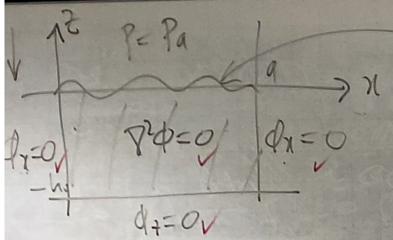


$\phi_z = \eta_t$ ✓

$\phi_t + g\eta = 0$

separable sol^{n}s

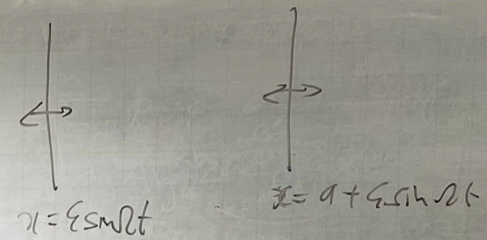
$$\phi(x,y,t) = e^{-i\omega t} \underbrace{X(x)}_{\cos(k_n x)} Z(z)$$

$$\eta(x,t) = A\,X(x)\,e^{-i\omega t}$$

(Re assumed)

$$\eta = Re\{C e^{-i\omega t}\cos(k_n x)\}$$

$$\phi = Re\left\{C(-i\omega e^{-i\omega t}\cos(k_n x)\frac{\cosh(k_n(z+h))}{k_n \sinh(k_n h)}\right\}$$



$$\dot{x} = \xi\Omega\cos\Omega t = \phi_x \text{ at } x = \cancel{\xi\sin\Omega t}\ 0$$

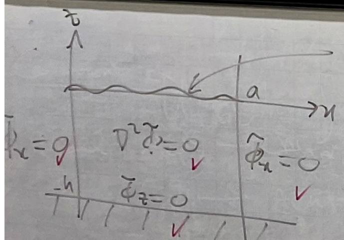
$$\phi_x(\xi, z, t) = \phi_x(0,z,t) + \cancel{\xi\phi_{xx}(0,z,t)}$$

$$\boxed{\phi_x = Re(\xi\Omega e^{-i\Omega t}) \text{ at } x = 0, a}$$

write

$$\phi = Re\{\xi\Omega(x - \tfrac{1}{2}a)e^{-i\Omega t}\} + \tilde{\phi}$$

$$\phi_x = Re\{\xi\Omega e^{-i\Omega t}\} + \tilde{\phi}_x$$



$\tilde{\phi}_z = \eta_t$

$$\tilde{\phi}_t + g\eta = Re\{i\xi\Omega^2(x - \tfrac{1}{2}a)e^{-i\Omega t}\}$$

Gen sol^n

(Re assumed)

$$\eta = \sum_{n=1}^{\infty} C_n e^{-i\Omega t}\cos(k_n x)$$

$$\phi = \sum_{n=1}^{\infty} -i\Omega C_n e^{-i\Omega t}\cos(k_n x)\frac{\cosh(k_n(z+h))}{k_n\sinh(k_n h)}$$

and BC =>

$$\sum_{n=1}^{\infty}\left[-\frac{\Omega^2\coth(k_n h)}{k_n} + g\right]C_n\cos(k_n x) = i\xi\Omega^2(x - \tfrac{1}{2}a)$$

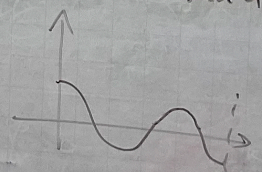
$\tilde{k}_m = k_{2m-1}$

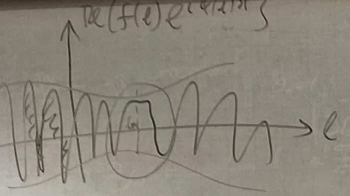
So $C_n = 0$ if n even

$$C_{2m-1} = \frac{4i\xi\Omega^2}{a\tilde{k}_m^2\left[-g + \frac{\Omega^2\coth(\tilde{k}_m h)}{\tilde{k}_m}\right]}$$

recall $g\hat{k}_m\tanh(\hat{k}_m h) = \omega_n^2$

$\cos\left(\frac{3\pi x}{a}\right)$





$\nabla^2\phi = 0 \quad z<0$

$\phi_z = U\eta_x,\ U\phi_x + g\eta = 0 \quad z=0 \quad x>0$

$\phi_z \to 0 \quad z \to -\infty$

$\eta = \eta_0(y),\ \frac{\partial \eta}{\partial x} = 0 \quad x=0$

$\hat{\phi}_{xx} + \hat{\phi}_{zz} = l^2 \hat{\phi}$

$\hat{\phi}_z = U\hat{\eta}_x,\ U\hat{\phi}_x + g\hat{\eta} = 0 \quad z=0$

$\boxed{\hat{\phi}_z \to 0} \quad z \to -\infty$

$\hat{\eta} = \hat{\eta}_0,\ \boxed{\hat{\eta}_x = 0} \quad x=0$

Separable sol^n $(m>0)$

Try $\hat{\phi}(x,z,l) = B\sin(kx)\, e^{mz}$, $\hat{\eta}(x,l) = A\cos(k$

Laplace : $\boxed{m^2 - k^2 = l^2}$

F.S. conditions $\begin{cases} Bm = -kUA \\ UkB + gA = 0 \end{cases}$ $\begin{pmatrix} kU & m \\ g & Uk \end{pmatrix}\begin{pmatrix} A \\ B \end{pmatrix} = 0$

Nontrivial sol^{n}s $\iff$ $\boxed{U^2k^2 = mg}$

quadratic for k^2: $\frac{U^4k^4}{g^2} - k^2 - l^2 = 0$

quadratic formula $\rightarrow$ answer... $k = k(l)$

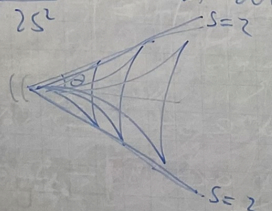
NB $k^2 > 0$ so $m > 0$

$= \pm \frac{dk}{dl} = \pm\sqrt{\frac{(s-1)}{2s^2}}$ max at $s=2$

$\max|\lambda| = \frac{1}{2\sqrt{2}}$

$\tan\theta = \frac{1}{2\sqrt{2}}$

$\sin\theta = \frac{1}{3}$

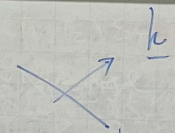


$\eta(x,y) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \hat{\eta}_0(l)\, e^{ily} \cos(k(l)x)\, dl$

At boundary of wave $(s=2)$

$k = \frac{g}{U^2}\sqrt{\frac{3}{2}}, \quad l = \frac{g}{U^2}\frac{\sqrt{3}}{2}$

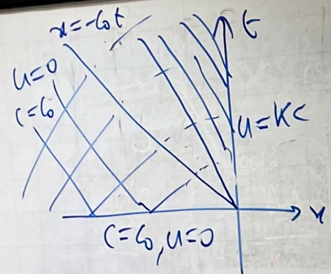
$\begin{pmatrix} k \\ l \end{pmatrix} = \frac{3g}{2U^2}\begin{pmatrix} \sqrt{\frac{2}{3}} \\ \frac{1}{\sqrt{3}} \end{pmatrix}$



wave crests orthogonal to $\underline{k}$

$\lambda = 2\pi/|\underline{k}| = \frac{4\pi U^2}{3g}$

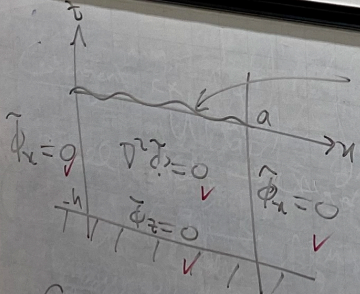
$$k = \frac{9}{u^2}\sqrt{\frac{3}{2}}, \quad l = \frac{9}{u^2}\frac{\sqrt{3}}{2}$$



$k<1$

In $c_0 t < x < 0$ still $\boxed{u + \frac{2c}{\gamma-1} = \frac{2c_0}{\gamma-1}}$

on the chars u, c both const. & satisfy $\boxed{u = kc}$



$\tilde{\Phi}_x = 0$ $\nabla^2\tilde{\Phi} = 0$ $\tilde{\Phi}_x = 0$ $\tilde{\Phi}_z = 0$

$\tilde{\Phi}_z = \eta_t$

$\tilde{\Phi}_t + g\eta = \mathrm{Re}\{ i\varepsilon\Omega^2 (x - \tfrac{1}{2}a) e^{-i\Omega t} \}$

Gen soln

Try Φ

Laplace

F.S. conditions { Br… uk…

Nonlin…

quadratic for k^2

qua…

$$\begin{pmatrix} k \\ l \end{pmatrix} = \left(\frac{3a\ldots}{2u\ldots} \right.$$

$\lambda = 2\pi / |k|$