

String Theory 1

Lecture # 16

6 D-branes

Last lecture: we defined a **D_p-brane** as a (p+1)-dimensional subspace of target space where the ends of open strings can end

(We refer to this subspace as the Dbrane world volume)

We saw how D-branes appear from T-duality

Strings with Neumann boundary conditions \longleftrightarrow Dirichlet boundary conditions

Today: a number of observations about Dbranes (mostly without proofs \rightarrow just an idea of what these important objects are in the context of string theory; see e.g. Zwiebach)

In this lecture course we studied:

- ▶ quantized strings (open & closed) in $\mathbb{R}^{1,25}$
a salient feature is that the massless sector includes
a graviton (from the closed sector)
gauge fields (from the open sector)

Note that we discussed OS with Neumann bcs only

- ▶ We also discussed quantized strings in $\mathbb{R}^{1,24} \times S^1$

↳ new features eg

- states have quantized momentum along S^1
and a winding quantum number

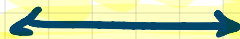
- T duality

OS more complicated; T-duality leads to the
notion of **D-branes**

Last lecture:

OS -

T-duality



open string with
Neumann boundary conditions
compactified on S^1_R

dual open string with
Dirichlet boundary conditions
compactified on $S^1_{\hat{R}}$, $\hat{R} = \alpha' / R$

D25 space-filling brane
↳ open string ends are
free to move on space-time

endpoints of the string
live on a D24 brane

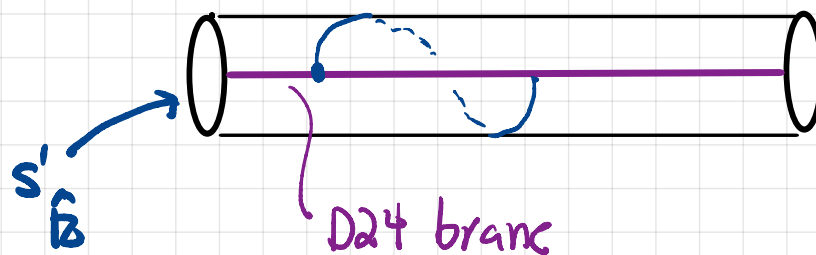
$$p^{25} = \frac{m}{R} \quad \text{quantized}$$

no winding

no translational symmetry
along $S^1_{\hat{R}}$

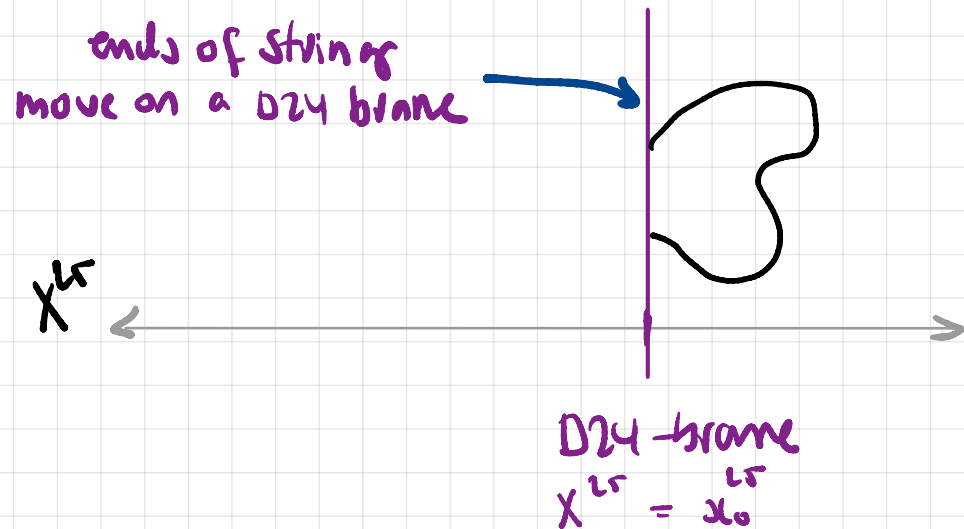
string can wind around $S^1_{\hat{R}}$

massless sector: (both sides)
25 dimensional $U(1)$ gauge field



6.1 Open strings with Dirichlet b.c.s in flat $\mathbb{R}^{1,25}$ (no compactification)

Consider an open string on $\mathbb{R}^{1,25}$ with **Dirichlet** boundary conditions in **one** direction (x^{25}) and **Neumann** boundary conditions in all other directions (x^i $i=0, \dots, 24$).



no translational symmetry along x^{25}
space-time symmetry
 $SO(1, 25) \rightarrow SO(1, 24)$

[More generally, one can consider an open string with Dirichlet boundary conditions in $26-(p+1)$ directions and Neumann boundary conditions in $(p+1)$ directions. In this case string ends move on a D_p brane and $SO(1, 25) \rightarrow SO(1, p) \times SO(25-p)$]

Mode expansion for $X^\mu(\bar{t}, \sigma)$:

Neumann
boundary
conditions

$$X^i(\bar{t}, \sigma) = x^i + 2\alpha' \bar{t} p^i + i\sqrt{2\alpha'} \sum_{n \neq 0} \frac{1}{n} \alpha_n^i \cos(n\sigma) e^{-in\bar{t}} \quad i=0, \dots, 24$$

Dirichlet
boundary
conditions

$$X^{25}(\bar{t}, \sigma) = x_0^{25} + \frac{1}{\sqrt{2\alpha'}} \frac{1}{\pi} (x_1^{25} - x_0^{25}) \sigma + \sqrt{2\alpha'} \sum_{n \neq 0} \frac{1}{n} \alpha_n^{25} e^{-in\bar{t}} \sin(n\sigma) e^{-in\bar{t}}$$

$$X^{25}(\bar{t}, 0) = X^{25}(\bar{t}, \pi) = x_0^{25}$$

no α_0^{25} mode.

\Rightarrow no momentum along X^{25}
(a term $p^{\mu} \bar{t} \Rightarrow$ endpoints would not stay at x_0^{25} when $\bar{t} \neq 0$)

quantizing the string: mostly as before except

$x_0^{L\bar{r}}$ remains a number

($x_0^{L\bar{r}}$ is not a parameter, it represents the location of a fixed D-brane)

Virasoro operators as before.

Mass-shell condition: $L_0 - 1 = (\alpha' p^2 + N) - 1$

becomes $\alpha' M_{L\bar{r}}^2 = -\alpha' |p|^2 = N - 1$, $|p|^2 = p \cdot p$ inner product on $\mathbb{R}^{1,24}$

Ground level ($N = 0$): tachyon on the D-brane $\alpha' M_{L\bar{r}}^2 = -1$

Massless spectrum:

must be level $N=1$

ground state
($N=0$)

$$|\mathcal{S}, \eta; K\rangle = (\mathcal{S} \cdot \alpha_{-1} + \eta \alpha_{-1}^{25}) |0; K\rangle$$

\uparrow
25 dim
momentum

\uparrow
(1+24)-dim
polarization vector

\uparrow
spacetime scalar

Imposing $L_1 |\mathcal{S}, \eta; K\rangle = 0 \quad (\Rightarrow L_m |\phi\rangle = 0, m \geq 2)$

we find that $|\mathcal{S}, \eta; K\rangle$ is physical if $\mathcal{S} \cdot K = 0$
with η unconstrained.

$$\begin{aligned} L_1 |\mathcal{S}, \eta; K\rangle &= (\mathcal{S}_i ([L_1, \alpha_{-1}^i] + \alpha_{-1}^i L_1) + \eta ([L_1, \alpha_{-1}^{25}] + \alpha_{-1}^{25} L_1)) |0; K\rangle \\ &= (\mathcal{S} \cdot \alpha_0 + \eta \alpha_0^{25} + (\mathcal{S} \cdot \alpha_{-1} + \eta \alpha_{-1}^{25}) L_1) |0; K\rangle \\ &= (\mathcal{S} \cdot K + (\mathcal{S} \cdot \alpha_{-1} + \eta \alpha_{-1}^{25}) (\alpha_{-1} \cdot \alpha_0)) |0; K\rangle = (\mathcal{S} \cdot K) |0; K\rangle \end{aligned}$$

null states at level one of the NSM $L_{-1}|0; K\rangle$:

$$L_{-1}|0; K\rangle = K \cdot \alpha_{-1}|0; K\rangle$$

$$\text{with } K \cdot K = 0 \quad (\text{Norm } (L_{-1}|0; K\rangle) = 0)$$

Then we have the massless physical states

- ▶ 25-dimensional photon $S \cdot \alpha_{-1}|0; K\rangle$ $\alpha_{(1)}$ physical state
 $S \cdot K = 0$

so the D-brane has a $U(1)$ field on its worldvolume
(true for any D_p brane)

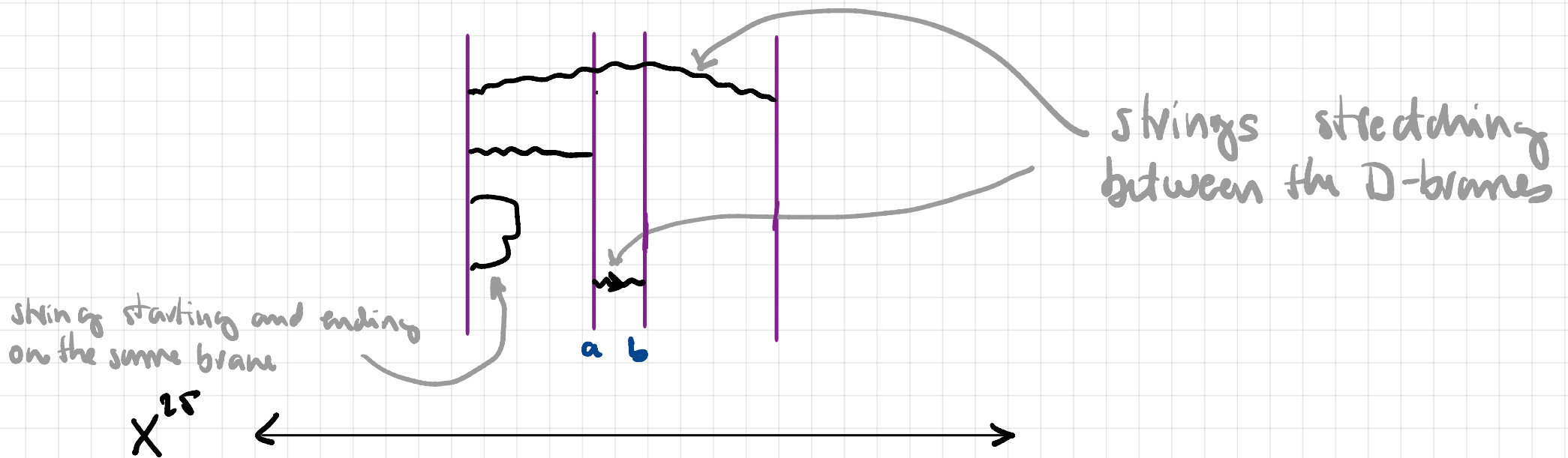
- ▶ scalar field $\varphi = \eta \alpha_{-1}^{25}|0; K\rangle$

more generally a D_p brane has a massless scalar for each normal direction.

φ can be identified with fluctuations in the position of the D-brane along the transverse x^{25} direction (no proof here!)
see B Zwiebach

6.2 Stretched strings

One can also have **systems** of D-branes with different classes of open strings (open string sectors)



Consider a string stretched between two parallel D24 branes located at $x_0^{25} = x_a^{25}$ and $x_0^{25} = x_b^{25}$

String endpoints

$$X_{ab}^L(\bar{0}, \sigma=0) = x_a, \quad X_{ab}^L(\bar{0}, \sigma=\pi) = x_b$$

$$X_{ab}^{2\sigma} = x_a^{2\sigma} + \frac{1}{\pi} \underbrace{(x_b^{2\sigma} - x_a^{2\sigma})}_{\Delta X_{ab}} \sigma + \sqrt{2\alpha'} \sum_{n \neq 0} \frac{1}{n} \alpha_n^{2\sigma} e^{-in\sigma} \sin(n\sigma)$$

$$\alpha_0^{2\sigma} = \frac{1}{\sqrt{2\alpha'} \pi} (x_b^{2\sigma} - x_a^{2\sigma})$$

mass-shell condition: $M_{ab}^2 = -p \cdot p = \underbrace{\left(\frac{x_b^{2\sigma} - x_a^{2\sigma}}{2\pi\alpha'} \right)^2}_{\text{shift of mass-levels}} + \frac{1}{\alpha'} (N-1)$

→ shift of mass-levels: $\left(\frac{\Delta x}{2\pi\alpha'} \right)^2 = [T \Delta x]^2 = \text{mass}^2$ of a string stretched between the branes

spectrum of the stretched string:

• $N=0$

$|K, ab\rangle$

information about which
D-brane string ends live
Chan-Paton levels

Labels

$[a, b]$

a denotes brane on which
end $\sigma=0$ lives
 b denotes brane on which
end $\sigma=\pi$ lives

in our case a, b take values
 $1, 2$; more generally, for N D-branes
they take values $1, \dots, N$

(For a string with ends on the same brane $a=b$)

mass shell condition:

$$M_{ab}^2 = -\frac{1}{\alpha'} + \left(\frac{\Delta X_{ab}}{2\pi\alpha'}\right)^2$$

tachyon if $|\Delta X_{ab}| < 2\pi\sqrt{\alpha'}$

$$\cdot \underline{N=1}$$

$$M_{ab}^2 = \left(\frac{\Delta X_{ab}}{2\pi\alpha'} \right)^2$$

$$S \cdot \alpha_{-1} |K, ab\rangle \quad (K \cdot P = 0)$$

$$\eta \alpha_{-1}^{25} |K, ab\rangle$$

← massive vector
on 25 dim space time

null states

$$L_{-1} |0; K; ab\rangle = K \cdot \alpha_{-1} |K; K; ab\rangle + \frac{\Delta X_{ab}}{\sqrt{2\alpha'} \pi} \alpha_{-1}^{25} |\eta; K; ab\rangle$$

with $K \cdot K = 0$ (from $L_{-1} = 0$ condition)

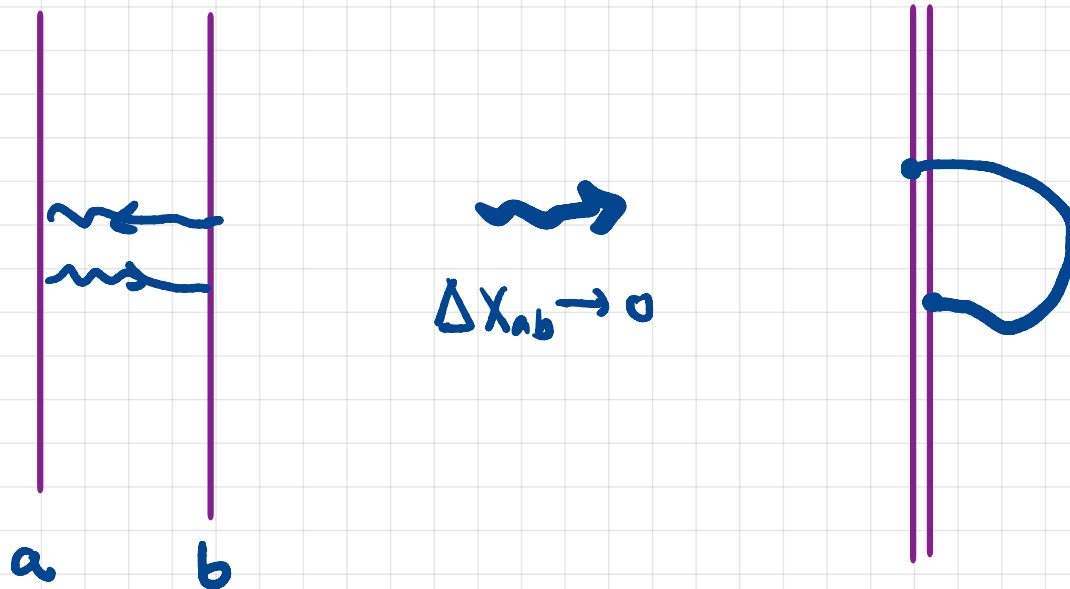
Longitudinal mode & scalar are null!

null states

$$\begin{aligned}L_{-1} |0; K; ab\rangle &= \frac{1}{2} \sum_n \alpha_{-1-n} \cdot \alpha_n | \dots \rangle \\&= \frac{1}{2} \left(\alpha_{-1} \cdot \alpha_0 + \sum_{n>0} (\cancel{\alpha_{-1-n} \cdot \alpha_n} + \alpha_{-1+n} \cdot \alpha_{-n}) \right) | \dots \rangle \\&= \frac{1}{2} \left(\alpha_{-1} \cdot \alpha_0 + \alpha_{-1} \cdot \alpha_0 \right) | \dots \rangle = (\alpha_{-1} \cdot \alpha_0) | \dots \rangle \\&= (\alpha_{-1} \cdot K + \alpha_{-1}^{2r} \alpha_0^{2r}) | \dots \rangle \\&= K \cdot \alpha_{-1} | \dots \rangle + \frac{\Delta X}{\sqrt{2\alpha'} \pi} \alpha_{-1}^{2r} | \dots \rangle \quad \left(\alpha_0^{2r} = \frac{1}{\sqrt{2\alpha'} \pi} \Delta X \right)\end{aligned}$$

with $K \cdot K = 0$ (Norm $(L_{-1} |0; K\rangle) = 0$)

Coincident limit: suppose we have N D-branes



stretched string states between
D-brane at x_a & D-brane at x_b

N massive vector fields

$|g; K, ab\rangle$

N^2 sectors

\Rightarrow
 $\Delta X_{ab} \rightarrow 0$

massless gauge fields

$|g; \alpha, K; ab\rangle + \text{scalars}$
 $a, b = 1, \dots, N$
Chan-Paton labels

One can show that the spectrum has a $U(N)$ global symmetry and that these (N^2) states transform in the adjoint representation of $U(N)$

One can choose a basis for these states

$$|S, K; A\rangle = \sum_{a,b} (t^A)^a_b |S, K; ab\rangle$$



$$A = 1, \dots, N^2$$



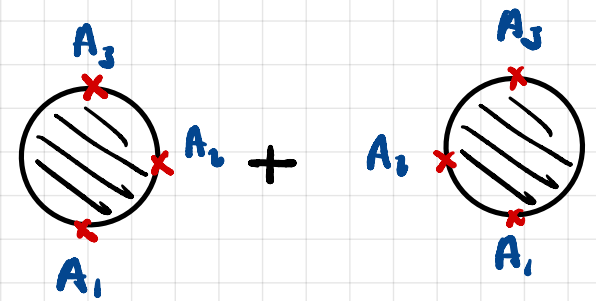
hermitian basis of $u(N)$

$$N(t^A t^B) = \delta^{AB}$$

Chan-Paton factors

$$\text{Null states: } L_{-1}|0, \underline{K}; ab\rangle = |\underline{K}; \underline{K}; ab\rangle + \frac{\Delta X_{ab}}{\sqrt{2} \pi} |\eta; \underline{K}; ab\rangle$$

3-point coupling of massless vectors



$$A(S_1, K_1, A_1; S_2, K_2, A_2; S_3, K_3, A_3)$$

$$\sim g_0 \delta(K_1 + K_2 + K_3) \left\{ S_1 \cdot K_3 S_2 \cdot S_3 + S_2 \cdot K_3 S_1 \cdot S_3 + S_3 \cdot K_2 S_1 \cdot S_2 \right. \\ \left. + \frac{\alpha'}{2} S_1 \cdot K_{23} S_2 \cdot K_{31} S_3 \cdot K_{12} \right\} \times \text{tr}(t^{a_1} [t^{a_2}, t^{a_3}])$$

gives the 3-point vertex for the $U(N)$ non-Abelian gauge theory

EFT action on D24 brane

$$\mathcal{L} = -\frac{1}{4} \text{Tr}(F_{\mu\nu} F^{\mu\nu}) - \frac{2i\alpha'}{3} \text{Tr}(F_{\mu\nu} F_{\nu\rho} F_{\rho\mu}) + \text{scalars}$$

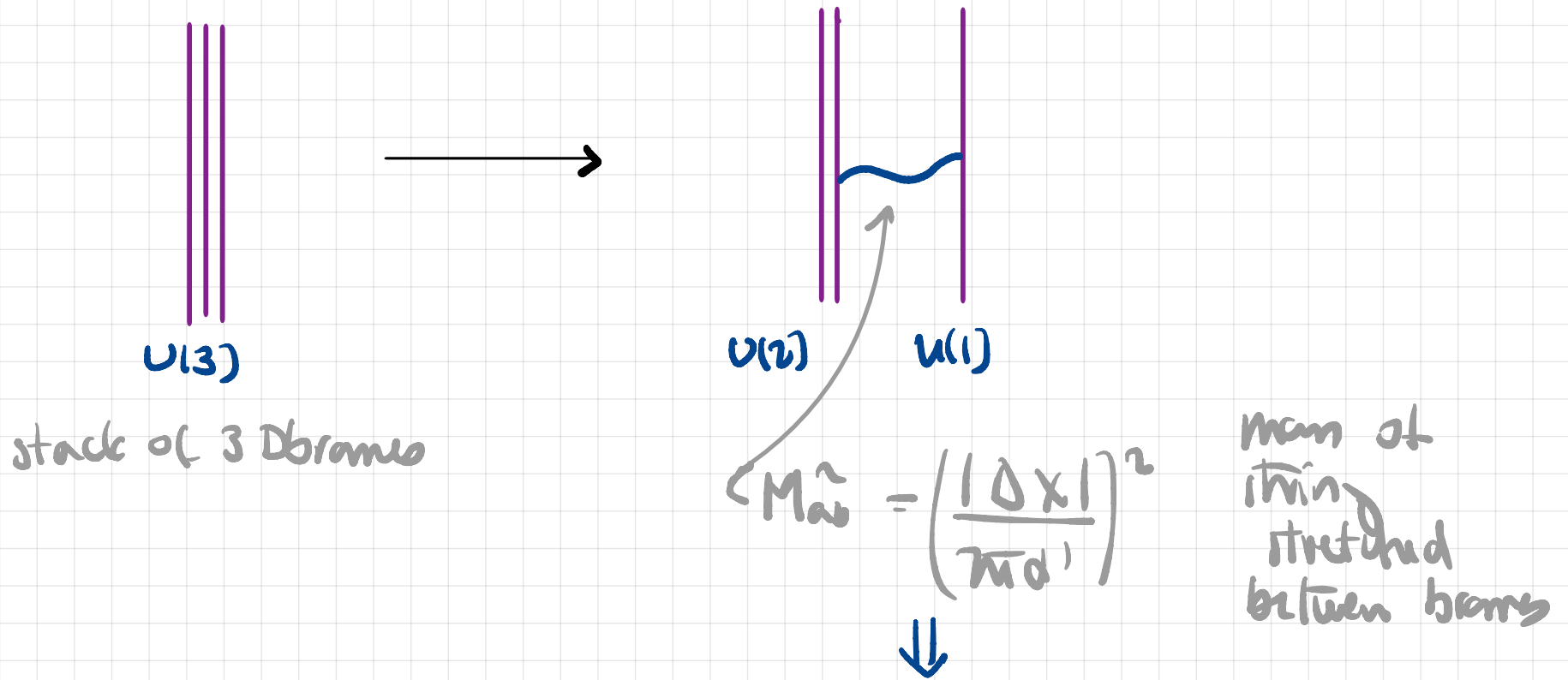
Yang-Mills

α' corrections

One can also obtain this from the β -function

(needs boundary couplings and boundary renormalization (bwr))

D-brane picture of non-abelian gauge theory leads to a rather nice picture of the Higgs mechanism.



$$L_{-1}|0, \underline{k}; ab\rangle = |k_i, \underline{k}; ab\rangle + \frac{\Delta x_{ab}}{\sqrt{2\pi}\alpha'} |\eta_i, \underline{k}; ab\rangle$$

longitudinal W 's \rightarrow W 's "eat" the Higgs \uparrow

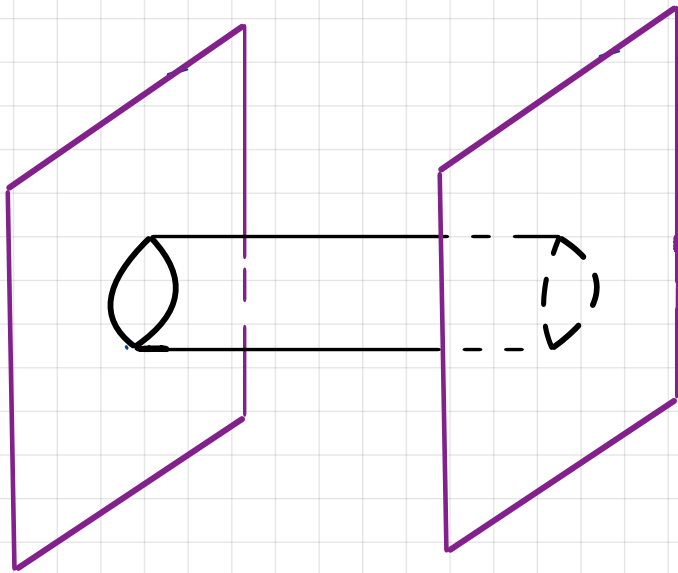
Epilogue:

D-branes as dynamical objects?

If they are, maybe they need to be included in the perturbative description of strings? how?

Estimate mass scales relevant to D-branes by computing its tension

closed string exchange between two D-branes



gravitational coupling $\kappa \sim g_0^2$

$$A \sim \kappa^2 \tau_p^2 \sim (g_0^4)$$

D-brane tension grav. o

$$\tau_p \sim \frac{1}{g_0^2}$$

D-branes are massive "non perturbative" objects

Polchinski 1995, "duality revolution"

Final remarks:

We have seen that the theory of quantised strings has a very rich structure

- ▶ quantised gravity (at low energies we obtain Einstein's gravity)
- ▶ gauge fields
- ▶ consistency of the theory \rightarrow fixes dimension of space time

► compactifications (strings in (nontrivial) background fields

↳ this covers: $\mathbb{R}^{1,2,5}$, $\mathbb{R}^{1,2,4} \times S^1_{12}$

More generally: • $\mathbb{R}^{1,d-1} \times M^{D-d}$ ← M Ricci flat to leading order in α'
(from $\beta=0$)

• $X^{1,d-1} \times M^{D-d}$ ← geometry dictated by $\beta=0$
eg max symmetric ↗

eg $AdS^3 \times S^7$, $AdS^5 \times S^5$, etc ...

• or even more general setups

► T duality

► emergence of (non-perturbative) Pbranes

More to learn

- ▶ dualities (Mirror symmetry, AdS/CFT)
- ▶ emergence of non-perturbative Branes
- ▶ CFT & AdS/CFT
- ▶ Strong coupling regime
- ▶ Black hole physics
- ▶ realistic phenomenology
- ▶ mathematical structures
 - ↳ geometry (differential, algebraic, ...)
 - topology, number theory, algebra ...

Improvements:

- ▶ remove tachyons \rightarrow (ST2) superstrings
(fermions in 2dim NLSM: supersymmetric WS theory)
- ▶ spacetime fermions \rightarrow ST2
- ▶ superstring theory (ST2) \rightarrow spacetime dim = 10
- ▶
⋮
- ▶ dualities

End of String Theory I

Thanks!