

## Geometric Group Theory

### Problem Sheet 0

1. Show that a subgroup of index 2 is normal.
2. Let  $A, B$  be finite index subgroups of  $G$ . Show that  $A \cap B$  is a finite index subgroup of  $G$ .
3. Let  $G$  be a finitely generated group and let  $H$  be a subgroup of  $G$  of finite index. Show that  $H$  is finitely generated.
4. Show that if  $G$  is a finitely generated group such that every (non-trivial) element of  $G$  has order 2 then  $G$  is finite.
5. Let  $H$  be a finite index subgroup of  $G$ . Show that there is a normal finite index subgroup  $N$  of  $G$ , such that  $N \subset H$ .
6. Let  $G$  be a finitely generated group. Show that  $G$  has finitely many subgroups of index  $n$ . (*hint*: use the previous exercise).