BO1.1. History of Mathematics

Sheet 0 — HT24

Reading Course: Georg Cantor, the infinite, and the origins of set theory. Christmas Vacation Reading

Outline

In this reading course, we will investigate the origins of the formalised notion of a set. Our starting point will be the questions asked by Bernard Bolzano about the nature of infinite sets that motivated this process of formalisation in the late nineteenth century. As we have seen, it was Georg Cantor who gave the very first explicit definition of a set, and much of the basic set theory that we now use originates with him. Around the same time that Cantor was working, we also find Richard Dedekind writing (about the nature of numbers) in an explicitly set-theoretic language. At the end of the nineteenth century and the beginning of the twentieth, several authors (such as Ernst Zermelo) responded to Cantor's writing, for example by axiomatising the new set theory, and we will read selections from their papers.

Main texts

The following are the main primary sources that we will be using during the reading course. Ahead of our first class in HT, please make sure that you have access to all of these — if you have any difficulties, please let me know.

- Bernard Bolzano, *Paradoxien des Unendlichen*, Leipzig: Verlag von Felix Meiner, 1851. The original German text can be found online here, as part of a website that collects together all of Bolzano's works. There are several editions and translations of Bolzano's *Paradoxien des Unendlichen*. The recommended English translation is that which appears as *Paradoxes of the infinite* in Steve Russ, *The mathematical works of Bernard Bolzano*, Oxford: Oxford University Press, 2004 (available via SOLO).

- Georg Cantor, 'Beiträge zur Begründung der transfiniten Mengenlehre', *Mathematische Annalen* 46(4) (1895), 481–512; *ibid.* 49(2) (1897), 207–246. The German originals can be found by searching for the journal on SOLO. We will use the English translations of these papers that are available in Philip E. B. Jourdain, *Contributions to the founding of the theory of transfinite numbers*, Chicago and London: Open Court, 1915 (available from the Internet Archive).

- Richard Dedekind, *Was sind und was sollen die Zahlen?*, Braunschweig: Vieweg, 1888. Again, the German original can be found via SOLO. We will use the English version contained in W. W. Beman's translation of some of Dedekind's works: *Essays on the theory of numbers*, Chicago: Open Court, 1901 (available from the Internet Archive).

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- Texts by other contributors to the development of set theory (e.g., Ernst Zermelo), depending on availability of English versions.

Additional sources that may be useful

- William Bragg Ewald, *From Kant to Hilbert: a source book in the foundations of mathematics*, 2 vols., Oxford: Clarendon Press; New York: Oxford University Press, 1999/2005.
- José Ferreirós Domínguez, *Labyrinth of thought: a history of set theory and its role in modern mathematics*, 2nd rev. ed., Basel: Birkhäuser, 2007.
- Ivor Grattan-Guinness, *The search for mathematical roots, 1870–1940: logics, set theories and the foundations of mathematics from Cantor through Russell to Gödel*, Princeton, NJ; London: Princeton University Press, 2000.

Vacation reading

As preparation for the reading course, please review the MT material on the history of set theory, and revisit the relevant sections of Katz. You should look in particular at attitudes towards the infinite prior to the nineteenth century. To this end, please read Katz's section on Zeno's Paradoxes (§2.3.3 in the 3rd edition) in combination with Aristotle's original discussion of these, which you will find in his *Physics*, Book VI, §9, paragraph 239b. There are many English translations of Aristotle's *Physics*, but a recommended version is that by Robin Waterfield, which you can find via SOLO. You may also recall Galileo's Paradox in connection with the infinite. This appeared in his *Discorsi e dimostrazioni matematiche intorno a due nuove scienze* of 1638. The Italian readers among you will find the original discussion on pp. 32–34 of Galileo's original text, but otherwise please read the English translation that has been given most recently on pp. 27–29 of Alessandro De Angelis, *Galileo Galilei's "Two New Sciences" for modern readers*, Cham: Springer, 2021.

In addition to the above, please do a little biographical reading on the main people mentioned above (namely Bolzano, Cantor, and Dedekind), and see whether you can find any other detailed discussions of the infinite prior to Bolzano's

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