Section III.3: The fundamental group of the circle

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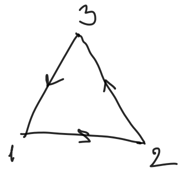
<u>Theorem III.32</u> $\pi_1(S^1) \cong \mathbb{Z}$

basepoint $1 \in S^1$

$l(t) = e^{2\pi i t}$, $t \in [0,1]$ generator of $\pi_1(S^1, 1)$

<u>Proof</u>



$E(K,1) \cong \mathbb{Z}$

$\alpha = (b_0, b_1, \ldots, b_n)$ edge loop based at 1.

If $b_i = b_{i+1}$, then perform elementary contraction.

If we traverse an edge and its reverse, contract.

So a shortest loop equivalent to α traverses all edges with the same orientation, and is hence l^n, $n \in \mathbb{Z}$.

$l^n \sim l^m \Rightarrow n = m$

<u>winding number</u> of an edge loop is the number of times it traverses $(1,2)$ in positive direction minus the number of times it goes along negative direction

Invariant under elementary contractions and expansions

So $\forall$ loop is equivalent to l^n for unique n

$E(K,1) \to \mathbb{Z}$ isomorphism: $l^n \cdot l^m = l^{n+m}$ $\square$