### BO1.1. History of Mathematics Lecture XVI Concluding miscellany

MT23 Week 8

### Summary

- The exam (briefly)
- ▶ Points to ponder
- ► The history of the history of mathematics\*
- ▶ Hilary Term reading course

### Structure of the exam paper

#### Section A

- Six extracts given
- Choose two and comment on the context, content, and significance
- Each extract is worth 25 marks
- Each extract is typically one short paragraph it will relate to a topic that we have studied, though you may not have seen the precise extract before
- By way of practice, choose any quotation or short extract that has appeared on the lecture slides

#### Section B

- Three essay topics given
- Choose one
- Answer worth 50 marks

### Typical exam questions (Section B)

Q. Discuss, with reference to specific examples, how concept X (or terminology Y, or notation Z, ...) has developed between 1600 and 1900.

Q. Discuss with reference to specific examples, how attitudes towards X have changed between 1600 and 1900.

Q. Discuss the significance of text X.

Q. Describe some aspects of the work of major figure X.

### Points to ponder (1)

What is the history of mathematics?
What does it mean to study the history of mathematics?
What is mathematics?

### Points to ponder (2)

What do you think the words 'mathematics' and 'mathematician' have meant throughout this course?

Have they had the same meanings throughout?

More generally, have they had the same meanings throughout history?

### Points to ponder (3)

If we choose to understand the word 'mathematics' differently, how does this change our view of the history of mathematics?

How could a revised definition of 'mathematics' change the selection of people and cultures who appear in the story?

What does the study of the history of mathematics have to tell us about the way in which we approach mathematics nowadays?

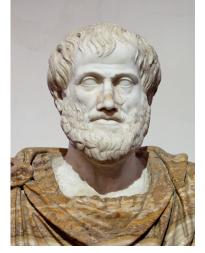
### Historiography of mathematics

According to the *OED*:

historiography, n.

- 1. The writing of history; written history.
- 2. The study of history-writing, esp. as an academic discipline.

### Ancient histories of mathematics

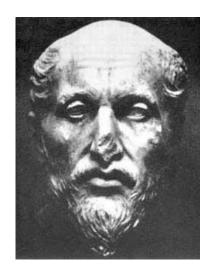


Aristotle (384–322 BC)

### Eudemus (4th century BC)

- Student and editor of Aristotle
- ► History of Arithmetic
- History of Geometry
- History of Astronomy

### Biographical background



Proclus's commentary on Euclid's *Elements* (5th century AD)

- (Spurious?) biographical details
- Built on anecdotes provided by Pappus (4th century AD)

### Later historical attributions



a full understanding of geometry "requireth diligent studie and reading of olde auncient authors"

### Renaissance humanist attitudes

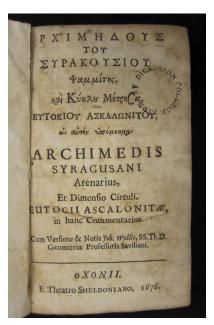


Sir Henry Savile (1549–1622)

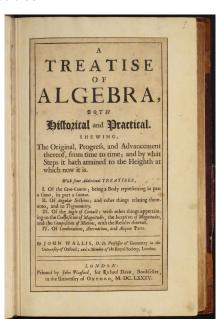
The teaching of mathematics should be founded on humanist principles:

- it should stem from the works of classical antiquity;
- scholars ought to have a concern for the history of their subject;
- they should actively seek to restore and edit surviving texts.

### Renaissance humanist attitudes



### Nationalist attitudes



### Comprehensive histories of mathematics

### HISTOIRE

D E S

### MATHEMATIQUES,

Dans laquelle on rend compte de leurs progrès depuis leur origine jusqu'à nos jours; où l'on expose le tableau & le développement des principales découvertes, les contestations qu'elles ont fait naître, & les principaux traits de la vie des Mathématiciens les plus célebres.

Par M. MONTUCLA, de l'Académie Royale des Sciences.

Multi pertransibunt & augebitur scientia. Băom

TOME PREMIER.



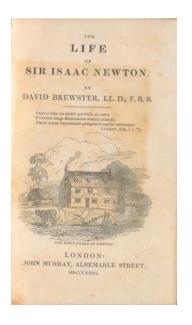
A PARIS.

Chez CH. ANT. JOMBERT, Imprimeur-Libraire du Roi pour l'Artillerie & le Génie, rue Dauphine, à l'Image Notre-Dame.

M. D.C.C. L.VIII.

Avec Approbation & Privilege du Ro

### Lauding the great mathematicians

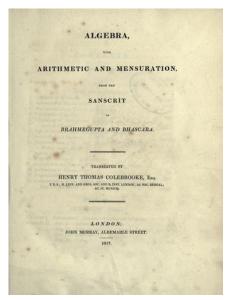


### Adding greater nuance



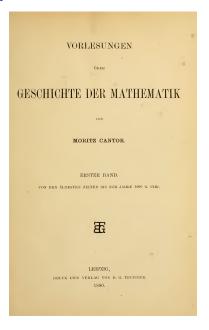
See: Adrian Rice, 'Augustus De Morgan: historian of science', *History of Science* 34 (1996), 201–240

### Awareness of mathematics beyond Europe



See: Ivahn Smadja, 'Sanskrit versus Greek 'proofs': history of mathematics at the crossroads of philology and mathematics in nineteenth-century Germany', Revue d'histoire des mathématiques 21(2) (2015) 217–349

### Anecdotal history



### Who studies the history of mathematics?

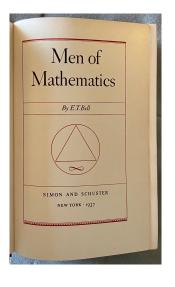
- Mathematicians?
  - Because mathematical knowledge is key?
  - Because only mathematicians will read it?
  - Suitable retirement project?
- Historians?
  - Because only an understanding of historical context makes it history?
  - ▶ For better integration into wider historical scholarship?
- Scholars who are somewhere in between?

### Professionalisation



Otto Neugebauer (1899–1990)

### Popularisation



- Humanisation of mathematics
- Mathematical myth-making
- Mathematical anecdotes for community-building
- Drawback: reinforcement of a particular view of the subject

until you obtain 2. It was therefore indeed a new idea to duplicate it by dividing 2 by n.

We do not know in whose brain this thought arose for the first time, nor when this happened. It certainly occurred long before the era of our texts, for the (2 : n)-table of the Rhind papyrus, which includes all the odd numbers from n = 3 to n = 101, was not constructed all at one time; its separate parts were computed by different methods. The oldest section contains the denominators which are divisible by 3; without exception, they all proceed according to the same rule:

$$2: 9 = \overline{6} + \overline{18}$$
  
 $2: 15 = \overline{10} + \overline{30}$   
 $2: 21 = \overline{14} + \overline{42}$ 

In these cases the division 2: 3k is simply a confirmation of a known result. In the other cases (certainly from n = 11 on), the duplication appears to have been obtained by actually carrying out the division of 2 by s. The text exhibits the divisions more or less explicitly, as in the following examples (2:5 and 2:7)

What part is 2 of 5? 3 is 1 + 3. 15 is 3.

i.e. a third of 5 is 1 + 3, a fifteenth is 3; these add up to 2. The result of the division is therefore 3 + 15; the terms 3 and 15 are clearly visible because they are written in red. In our "translation" the red symbols have been printed in bold-face type

In this manner the work proceeds. In dividing 2 by 5, 9, 11, 17, 23, 29 and a few of the larger integers, the 3 sequence is used, i.e. the sequence of fractions 3, 3, 6, 12, ...; but the division by 7 and 13 employs only the 2-sequence (2, 4, 8, . . ). It turns out that only in these two cases the 2-sequence produces a simpler result than the 3-sequence. For instance, the use of the 2-sequence would, in calculating 2:11, lead to the result 2:11=8+22+88, while the 3sequence gives  $2:11=\overline{6}+\overline{66}$ , which, having fewer terms and smaller de-

nominators, is obviously to be preferred

The calculations which have been reproduced here certainly tell their own story. In the case 2:7, the number 4, placed in front of 28, indicates where 28 comes from, viz. from 4 x 7, the further details being shown in an auxiliary column. THE EGYPTIANS

The results of the divisions 2: n are summarized in the following table, which does not include divisors that are divisible by 3, all of which follow the rule  $2:3k=2k+\overline{6k}$ 

```
2:5-3+15
                                                         2: 53 = 30 + 318 + 795
2: 7 - 4 + 28
                                                         2: 55 = 30 ± 330
2:11 = 6 + 66
                                                         2: 59 = 36 + 236 + 531
2:13 = 8 + 52 + 104
                                                         2: 61 - 40 + 244 + 488 + 510
2:17 = \overline{12} + \overline{51} + \overline{68}
                                                         2: 65 - 39 + 195
2:19 = \overline{12} + \overline{76} + \overline{114}
                                                         2: 67 = 40 + 335 + 536
2:23 = \overline{12} + \overline{276}
                                                         2: 71 = 40 + 568 + 710
2:25 = \overline{15} + 75
                                                         2: 73 = \overline{60} + \overline{219} + \overline{292} + \overline{365}
2:29 = \overline{24} + \overline{58} + \overline{174} + \overline{232}
                                                         2: 77 = \overline{44} + \overline{308}
                                                         2: 79 = \overline{60} + \overline{237} + \overline{316} + \overline{790}
2:31 = \overline{20} + \overline{124} + \overline{155}
                                                         2: 83 = 60 + 332 + 415 + 498
2:35 = 30 + 42
                                                         2:85 = \overline{51} + \overline{255}
2:37 = \overline{24} + \overline{111} + \overline{295}
                                                         2:89 = \overline{60} + \overline{356} + \overline{534} + \overline{890}
2:41=\overline{24}+\overline{246}+\overline{328}
                                                         2: 91 = 70 ± 130
2:43 = \overline{42} + \overline{86} + \overline{129} + \overline{301}
                                                         2:95 = \overline{60} + \overline{380} + \overline{570}
2:47=\overline{30}+\overline{141}+\overline{470}
                                                         2: 97 = 56 + 679 + 776
2:49 = \overline{28} + \overline{196}
2:51 = 34 + 102
                                                        2:101 - \overline{101} + \overline{202} + \overline{303} + \overline{605}
```

Beginning with 2:31, the form of presentation changes; the calculations are given in abbreviated form. But, what is more important, the method of calculation changes; another idea is introduced. While up to this point, all divisions were carried out by means of the 2-sequence and the 3-sequence, the divisions 2:31 and 2:35 proceeded quite differently, as is seen from the following examples:

What part is 2 of 31?  $\overline{20}$  is  $1 + \overline{2} + \overline{20}$ ,  $\overline{124}$  is  $\overline{4}$ ,  $\overline{155}$  is  $\overline{5}$ Computation:

The start of the computation of 2:31 is easy to account for, since division of 31 by 10, and halving of the result shows that  $\frac{1}{20}$  of 31 is  $1 + \overline{2} + \overline{20}$ . This fraction is to be increased so as to produce 2. How did the calculator hit upon the idea that this requires 4 + 5? It checks; for the leather scroll has the relation

### Rewriting the history of mathematics

#### On the Need to Rewrite the History of Greek Mathematics

SABETAI UNGURU Communicated by W. HARTNER

'History is the most fundamental science, for there is no human knowledge which cannot loss its scientific character when men forget the conditions under which it originated, the questions which it answered, and the function it was created to serve, A great part of the mysticism and superstition of cluented men consists of knowledge which has broken loose from its historical moorings.'

BRALLINE EXPRESSIVES.

"It would not occur to the modern mathematician, who uses algebraic symbols, that one type of geometrical progression [Le., 1, 2, 4, 8] could be more perfect better deserving of the name than another. For this reason algebraic symbols should not be employed in interpreting such a passage as ours [Le, Plato, Thanacas, 32A, B, Thanacas,

'Any historian of mathematics conscious of the perils and pitfalls of Whig history quickly discovers that the translation of past mathematics into modern symbolism and terminology represents the greatest danger of all. The symbols and terms of modern mathematics are the bearers of its concepts and methods. Their application to historical material always involves the risk of imposing on that material, a content it does not in fact possesses.'

The previous string of quotations is (most certainly) not illustrative of the ways in which the history of mathematics has traditionally been written. The authors of the quotations themselves have not always practiced what they occa-

Greek Science Its Meaning For Ux (Harmondsworth: Penguin Books, 1953), 311.
 Plato's Cosmology (New York: The Liberal Arts Press, 1957) 49.

Sabetai Unguru, 'On the need to rewrite the history of Greek mathematics', *Archive for History of Exact Sciences* 15 (1975), 67–114

<sup>5</sup> The Mathematical Curves of Pierre de Fermal (1601–1665) (Princeton, N.J.: Princeton University Press, 1973), XII–XIII.

### A broader perspective

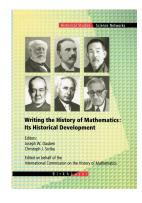


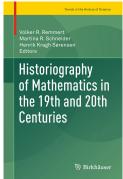
### A broader perspective

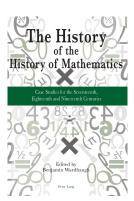


Brigitte Stenhouse, *Mary Somerville: Being and Becoming a Mathematician*, PhD thesis,
The Open University, 2021

### Historiography of mathematics: references







# How do we know what we know about ancient Egyptian mathematics?

#### 8 Chapter 1 Ancient Mathematics

numbers developed to the point where the same digit it represented 60 is well. We do not have why the fibely-learis decided to have one large unit represent 60 small under the adapt this method for their numeration systems. One conjecture is that 60 is evenlad when below the most conjecture is that 60 is evenlad the adapt of their larger wine conjecture of the conjecture of the state of the conjecture is that 60 is evenlad the system is still in such its or main for might and time measurement, units preserved over the contained in astronomical contexts and others also represent the part of world culture.

There is no record of the writin number system of accised belia, but then is literally colorise that muscles symbols defect in it. In region from about the third colory a cr., that examples of written numbers are available. Originally, the system was mixed. There was explored system mixed in the hearist with spears systemly to the numbers 1 strough 9 and 10 frough 50. For larger morbers, the system was a miniplicative one similar to 9 and 10 frough 50. For larger morbers, the system was a miniplicative one similar to the few 100 and 100 mixed 100 mixed 50. The system was a miniplicative one similar to discuss the few 100 and 100 mixed to 100 mixed with the system was a miniplicative one similar to the few 100 mixed 100 mixed to 100 mixed 100 mixe

#### 1.3 ARITHMETIC COMPUTATIONS

Once their system of writing numbers came into existence, all of the civilizations under discussion devised rules for the basic arithmetic operations-addition, subtraction, multiolication, and division-and as a consequence of the last operation, rules for writing and operating with fractions. These rules may be considered as some of the earliest algorithms. An algorithm is an ordered list of instructions designed to produce an answer to a given type of problem. Ancient peoples produced algorithms of all sorts to handle many different problems. In fact, ancient mathematics can be characterized as algorithmic in nature, as opposed to the Greek mathematics, which emphasized theory. In most of the available documents of ancient mathematics, the author describes a problem to be solved and then proceeds to use an algorithm, either explicit or implicit, to obtain the solution. There is little concern in the documents as to how the algorithm was discovered, why it works, or what its limitations are. Instead, we simply are shown many examples of the use of the algorithm, often in increasingly complex situations. Nevertheless, in our discussion of these algorithms, we will describe the possible origins and justifications of each one and will present the possible answers that the Babylonian, Chinese, or Egyptian scribes gave to their students who asked the eternal question "why?"

#### 13 Arithmetic Commutations

Such a simple algorithm for addition and subsection is not possible in the hieratic system. For those operations, the malternatical papers in one provide much reinfance; the answers to addition and subsection problems are merely written down. Most probably, the services and addition tables. At some point these would have existed in writines form, but a composest sorthe would, of course, have memorized them. The sorthest presumably used the addition tables in overgree for variation mechanism.

The Egyptian algorithm for multiplication was based on a continual doubling process. For multiply two multers as and, bit encisive would first write down the pirt. I.e. Bit would then double each number in the pair repeatedly, until the next doubling would cause the first element of the pair to exceed or. Then, having determined the powers of 2 shat add to a, the sorthe would add the corresponding multiples of b to get the answer. For example, to multiply 12 by 3 the scribe would add own the following lines:

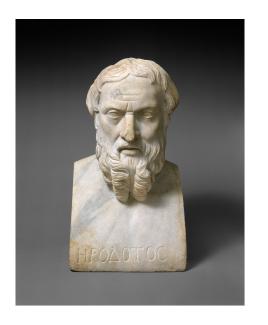
At this point, he would notice that the next doubling would produce 16 in the first column, which is larger than 13. He would then check off those multipliers that added to 13, namely 1, 4, and 8, and add the corresponding numbers in the other column. The result would be written on Taxis. 13. 156.

As before, there is no record of how the surbe did the doubling. The answers are imply written down. Petugo the scribe that merchand an extensive 2 times table. In that, there is some relations that adoubling was a standard method of compensation in zero of Africa to the could be Egypt as croble search of from their southern to the scott of Egypt, as it is likely that the Egyptian croble search of from their southern collabague. In addition, the scribes were southout some that over positive integer could be to suitagely expressed as the sum of powers of 2. The fact provides the justification of for the procuber. How was it discovered? Our best passes is that it was discovered by experimentation and the passed down as rathing.

Because devision is the inverse of multiplication, a problem such as 156 ± 12 would be stude at "multiply 12 so as to get 156." The critic would then write down the same lines listed before. This time, however, he would check off the lines having the numbers in the right-hand column that sum to 156; in this case, 12, 48, and 95. Then the sum of the corresponding numbers on the left, namely 1, 4, and 8, would give the answer 13. Of corner, division does not always "one out even." When it did not the Egyptian used

The last of fraction that the Egyptians used were unit fractions, or "part," (fraction with namerican Vis. with the single experience of 22), pulsaps became there fractions are the most "nameral." The fraction 1/6 (the set) part) is represented in his egyptic to by the symbol for the images or with the eyesther a shown. In the historia is do it used instant. Thus 1/1 is denoted in historylaptic by #\frac{1}{2}\$ and in the hearist by \$\prec{\psi}\$. The single exception, 22), had a special notwide: "In historylaptic and 1's in historia, fifth for forcer symbol is indicative of the recipeoual of 11/23, in the remainder of this text, however, the notation if with leading to the part of the part of the recipeous of the part of the pa

### Herodotus (5th century BC)



### Herodotus on Egyptian geometry

It was this king, moreover, who divided the land into lots and gave everyone a square piece of equal size, from the produce of which he exacted an annual tax. Any man whose holding was damaged by the encroachment of the river would go and declare his loss before the king, who would send inspectors to measure the extent of the loss, in order that he might pay in future a fair proportion of the tax at which his property had been assessed. I think this was the way in which geometry was invented, and passed afterwards into Greece . . .

### Mediaeval Islamic Egypt



### Montucla on ancient Egyptian geometry

### HISTOIRE

### MATHEMATIOUES.

Dans laquelle on rend compte de leurs progrès depuis leur origine jusqu'à nos jours; où l'on expose le tableau & le développement des principales découvertes, les contestations qu'elles ont fait naître, & les principaux traits de la vie des Mathématiciens les plus célebres.

Par M. MONTUCLA, de l'Académie Royale des Sciences & Belles-Lettres de Pruffe.

Multi pertransibunt & augebitur scientia. Băcon-

TOME PREMIER.



Chez CH. ANT. JOMBERT, Imprimeur-Libraire du Roi pour l'Artillerie & le Génie , rue Dauphine , à l'Image Notre-Dame,

M. DCC. LVIII.



DES MATHÉMATIOUES, Part. I. Liv. II. 11

assigner à chacun une portion de terre égale à celle qu'il possédoit avant l'inondation. Telle fut, dit-on, l'origine de l'arpentage, premiere ébauche de la Géometrie, à laquelle néanmoins elle a donné le nom : car Géometrie, fignific en Grec, melure de la serre, ou des terrains. Je remarque en passant que c'est assez gratuitement qu'on suppose que le Nil confondoit ainsi les limites des possessions; il n'étoit pas bien difficile de lui en oppofer d'affez stables ou d'affez profondes pour subsister malgré l'inondation. On ne scauroit se persuader que l'Egypte sût chaque année ravagée par les eaux : cela s'accorderoit mal avec l'idée d'un pays délicieux, comme celle que nous en donne l'antiquité.

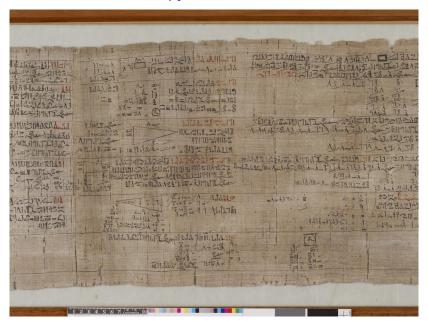
Quelques Ecrivains, parmi lesquels est Hérodote, fixent la naissance de la Géometrie au tems où Sesostris (k) coupa l'Egypte par des canaux nombreux, & en fit une forte de répartition générale entre ses habitans. M. Newton (1) en adoptant le fentiment d'Hérodote, dit que ce partage fut fait par le conseil de Thot, le Ministre de Sesostris, qui est suivant lui Osiris. Cette conjecture sur l'emploi & la nature de ce personnage célébre, n'est pas destituée d'autorités anciennes, & s'accorde parfaitement avec l'opinion dont on a parlé ailleurs, que Theut étoit l'inventeur des nombres, du calcul & de la Géometrie. En effet, on peut dire que le partage projetté par Selostris exigeant des connoissances Géometriques, son Ministre en jetta à cette occasion les fondemens. Ceci s'accorde encore avec le sentiment qui attribue ces inventions à Hermes, autrement le fameux Mercure Trismegisle; car tous ces hommes font probablement les mêmes. Un Ecrivain (m) raconte que ce Mercure grava les principes de la Géometrie sur des colonnes qui furent dépofées dans de valtes souterrains, & le fabuleux Jamblique (n) dit que Pythagore profita beaucoup de la vûc de ces monumens. Un Auteur enfin cité par Diogene Laerce, (o) dit que Maris, apparemment ce Prince qui fit creufer le fameux lac de ce nom, pour fervir de décharge au Nil, avoit inventé les principes de la Géometrie. On voit facilement le motif de sa conjecture.

(k) Herod. I. 11.

### Augustus De Morgan (1838)

There is a *stock history* of the rise of geometry ... that the Egyptians, having their landmarks yearly destroyed by the rise of the Nile, were obliged to invent an art of land-surveying in order to preserve the memory of the bounds of property; out of which art geometry arose. ... There is no proof whatever ...

### Rhind Mathematical Papyrus



### Reconstructing ancient Egyptian mathematics

- Is it valid to use modern mathematical ideas to reconstruct ancient mathematics?
- What do you do if the mathematical and linguistic evidence point in different directions?
- What story were people trying to tell?
- Where does ancient Egyptian mathematics sit within wider stories?

### HT reading course: content

## Early set theory in the works of Cantor, Dedekind, and others

The reading course will consist of three parts: the detailed reading and analysis of portions of

- 1. Bolzano's *Paradoxes of the infinite* (1851), as a glimpse of early 19th-century investigations of the infinite and associated embryonic set-theoretic ideas;
- 2. Dedekind's writings on real numbers, with their implicit use of a set-theoretic language;
- 3. Cantor's investigations of the infinite, which led to his formalisation of the set concept.

As during the lecture course, the emphasis will be on the use of original sources — not only those mentioned above, but also any other relevant materials that may arise.

### HT reading course: arrangements

**Seminars:** weekly classes of an hour and a half each (Note that these will timetabled with the lectures as two sessions per week, but you only need to attend one of these — sign up as you would for intercollegiate classes.)

**Essays:** up to 2,000 words to be submitted in advance for discussion in the seminars in weeks 3, 5 and 7 (Further details will appear online during the vacation.)

**Assessment:** extended essay (3,000 words), details of which will be announced on Monday of week 7. To be submitted by 12 noon on Monday of week 10.

### HT reading course: vacation work

Vacation reading for discussion in first-week seminar is now given on the course webpage.

### **BSHM**

The British Society for the History of Mathematics:

www.bshm.ac.uk

### BSHM undergraduate essay prize

http://www.bshm.ac.uk/undergraduate-essay-prize

### See you next term...

