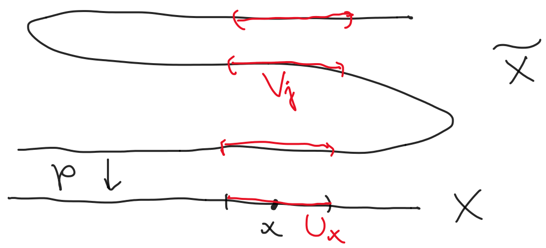
Section VI.1: definitions

<u>Def</u> A continuous map $p: \tilde{X} \to X$ is a <u>covering map</u> if $X, \tilde{X}$ are non-empty, path-connected, and $\forall x \in X$ has a neighbourhood $U_x \ni x$ such that $p^{-1}(U_x) \cong \coprod_{j \in J} V_j$, V_j open in $\tilde{X}$, and $p|_{V_j} : V_j \to U_x$ is a homeomorphism.

U_x : <u>elementary open sets</u>

$\tilde{X}$ is a <u>covering space</u> of X

$b, \tilde{b}$ are basepoints of $X, \tilde{X}$, resp., such that $p(\tilde{b}) = b$, then $p : (\tilde{X}, \tilde{b}) \to (X, b)$ a <u>based covering map</u>.



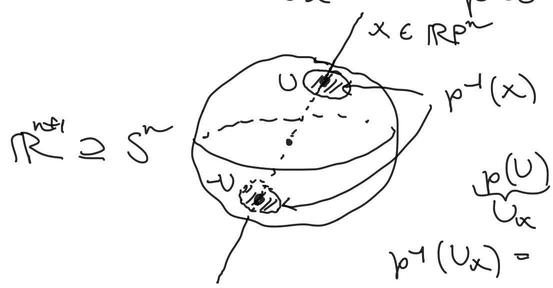
<u>Rem</u> J a priori depends on x, but we'll see that $|J|$ is independent of x.

<u>Example</u>
$$p : \mathbb{R} \to S^1$$
$$t \mapsto e^{2\pi i t}$$

$x \in S^1$, take U_x to be the open semi-circle centred at x.
<u>e.g.</u> $p^{-1}(U_1) = \bigcup_{n \in \mathbb{Z}} (n - \frac{1}{4}, n + \frac{1}{4})$.

<u>Example</u>
$$S^1 \to S^1 \qquad (S^1 \subset \mathbb{C}) \text{ is a covering, } |J| = n.$$
$$z \mapsto z^n$$

<u>Example</u> $\mathbb{R}P^n :=$ set of 1-dim'l subspaces of $\mathbb{R}^{n+1}$.
$$p : S^n \to \mathbb{R}P^n$$
$$v \mapsto \langle v \rangle$$
$\forall x \in \mathbb{R}P^n$, $|p^{-1}(x)| = 2$ since $\langle v \rangle = \langle w \rangle \Leftrightarrow w = \pm v$.
endow $\mathbb{R}P^n$ with quotient topology.
If U_x is a small open set around $x \in \mathbb{R}P^n$, then $p^{-1}(U_x)$ is two copies of U_x, and restriction of p to each is a homeo onto U_x. So p is a covering.



$\underset{U_x}{p(U)}$ open neighbourhood of x

$p^{-1}(U_x) = U \amalg -U$