BO1.1. History of Mathematics
Sheet 1 — HT23
Reading Course: Georg Cantor, the infinite,
and the origins of set theory. Reading and Essays

Under the headings below, I set out the reading required as preparation for the classes each week. There will be three essays to complete during the term, to be handed in on the **Mondays of weeks 3, 5 and 7**: please see the further details below. For references that are otherwise unspecified, please see the Christmas Vacation Reading sheet.

Details of the assessed extended essay will be sent to you on the Monday of week 7. We will be able to discuss this a little in the class in week 7, but then the class in week 8 will be given over entirely to discussion and advice about the extended essay.

Week 1: Preliminaries (biographies and the infinite)

Please see the Christmas Vacation Reading sheet.

Week 2: Bolzano and the infinite

Preliminary reading

- Ewald (1999/2005), vol. 1, pp. 168–172, 249–250.

Main reading

- Bolzano (1851), §§1–37, as found either in Russ (2004), pp. 595–642, or in Ewald (1999/2005), pp. 250–292.

Essay

What form did (mathematical) discussions of the infinite take prior to the mid-nineteenth century? How did Bolzano's approach to the infinite differ? What were his motivations, and what was the technical content of his work? Where did Bolzano's ideas on the infinite sit within wider trends in nineteenth-century mathematics?

(2000 words, to be handed in by 5pm on Monday of week 3)

Week 3: Cantor, part 1

Preliminary reading

- Ewald (1999/2005), vol. 2, pp. 838–839, 878–881.

Main reading

- Georg Cantor, 'Ueber eine Eigenschaft des Inbegriffes aller reellen algebraischen Zahlen', *Journal für die reine und angewandte Mathematik* 77 (1874), 258–262. The original paper can be found via SOLO, and an English translation is available in Ewald (1999/2005), vol. 2, pp. 839–843.

- Georg Cantor, *Grundlagen einer allgemeinen Mannigfaltigkeitslehre*, Leipzig: Teubner, 1883. This is the stand-alone publication of an essay that originally appeared under a different title as the fifth in a series of papers on set theory ('Ueber unendliche, lineare Punktmannichfaltigkeiten (5)', *Mathematische Annalen* 21(4) (1883), 545–591). The German paper may be found via SOLO (under its original title), and an English translation is available in Ewald (1999/2005), vol. 2, pp. 881–920.

Other useful sources

- Joseph Warren Dauben, *Georg Cantor: His mathematics and philosophy of the infinite*, Princeton, NJ: Princeton University Press, 1991.

- Joseph W. Dauben, 'Georg Cantor, paper on the "Foundations of a general set theory" ', in I. Grattan-Guinness (ed.), *Landmark writings in western mathematics 1640–1940*, Amsterdam; Oxford: Elsevier, 2005, pp. 600–612.

- Grattan-Guinness (2000), chapter 3.

- G. H. Moore, 'Towards a history of Cantor's Continuum Problem', in J. McCleary and D. Rowe (eds.), *The history of modern mathematics. Volume I: Ideas and their reception*, San Diego: Academic Press, 1989, pp. 79–121.

Week 4: Cantor, part 2

Ongoing reading

- Jourdain (1915), Introduction.

Main reading

- Georg Cantor, 'Beiträge zur Begründung der transfiniten Mengenlehre (1)', *Mathematische Annalen* 46(4) (1895), 481–512. The German original may be found via SOLO, and an

English translation is available in Jourdain (1915), pp. 85–136.

Essay

Discuss the context, content, and significance of Cantor's early work on set theory. What motivated his development of this framework for doing mathematics? Which particular problems were of interest to him? What was the technical content of his work? What were the immediate responses to Cantor's ideas?

(2000 words, to be handed in by 5pm on Monday of week 5)

Week 5: Cantor, part 3

Ongoing reading

- Jourdain (1915), Introduction.

Main reading

- Georg Cantor, 'Beiträge zur Begründung der transfiniten Mengenlehre (2)', *Mathematische Annalen* 49(2) (1897), 207–246. The German original may be found via SOLO, and an English translation is available in Jourdain (1915), pp. 137–208.

Week 6: Dedekind and set theory

Main reading

- Dedekind (1888); an English translation appears under the title 'The nature and meaning of numbers' in Beman (1901), pp. 44–115; a revised English translation is given by Ewald (1999/2005), vol. 2, pp. 790–834.

Other useful sources

- Ewald (1999/2005), vol. 2, pp. 787–790 and pp. 843–878, 923–940 [the latter two page ranges contains transcriptions of Cantor's correspondence with Dedekind].

- José Ferreirós, 'On the relations between Georg Cantor and Richard Dedekind', *Historia Mathematica* 20 (1993), 343–363.

- I. Grattan-Guinness, 'The rediscovery of the Cantor–Dedekind correspondence', *Jahresbericht der Deutschen Mathematiker-Vereinigung* 76 (1974), 104–139.

Essay

How did Dedekind employ set-theoretic ideas in his writings on the nature of numbers? What was his connection to Cantor, and how was he influenced by him? How did Dedekind's attitude towards Cantor's ideas compare with other responses that you have seen?

(2000 words, to be handed in by 5pm on Monday of week 7)

Week 7: Other responses to Cantor's set theory

Main reading

- Ernst Zermelo, 'Untersuchungen über die Grundlagen der Mengenlehre I', *Mathematische Annalen* 65(2) (1908), 261–281. The German original can be found via SOLO. An English translation is available in Jean van Heijenoort, *From Frege to Gödel: A source book in mathematical logic, 1879–1931*, Cambridge, MA: Harvard University Press, 2002, pp. 199–215. The original German can also be found, along with a different English translation, in Ernst Zermelo, *Collected works/Gesammelte Werke: Volume I – Set theory, miscellanea/Band I — Mengenlehre, Varia* (Craig G. Fraser, Akihiro Kanamori, Heinz-Dieter Ebbinghaus, eds.), Berlin; Heidelberg: Springer, 2010, pp. 188–229.

- You will by now have seen that there were several other responses to Cantor's set theory. Please select one that interests you, and be prepared to discuss it in the class.

Other useful sources

- I. Grattan-Guinness, 'The correspondence between Georg Cantor and Philip Jourdain', *Jahresbericht der Deutschen Mathematiker-Vereinigung* 73 (1971), 111–130.

- Grattan-Guinness (2000), chapter 4 and §8.7.6.

- Gregory H. Moore, *Zermelo's axiom of choice: Its origins, development, and influence*, New York: Springer, 1982 (Dover reprint 2013).

- Zermelo, *Collected works*, pp. 80–114, 160–188.

Week 8

No reading is assigned for week 8: the class in week 8 is given over entirely to discussion of the extended essay.