Section IV.1: Free groups - definition

S set : alphabet

$S^{-1} \cap S = \emptyset$

$x \in S \rightsquigarrow x^{-1} \in S^{-1}$, $(x^{-1})^{-1} = x$

words in $S \cup S^{-1}$ $\quad \ldots xx^{-1} \ldots \rightsquigarrow \ldots \cancel{xx^{-1}} \ldots$

Def A word w is a finite sequence $x_1, \ldots, x_m$ for $m \in \mathbb{N}$, where $x_i \in S \cup S^{-1}$. We write w as $x_1 \ldots x_m$. When $m = 0$, we have the empty sequence: $\emptyset$

Def Concatenation of $x_1 \ldots x_m$ and $y_1 \ldots y_n$ is $x_1 \ldots x_m y_1 \ldots y_n$.

Def w' is an elementary contraction of w, we write $w \searrow w'$, if $w = y_1 x x^{-1} y_2$ for words y_1, y_2; $x \in S \cup S^{-1}$, and $w' = y_1 y_2$. $w' \nearrow w$, we say that w is an elementary expansion of w'.

Def Words w, w' are equivalent, $w \sim w'$, if $\exists$ words $w_1, \ldots, w_n$ with $w_1 = w$ and $w_n = w'$, and $\forall i$, $w_i \nearrow w_{i+1}$ or $w_i \searrow w_{i+1}$. The class of w is denoted by $[w]$.

Def The free group on the S, denoted $F(S)$, consists of equivalence classes of words in the alphabet S.
The product of $[w]$ and $[w']$ is $[ww']$.
The identity is $[\emptyset]$.
Inverse of $[x_1 \ldots x_n]$ is $[x_n^{-1} \ldots x_1^{-1}]$.
Well-defined: $w_1 \sim w_1'$, $w_2 \sim w_2'$, then $w_1 w_2 \sim w_1' w_2'$.

Def $i : F(S) \xrightarrow{\simeq} G$ isomorphism. Then $i(S)$ is a free generating set of G.