Comment on Problem Sheet 4, question 4, part (c)

In this question, we are asked to show the following:

(c) In the ejection phase, explain why $p_{\mathrm{LV}} \approx p_a$, and hence show that

$$p_a^* = \frac{C_d p_{\mathrm{LV}}^0 + C_a p_a^0}{C_a + C_s}.$$

Show that the evolution of p_a and p_v are then governed approximately by

$$(C_a + C_s)\dot{p}_a = -\frac{p_a}{R_C}, \qquad \dot{p}_v = \frac{p_a}{R_c C_v}.$$

Hence solve for p_a and p_v in the ejection phase. Show that at the end of this phase,

$$p_a = p_{\mathrm{LV}} = p^\dagger, \qquad \text{and} \qquad p_v = p^*,$$

where

$$p^\dagger = p_a^0 \exp\left(-\frac{\Delta t_F}{R_c(C_a + C_S)}\right),$$
$$p^* = p_v^0 - \frac{R_c(C_a + Cs)p_a^0}{R_c C_v}\left[\exp\left(-\frac{\Delta t_F}{R_c(C_a + C_s)}\right) - 1\right].$$

The part in red is true if this phase occurs very quickly. However, the part in blue does not make this assumption. These two parts are therefore two possible solutions to the same question that cannot be reconciled. I have updated the solutions to reflect the answers to the two possible parts, but caveat this with the fact that a corrected version of this problem would not have the red part and instead would just ask the blue part of the question.