String Theory 1

Lecture #1

Welcome to String Theory 1 !

→ delaossa@maths.ox.ac.uk

→ reading : see webpage of the course

→ lecture notes : Ling Lin's from HT 2023

(note differences in conventions)

- My own notes will be uploaded to Moodle
- Sometimes I will use slides ← uploaded ahead of time (so you can annotate them)
- ∃ Index (table of contents) in Moodle

→ classes 3 2hour classes for each group

Chapter 0

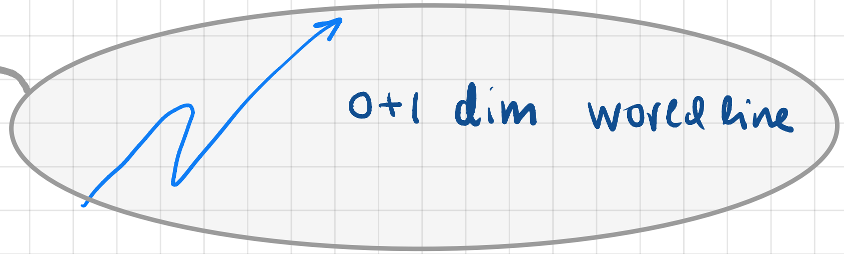
Introduction as motivation

(Let's begin with a brief introduction as a way of motivation)

The starting point of string theory is that it is a theory of fundamental quantum mechanical **strings**

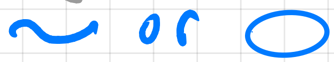
QFT : we model fundamental particles as $\longleftrightarrow$ point-like objects •

particles trace trajectories in spacetime (the worldline) and we study the QFT on the WL (actions describing the physics of these particles)



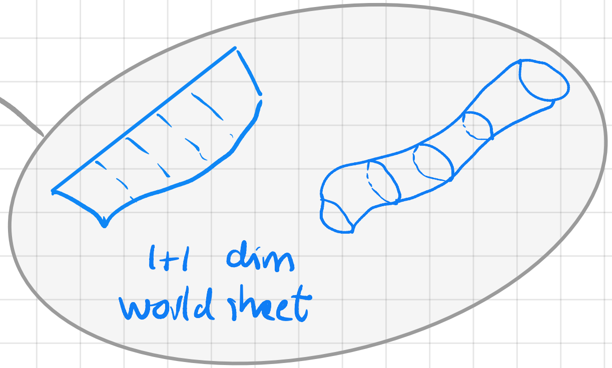
String Theory : fundamental objects $\longleftrightarrow$ strings
(we postulate that fundamental objects are 1 dim)

two topologies:



string sweeps out a 1+1 dim world sheet in spacetime and we study QFTs on the WS of the string

↳ 2 dim 1+1



Before string theory (before 1985)

modern physical theories of nature are { Standard model / GR

- **standard model**: QFT of EM + weak forces + strong forces

  gauge theory with $U(1) \times SU(2) \times SU(3)$ symmetries

  - describes elementary particles and their fundamental interactions
  - ignores gravity : quantum mechanics + special relativity $M = \mathbb{R}^4$

SM works at energy scales of $\sim 10^{-13}$ cm while gravity only becomes significant at much smaller scales $\sim 10^{-33}$ cm

(A)

each particle modeled by a point → is associated to a **field**

$\psi(x)$

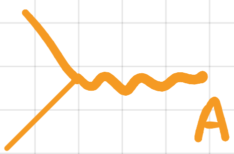
field

intrinsic properties (mass, e.m. charges, ...)

(B)

interactions between particles also described in terms of a particle $A_\mu(x)$ $(\mu = 0, 1, 2, 3)$

← spin 1 particle



math: interaction is localized in space time

↳ The standard model has been extremely successfull in its predictive power and so far consistent with experimental observations

↳ QFT is very rich in mathematical content (involves YM theories, Lie groups, ...)

Given its success mathematicians are trying to create a rigorous mathematical frame for QFT
(axiomatic QFT ⇝ functional analysis, operator algebras, ...)

- **General relativity**: Einstein's theory of gravity

  ↳ gravitational forces come from the curvature of spacetime caused by the presence of energy and matter

  ↳ GR describes the phenomena associated to the large scale of the universe

Mathematically intimately related to <u>differential geometry</u> where the gravitational field is the metric on spacetime & the theory is ruled by the principle of general covariance

Together (SM+GR): extremely successful theories of particle physics & gravity consistent with experimental observations

However there are many problems

- SM leaves many questions unanswered:
  - The Lagrangian describing this model has too many arbitrary (dimensionful) parameters (~20, coupling constants, mass of particles) and there is no explanation for the values they take.

- Naturalness problem: nuclear force $\sim 10^{-13}$ cm; gravitational force $\sim 1.6\times10^{-33}$ cm

  no explanation for the disparate scales between SM & gravity

  $L_P = \left(\frac{\hbar G_N}{c^3}\right)^{1/2}$ $\quad M_P = \left(\frac{\hbar c}{G_N}\right)^{1/2} = 1.1\times10^{19}\,\frac{GeV}{c^2}$

- QFT (perturbative formulation) is incomplete: plagued by UV loop divergences when computing perturbative scattering amplitudes:
  - ↳ puts in doubt the validity of QFT at high energies
  - however ↳ we resort to regularisation and renormalisation (for renormalisable theories)

The standard model is a renormalisable QFT

- a quantum theory of gravity seems inconsistent (on the other hand)
  (UV divergences!)

In fact the perturbative quantum field theory of gravity is **NOT** renormalisable

ST arose as an attempt to find a <u>consistent</u> quantum theory of gravity

Note however that S.T. was developed in the 60's to try to understand the behaviour of hadrons, in particular the large proliferation of hadronic resonances with higher & higher spin. This was abandoned in favor of QCD (now part of the S.M.). Also S.T predicted a massless spin 2 particle that wasn't observed in the hadronic spectrum

This is a first (very introductory) course on string theory and we aim to discuss a few of its key features

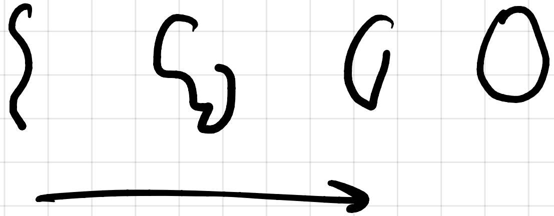
- consistently incorporates gravity (GR) with quantum mechanics so maybe string theory is a theory of quantum gravity

In fact, all string theories contain in their massless spectrum at least one spin 2 boson that can interpreted as the graviton, ie the particle that mediates gravitational interactions

↳ the closed string sector

- It incorporates other interacting & phenomenologically relevant ingredients from QFT & particle physics
  - non-Abelian gauge symmetries with chiral matter from the open string sector

Note that a theory of open strings necessarily contains closed strings since open strings can close up



∴ quantum-gravity & YM theories "unified"

↳ ST contains all known forces of nature

- spacetime supersymmetry (ST 2!)

- "unique" theory (this is very subtle; relates to dualities)

  There are no free parameters

- Extra dimensions
  - $D=26$ bosonic string
  - $D=10$ superstrings

- extended objects (D-branes for example)

But many unresolved issues

While it might be controversial to claim that ST is the fundamental theory of nature we still have many benefits from learning it:

- powerful tools to study strongly coupled field theories eg holography

  holographic principle ⇝ info re spectrum & dynamics of theory inside a volume V encoded in the degrees of freedom on ∂V

  AdS/CFT correspondence   weak-strong coupling duality

  ↳ further courses ?

- ST has inspired and predicted new mathematics

eg Mirror symmetry first discovered in ST
very surprising mathematically

⋮ new branch of mathematics

This course: we will develop

- bosonic string theory
  - ↳ missing some of the features mentioned above and suffers from serious defects & inconsistencies eg contains tachyons
  - ↳ however illustrates key ideas & techniques in a relatively clean way.

If you can continue to

ST II: learn superstring theory
This is a more exciting theory

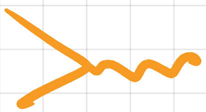
Contents

① Classical relativistic string

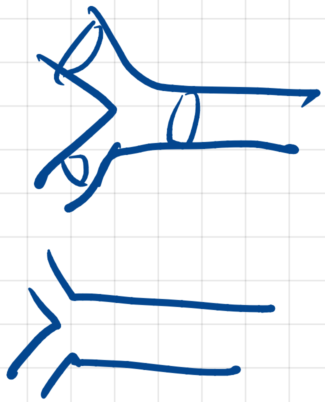
② Quantise
- spectrum ⊃ graviton
- space time dimensions: $D=26$ as a consistency constraint of the quantum theory

2 dim theory on WS
⇝ 2 dim CFT
← TT caum

③ Scattering of strings ⇝ interactions



gets replaced by



④ Strings in background fields

└ so far only WS 2dim theory
→ now a D=26 dim gravity theory emerges
↑ quantum

⑤ Compactifications & T-duality D → d

↳ obtain a d<26 dim theory
reduce 26 dim theory to an effective d dim theory

Next

Chapter 1 Classical relativistic string

↳ study relativistic classical string propagating in a fixed spacetime M

⇝ 1.1 Classical relativistic point particle ↲ in a way that is generalisable to strings

1.2 Classical relativistic string: action principle

1.3 ...