Section III.2: A simplicial version

Def An edge path is a finite sequence of vertices $(a_0, \dots, a_n)$ of a simplicial complex K such that $\{a_{i-1}, a_i\}$ form a simplex of K (possibly $a_{i-1} = a_i$).

n length

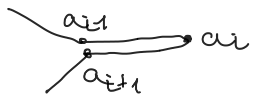
If $a_0 = a_n$, then this is an edge loop.

If $\alpha = (a_0, \dots, a_n)$, $\beta = (b_0, \dots, b_m)$ are edge paths, $a_n = b_0$, then their product $\alpha\beta := (a_0, \dots, a_n, b_1, \dots, b_m)$.

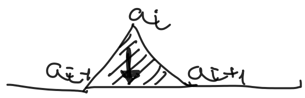
Def α edge path, an edge path obtained from α using one of the following moves is an elementary contraction of α:

(0) $a_{i-1}, a_i \rightsquigarrow a_{i-1}$ if $a_{i-1} = a_i$

(1) $a_{i-1}, a_i, a_{i+1} \rightsquigarrow a_{i-1}$ if $a_{i-1} = a_{i+1}$



(2) $a_{i-1}, a_i, a_{i+1} \rightsquigarrow a_{i-1}, a_{i+1}$ if (a_{i-1}, a_i, a_{i+1}) is a 2-simplex of K



If α is an contraction of β, then β is an elementary expansion of α.

α is equivalent to β ($\alpha \sim \beta$) if β can be obtained from α by a sequence of elementary expansions and contractions.

Rem If $\alpha \sim \beta$, then they the same initial and terminal vertices.

Def/Thm III.26 K simplicial complex, $b \in V(K)$

The equivalence classes of edge loops in K based at b form a group denoted by $E(K, b)$ called the edge loop group.

Proof The product is induced by product of edge loops.

well-defined: respects $\sim$

associative: product of edge loops is assoc.

identity: class of (b)

inverse of $(b, b_1, \dots, b_{n-1}, b)$: $(b, b_{n-1}, \dots, b_1, b)$