

Synopsis (by individual lecture)

0. Background material.

CHAPTER 1

1. Introductory lecture.

CHAPTER 2

2. Topological surfaces. Examples. Euler number. Orientability.

3. Classification theorem. Canonical surfaces. Identifying surfaces.

CHAPTER 3

4. Smooth surfaces in \mathbb{R}^3 . Examples. Tangent space. Gradient. Abstract smooth surfaces.

5. First fundamental form. Examples. The concept of a Riemannian 2-manifold. Isometries.

6. Second fundamental form. Weingarten map. Gaussian curvature.

7. Theorema Egregium.

CHAPTER 4

8. Geodesics.

CHAPTER 5

9. Local and Global Gauss-Bonnet Theorems and applications.

10. Poincaré-Hopf theorem. Morse theory.

CHAPTER 6

11. The hyperbolic plane, its isometries and geodesics.

12. Hyperbolic geometry and trigonometry.

13. Compact hyperbolic surfaces as surfaces of constant negative curvature.

CHAPTER 7

13-14. Riemann surfaces; examples, including the Riemann sphere, the quotient of the complex numbers by a lattice, and double coverings of the Riemann sphere.

15. Holomorphic maps of Riemann surfaces and the Riemann-Hurwitz formula.

16. Elliptic functions including Weierstrass \wp -function.

Recommended Texts

Elementary Differential Geometry, Andrew Pressley, Springer (2010)

Geometry of Surfaces, Graeme Segal, Mathematical Institute Notes (1986)

Geometry from a Differential Viewpoint, John McCleary, Cambridge (2012)

Surface Topology, Peter Firby, Cyril Gardiner, Woodhead (2001)

Complex Algebraic Curves, Frances Kirwan, Cambridge (1992)

Elementary Differential Geometry, Barrett O'Neill, Academic Press (2006)

Differential Geometry of Curves and Surfaces, Manfredo Do Carmo, Dover (2017)

Geometry of Surfaces, John Stillwell, Springer (1992)

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The majority of these notes is original material, but they also make substantial use of Segal's Institute notes and Ritter's lecture notes.