCaplet}(t_n)$$

$$\text{NextCaplet}(t_n) =$$

$$\max\left(1 - B(t_n, t_n + \delta t)\left(1 + \left(Y_{t_n} + \delta L\right)\delta t\right), 0\right)$$

③ After loop

return $X_{t_0} * N$