

B4.3 Distribution Theory

MT23

General Prerequisites: Part A *Integration* is essential. A good working knowledge of the Part A *Metric Spaces and Complex Analysis* is expected. Part A *Integral Transforms* and *Multidimensional Analysis and Geometry* are desirable but not essential.

Course Overview: Distribution theory can be thought of as the completion of differential calculus, just as Lebesgue integration theory can be thought of as the completion of integral calculus. It was created by Laurent Schwartz¹ in the 20th century, not long after Lebesgue's integration theory. Besides being an important part of Analysis it also has many applications. One of the main areas of applications is to the theory of partial differential equations, and a brief treatment, mainly through examples, is included in this course and its continuation *B4.4 Fourier Analysis*. A more systematic study is deferred to the Part C/OMMS courses *Functional Analytic Methods for PDEs*, *Fixed Point Methods for Nonlinear PDEs* and *Optimal Transport and PDEs*. The course also provides preparation for many other Part C/OMMS courses, including *Analytic Number Theory*, *Further Functional Analysis*, *Topics in Fluid Mechanics*, *Applied Complex Variables*, *Stochastic Analysis and PDEs*.

Reading List: In addition to the Lecture Notes for the course I strongly recommend the book:

[1] R.S. Strichartz: *A Guide to Distribution Theory and Fourier Transforms* (World Scientific, 1994. Reprinted: 2008, 2015)

Further Reading: The following books are more advanced, but also highly recommended for further reading and study.

[2] J.J. Duistermaat and J.A.C. Kolk, *Distributions, Theory and Applications* (Birkhäuser 2010)

[3] H. Brezis, *Functional analysis, Sobolev spaces and partial differential equations* (Springer 2011)

[4] L.C. Evans, *Partial Differential Equations* (Amer. Math. Soc. 1998)

[5] E.H. Lieb and M. Loss, *Analysis* (Amer. Math. Soc. 1997)

[6] E.M. Stein and R. Shakarchi, *Fourier analysis. An introduction* (Princeton Univ. Press 2003)

[7] E.M. Stein and R. Shakarchi, *Complex analysis* (Princeton Univ. Press 2003)

[8] E.M. Stein and R. Shakarchi, *Real analysis. Measure theory, integration, and Hilbert spaces* (Princeton Univ. Press 2005)

¹As with most theories and scientific discoveries many of the main ideas underpinning the general theory were the product of many people and had been considered before in a less general set-up. See J. Lützen: *The prehistory of the theory of distributions. Studies in the History of Mathematics and Physical Sciences 7. Springer, 1982.*