C5.2 Elasticity and Plasticity

Introduction

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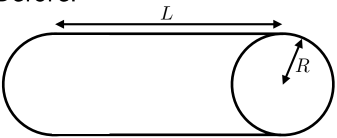
Michaelmas Term

Hooke's Law

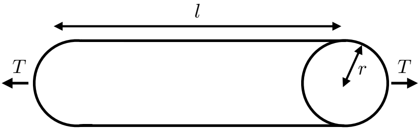
In a classical experiment, a solid rod is stretched to a tension T:

Before:



L = length
$A = \pi R^2$ = cross-section area

After:



ℓ = length
πr^2 = cross-section area

Hooke's Law states that tension $\propto$ extension:

$$T = k(\ell - L)$$

where constant of proportionality k measures stiffness of the rod.

Hooke's Law

Empirical observations:

(1) stiffness $k \propto A/L$ so

$$\left(\frac{T}{A}\right) = E\left(\frac{\ell - L}{L}\right)$$

- $T/A =$ *stress* i.e. force per unit area.
- $(\ell - L)/L =$ *strain* i.e. extension relative to the initial length.
- $E =$ *Young's modulus* is constant for any solid material.

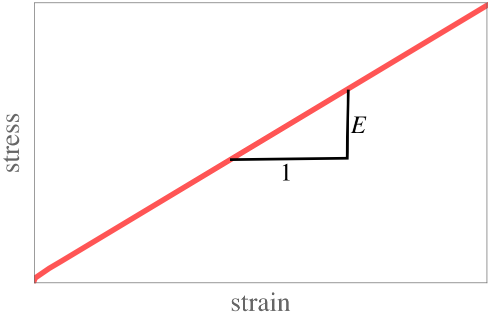
(2) cross-sectional contraction or expansion:

$$\left(\frac{r - R}{R}\right) = -\nu\left(\frac{\ell - L}{L}\right)$$

- contraction for $\nu > 0$, expansion for $\nu < 0$.
- $\nu =$ *Poisson's ratio* is constant for any solid material.

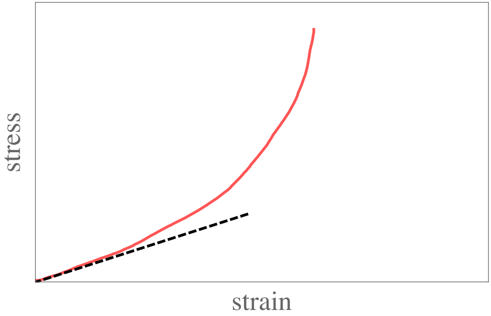
E and ν characterize a linear isotropic elastic material.

Hooke's law



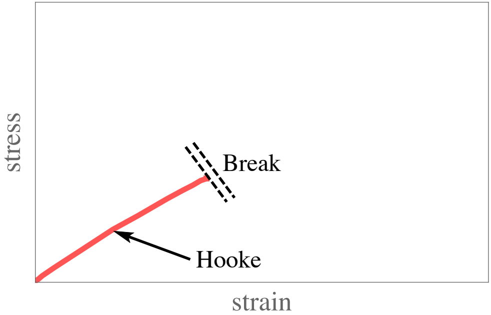
- Hooke's law underpins linear elasticity.
- It generally works well for sufficiently small strain but fails at larger strains for *three* main reasons.

(1) Nonlinear elasticity



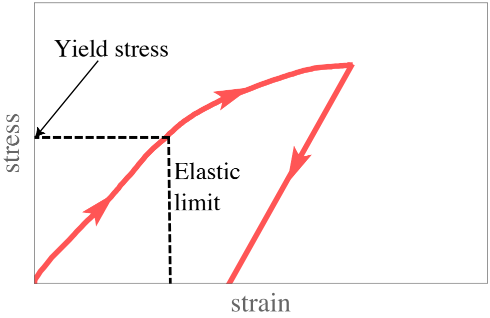
- Materials like rubber can undergo large strains with nonlinear stress–strain relationship.
- But most solids are not like rubber — elasticity breaks down before nonlinearity becomes important.

(2) Fracture



- Brittle solids fracture and break under excessive loading.
- Typically fracture occurs while material is still in linear régime.

(3) Plasticity



- Ductile solids undergo irreversible plastic deformation if applied stress exceeds a critical yield stress.
- When loading is removed, a permanent residual strain remains.

Lecture plan

Lectures	**Lecture notes**	**Sheet**
1–2	§1 Equations of linear elasticity	0–1
3 3-4	§2 Elementary steady solutions §3 Antiplane strain and torsion	1
5 6–7	§4 Plane strain §5 Elastic waves	2
8–9 10–11 11–12	§6 Models for thin structures §7 Contact §8 Fracture	2–3
13–16	§9 Plasticity	4