Visualising the filters and response, memory, in a CNN: deterministic multi-resolution frames and transfer learning

Theories of Deep Learning: C6.5,
Lecture / Video 11

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LeNET-5, an early Image processing DNN:

Network architectures often include fully connected and convolutional layers

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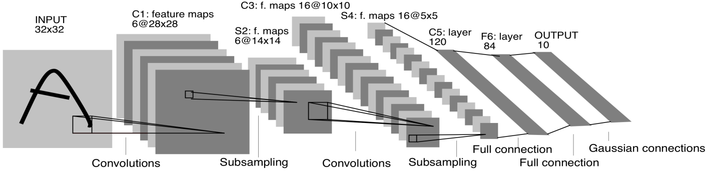

Fig. 2. Architecture of LeNet-5, a Convolutional Neural Network, here for digits recognition. Each plane is a feature map, i.e. a set of units whose weights are constrained to be identical.

C1: conv. layer with 6 feature maps, 5 by 5 support, stride 1.

S2 (and S4): non-overlapping 2 by 2 blocks which equally sum values, mult by weight and add bias.

C3: conv. layer with 16 features, 5 by 5 support, partial connected.

C5: 120 features, 5 by 5 support, no stride; i.e. fully connected.

F6: fully connected, $W \in \mathbb{R}^{84\times 120}$.

`http://yann.lecun.com/exdb/publis/pdf/lecun-98.pdf`

A simple two layer CNN (Papyan et al. 16')

Convolutional structure are the form of multi-resolution analysis

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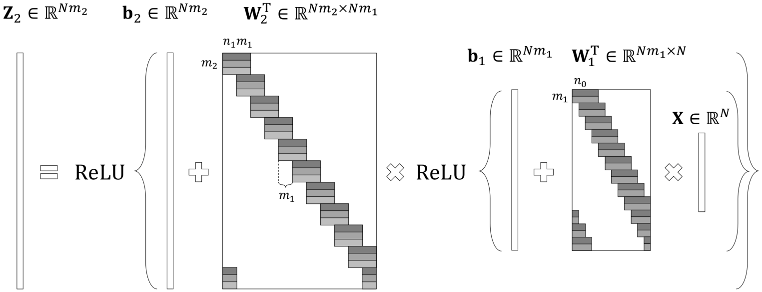
Consider a deep conv. net composed of two convolutional layers:



$$Z_2 = \sigma\left(b^{(2)} + (W^{(2)})^T \sigma\left(b^{(1)} + (W^{(1)})^T x\right)\right)$$

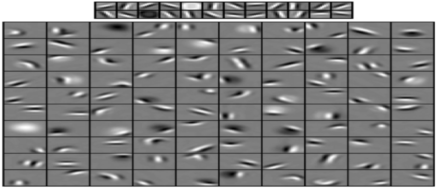
https://arxiv.org/pdf/1607.08194.pdf

Convolutional Deep Belief Networks (H. Lee et al. 11')

Localized Fourier, Wavelet, structure learned

We omit the details of this somewhat different architecture, which is stylistically similar to a deep CNN.

Figure 3. The first layer bases (top) and the second layer bases (bottom) learned from natural images. Each second layer basis (filter) was visualized as a weighted linear combination of the first layer bases.



http://www.cs.utoronto.ca/~rgrosse/cacm2011-cdbn.pdf
Display of the convolutional masks in layers 1 and 2, trained from Kyoto natural image database.
http://eizaburo-doi.github.io/kyoto_natim/

Convolutional Deep Belief Networks (H. Lee et al. 11')

Learned / memorized complex structure from data classes

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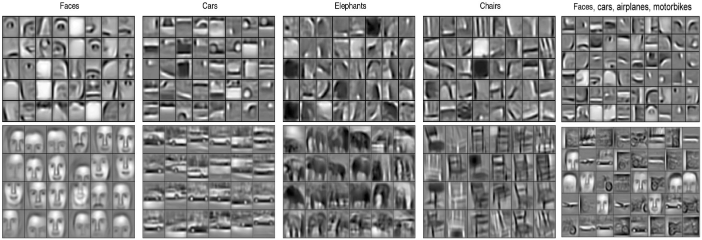
Figure 4. Columns 1–4: the second layer bases (top) and the third layer bases (bottom) learned from specific object categories. Column 5: the second layer bases (top) and the third layer bases (bottom) learned from a mixture of four object categories (faces, cars, airplanes, motorbikes).



http://eizaburo-doi.github.io/kyoto_natim/

The third and fourth layers develop bases which represent features or objects, trained on CalTech 101 dataset.

http://www.vision.caltech.edu/Image_Datasets/Caltech101/

Deep CNN, AlexNet (Krizhevsky et al. 12')

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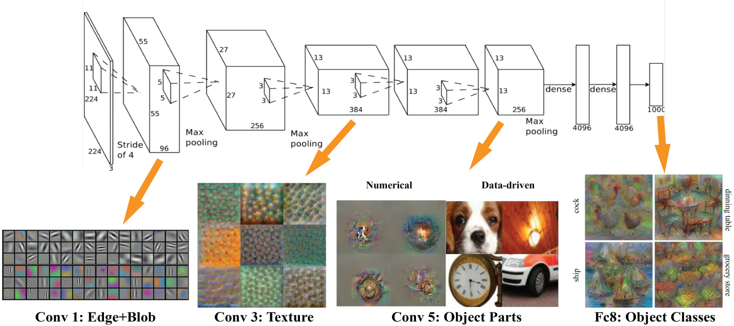

Images are those that maximize specific activation responses.
Layer 1 are masks, subsequent layers are their linear combinations.

`http://papers.nips.cc/paper/4824-imagenet-classification-with-deep-convolutional-neural-networks.pdf`

Deep CNN, VGG (Mahendran et al. 16')

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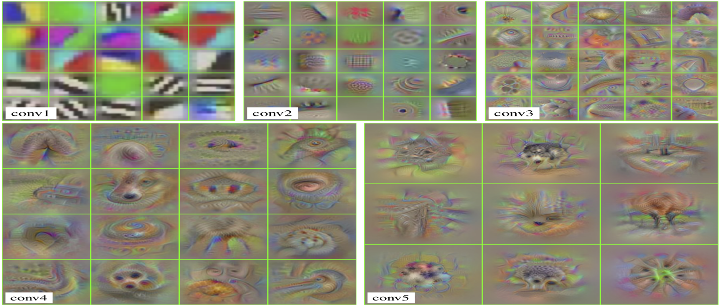


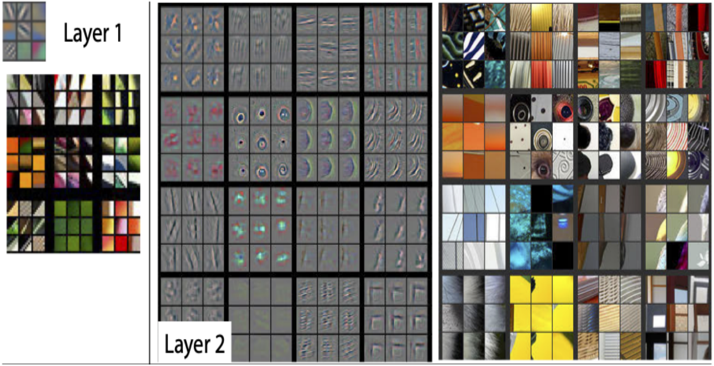
Figure 16: Activation maximization of the first filters of each convolutional layer in VGG-M.

Note, again we observe the same pattern, the initial filters are similar to Gabor/Wavelet filters and later layers are image components.

https://arxiv.org/abs/1512.02017

Deep CNN (Zeiler et al. 13')

Learned / memorized complex structure from data classes



Layer 1 are masks, subsequent layers are their linear combinations.

https://arxiv.org/abs/1311.2901

Deep CNN (Zeiler et al. 13')

Learned / memorized complex structure from data classes

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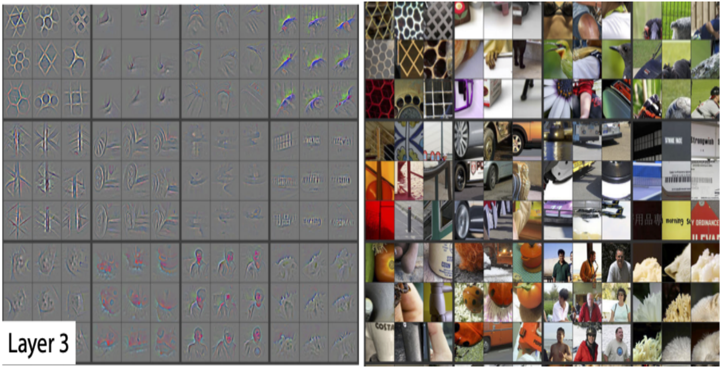

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Deep CNN (Zeiler et al. 13')

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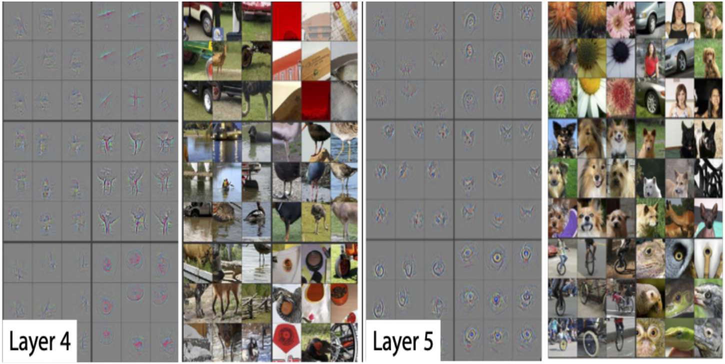

Layer 1 are masks, subsequent layers are their linear combinations.

https://arxiv.org/abs/1311.2901

Summary: similarity and importance of initial layers

Importance of training initial layers to develop representation

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We observe the initial layer of CNNs to be similar to one another, and to exhibit wavelet like representations. This is to be expected.

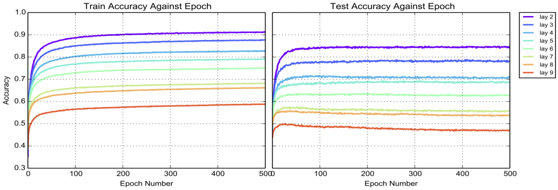
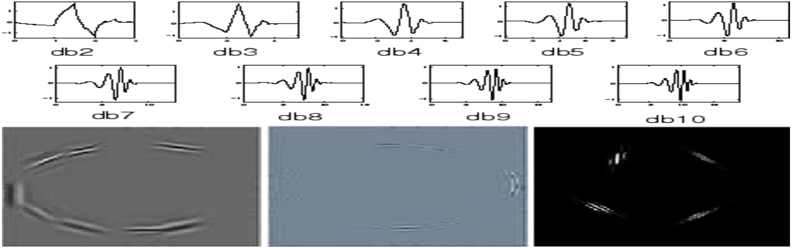


Figure 6: Demonstration of expressive power of remaining depth on MNIST. Here we plot train and test accuracy achieved by training exactly one layer of a fully connected neural net on MNIST. The different lines are generated by varying the hidden layer chosen to train. All other layers are kept frozen after random initialization.

Accuracy of a *random network* is improved most by training earlier layers (Raghu 16').

`https://arxiv.org/pdf/1611.08083.pdf`

Applied and computational harmonic analysis community developed representations with optimal approximation properties for piecewise smooth functions.



Most notable are the Daubechies wavelets and Curvelets/Contourlets pioneered by Candes and Donoho. While optimal, in a certain sense, for a specific class of functions, they can typically be improved upon for any particular data set.

Optimality of curvelets in 2D

Near optimality suggest a good initial CNN layer.

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Theorem (Candes and Donoho 02')

Let f be a two dimensional function that is piecewise C^2 with a boundary that is also C^2. Let f_n^F, f_n^W, and f_n^C be the best approximation of f using n terms of the Fourier, Wavelet and Curvelet representation respectively. Then their approximation error satisfy $\|f - f_n^F\|_{L^2}^2 = \mathcal{O}(n^{-1/2})$, $\|f - f_n^W\|_{L^2}^2 = \mathcal{O}(n^{-1})$, and $\|f - f_n^C\|_{L^2}^2 = \mathcal{O}(n^{-2}\log^3(n))$; moreover, no fixed representation can have a rate exceeding $\mathcal{O}(n^{-2})$.

http://www.curvelet.org/papers/CurveEdges.pdf

Initial layers can start as representations for the data class

Transfer learning: training only the final classification layer

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The first layer of a CNN can be initialized from a known representation for the data class. One can perform classification based on two layer net: layer 1: $h_2(x) = \sigma(W^{(1)}x + b^{(1)})$ where $W^{(1)}$ is a fixed transform of x to, say, the wavelet domain and $\sigma(\cdot)$ project to keep just the largest entries with hard or soft thresholding;

$$\sigma_{hard}(x;\tau) = \begin{cases} x & x > \tau \\ 0 & |x| \leq \tau \\ -x & x < -\tau \end{cases}, \quad \sigma_{soft}(x;\tau) = \begin{cases} x-\tau & x > \tau \\ 0 & |x| \leq \tau \\ -x+\tau & x < -\tau \end{cases}$$

layer2: $h_3 = \sigma(W^{(2)}h_2 + b^{(2)})$ with $W^{(2)}$ learned as the classifier based on the sparse codes h_2. However, h_2 does not build in invariance we would desire in classification, such as dilation, rotation, translation, etc... Depth remains important to generate these.