String Theory 1

Lecture #7

chapter 2 Old covariant quantisation

 \mathbf{N}

- Intro Inction 2.1
- 2.2 Hilbert space (without constraints)
- 2.3 Constraints, Normal ordering and the Viranoso algora
- 2.4 Constraints and physical states
- 2.5 Mass-shell and level matching conditions
- 2.6 Level 1 states & dealing with graves
- 2.7 level 2 states e dealing with ghosts
- 2.9 Null states and D=26
- 2.3 Physical states of the closed string

In the last four lectures we have been discussing the OCQ of the

Madivistic string.

Alphas constructed by imposing half the constraints

 $lm(\phi) = 0$ $\forall m > 0$, $(l_0 - a)(\phi) = 0$ worn normal order

What is then the vole of 2-m m217

The remaining L-m (m21) lead to redundanci is in the spectrum is monding to the associated gauge rometries.

La next: discuss these reclussdancies

Summany: Physical states: $(L_0-a)(\psi > = 0)$ $L_m(\psi > = 0, m > 1)$



(only need to check m=1,2)

Grand stato

 Grand stato
 Q<O</td>
 massive ground stato

 IK,OS
 a=O
 massless
 ground stato

 a>O
 tachyonic
 ground stato
 (!)

 $N=1 \qquad 5\cdot N=1 \qquad 5\cdot N=10; KS=15; KS \qquad 5\cdot N=1 \qquad 0-derves of freedom)$

 $a'K^{2} = a - i$: so K is { spacetike if a > i } ghosts! disht like if a = i ghosts! timelike if a < i so $a \le i$

5. K=0: Nonsuerse plani bation

S²>0: polavitation is Null or space like to avoid shorts Critical theory: common the case a=1 (threshold case)

N=0: (K, O) tachyon $[d'M^2=-1(OS)]$

 $N = (: |S;K) = S \cdot d_{-1}|O;K)$ is a massless state $(K^2 = 0)$

photon: D-2 degress of preedon as the Ionzitudinal states IK;K> decouples

> [orthogonal to all physical states 13, K'> <K, K13, K'> = 0]

Lo Virarora comphraints par a =1 previsely restrict the level 1 states to compistently describe space-time photos <u>Nemark</u>: the decompling of the longitudinal degree of Weedon is due to the fact that it corresponds to a state that is "pure gauge"

ie IK; K) is created by the action of L-1 (which is a generator of a conformal transformation). In this surge we say that IK; K) is pure going state

Indeel:

 $L_{-10;K} = \sqrt{2a^{2}K} \cdot a_{-10;K} = \sqrt{2a^{2}K} \cdot K$

 $L_{-1}(0; K) = \left(\frac{1}{3}\sum_{h=-\infty}^{\infty} d_{-h-1} \cdot d_{h}\right) |0, K\rangle = \frac{1}{a} \left(d_{-1} \cdot d_{0} + d_{0} \cdot d_{-1} + \sum_{h=2}^{\infty} d_{h-1} \cdot d_{-h}\right) |0, K\rangle$

 $= a_{-1} \cdot a_{-1} \cdot a_{-1} = \sqrt{2a''} \quad K \cdot$

more on this son



g.K=0 x g spacelik

(D-1 degress of Weedom) $g^2 > 0$ the norm states

marine rector born in D-lins.

2.7 level 2 states 2 dealing with ghosts

Level 3 state for a =1 (threshold value as is as there are aposts in the pany)

(os)

- Comider the level 2 state 1 \$, K> with momentum K
- $|\phi_{1}K\rangle = [C_{1}d_{-1}d_{-1} + C_{1}d_{-2}K + C_{3}(d_{-1}K)(d_{-1}K)]|O_{1}K\rangle$
- onshell mass condition d'IX= 1 <- d'M2=N-a
- $L_{1}[\phi, K] = 0$ iff $C_{1}+C_{2}-2C_{3}=0$
- $L_1(\phi, K) = 0$ iff $D_{C_1} 4C_2 2C_3 = 0$
- 80 $l\phi_{1}K > = c_{1}[d_{1}\cdot d_{1} + \frac{1}{5}(D-1)d_{-2}\cdot K + \frac{1}{10}(D+4)(d_{-1}\cdot K)(d_{-1}\cdot K)]lo_{1}K > \frac{1}{5}(D-1)d_{-2}\cdot K + \frac{1}{10}(D+4)(d_{-1}\cdot K)(d_{-1}\cdot K)[d_{-1}\cdot K)]lo_{1}K > \frac{1}{5}(D-1)d_{-2}\cdot K + \frac{1}{10}(D+4)(d_{-1}\cdot K)(d_{-1}\cdot K)(d_{-1}\cdot K)]lo_{1}K > \frac{1}{5}(D-1)d_{-2}\cdot K + \frac{1}{10}(D+4)(d_{-1}\cdot K)(d_{-1}\cdot K)[d_{-1}\cdot K]lo_{1}K > \frac{1}{5}(D-1)d_{-2}\cdot K + \frac{1}{10}(D+4)(d_{-1}\cdot K)(d_{-1}\cdot K)[d_{-1}\cdot K]lo_{1}K > \frac{1}{5}(D-1)d_{-2}\cdot K + \frac{1}{10}(D+4)(d_{-1}\cdot K)(d_{-1}\cdot K)[d_{-1}\cdot K]lo_{1}K > \frac{1}{5}(D+1)d_{-2}\cdot K + \frac{1}{10}(D+4)(d_{-1}\cdot K)[d_{-1}\cdot K]lo_{1}K > \frac{1}{5}(D+1)d_{-2}\cdot K + \frac{1}{10}(D+4)(d_{-1}\cdot K)[d_{-1}\cdot K]lo_{1}K > \frac{1}{5}(D+1)d_{-2}\cdot K + \frac{1}{10}(D+1)(d_{-1}\cdot K)[d_{-1}\cdot K]lo_{1}K > \frac{1}{5}(D+1)d_{-2}\cdot K + \frac{1}{10}(D+1)(d_{-1}\cdot K)[d_{-1}\cdot K]lo_{1}K > \frac{1}{5}(D+1)d_{-2}\cdot K + \frac{1}{10}(D+1)(d_{-1}\cdot K)[d_{-1}\cdot K]lo_{1}K > \frac{1}{5}(D+1)(d_{-1}\cdot K)[d_{-1}\cdot K]lo_{1}K > \frac{1}{5}(D+1)($
 - is a physical state for any D

Nom $\langle \phi; K | \phi; K' \rangle = -\frac{2}{25} |c_1|^2 (D-1) (D-2L) \delta(K-K')$

no phosts => 1 ≤ D ≤ 26 (3 ghosts for D>26)

mill' (physical-telo norm) states when D-26 (or D=1)

to when a=1 & D = 26 there one more "null" states (prolification of millitotes Q D=26 into of lorg gange (ponf.) mm Ing)

1372 Brower; Goddawd, Thorn No ghost theorem () For a=1 and D=26 theorem has no ghosts

Note: there are no ghosts for a <1 & 150 = 25

(this is a corollary of the no ghost theorem for the critical string)

BUT these thus in one in confistent at the level of string boops (need to look at one toop internations)

2.8 Null states and D=26

- Deprision:
 - A state 143 is called spurious if
 - · it is orthogonal to all physical states and
 - · obergs (L. a) 14 > -0

A null state is a spurious state which is also physical.

A null state has two norm as it is orthogonal to itself.

An example of a null state is 1K; K> at level 1 k a=1, and this state 13 alko pure sauge. (this is expected in gauge theories to find such states on in the

anpla-Blentw formulation of RED)

Null states are physical states that decouple from the dynamics.

These states are "quotiented out", that is

two physical states are equivalent if they differ by a null state

so we derine fleris = flephys / flemeSphysically distinct states

We expect: physical states of the form L-m/y> Vm>0 one null!

spurius states generated by the midual symmetries (conformal transformations)

Comrider the state

 $(\Psi) = \sum_{m>0} L_{-m} |\Psi_m\rangle$ st LolVm> = (a-m) 14-m>

Thm < ((14 > = 0 the > ∈ sleeping

 $(L_o-a) | \psi \rangle = 0$

 $\sum_{m>0} (L_0 - \alpha) L_m | V_m \rangle = \sum_{m>0} ([L_0, L_m] + L_m L_0 - \alpha L_m) | V_m \rangle = \sum_{m>0} L_m (m - \alpha + L_0) | V_m \rangle$ mlm

so 14> is spurius.

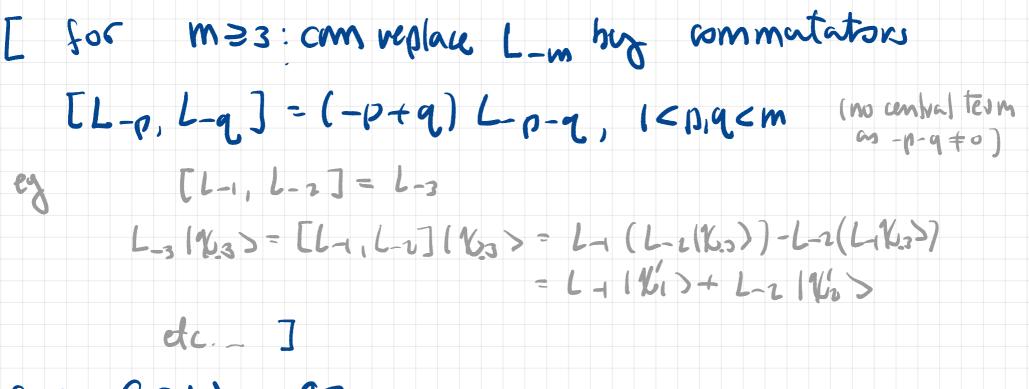
One can in fact place that any spining state is of this form, ic all spining states are "our sange". (GSW p 83)

[ouve gauge states ~ states zunerated by residual symmetries (conformal symmetries!)] this is expected in sanse there is

One can prove moreover that all spurious states one of the form

$|\Psi\rangle = L_{-1}|\chi_{-1}\rangle + L_{-2}|\chi_{-2}\rangle$

 $L_0 | \chi_{-1} \rangle = (a - 1) | \chi_{-1} \rangle$, $L_0 | \tilde{\chi}_{-2} \rangle = (a - 2) | \chi_{-2} \rangle$ with



SLE GSW p83

Null states: (ie spurius states which are physical) Convider first: $|\psi\rangle = L_{-1}|\chi\rangle$, $Lo|\chi\rangle = (a-1)|\chi\rangle$ with $Lm(\chi\rangle = 0$ \forall m>0.

 $(L_0-a)|\psi\rangle = 0$ by construction

• $L_1 | \psi \rangle = [L_1, L_1, I_1 | \psi \rangle = 2 L_0 | \psi \rangle = 2 (a-1) | \psi \rangle$

=0 iff $\alpha = 1$

- $L_2 | \Psi \rangle = [L_1, L_-] | \Psi \rangle = 3 L_1 | \Psi \rangle = 0$

So, for a=1 we get an infinite st of null states 142 = L-122, Lm 22 = 0 Am>0, Lo122 = 0

(zmraling the case Jid IK; K> = L-10; K>).

Now consider another example

 $|\Psi\rangle = (L_{-2} + \gamma L_{-1}^{2})|\psi_{2}\rangle, Lm|\psi_{1}\rangle = 0 \forall m>0, L_{0}|\psi_{1}\rangle = (a-2)|\psi_{1}\rangle$

- $L_{1} | \Psi \rangle = [L_{1}, L_{-1} + \delta L_{-1}] | \Psi_{12} \rangle = (3 L_{-1} + \delta 2 (L_{2}L_{-1} + L_{-1}L_{0})) | \Psi_{12} \rangle$ $= (3 L_{-1} + 2\delta [L_{0}, L_{-1}] + 4\delta L_{-1} (a-2)] | \Psi_{12} \rangle [L_{1}, L_{-1}] = [L_{1}, L_{-1}] L_{-1} + L_{-1} [L_{1}, L_{-1}]$ $= (3 L_{-1} + 2\delta [L_{0}, L_{-1}] + 4\delta L_{-1} (a-2)] | \Psi_{12} \rangle$
 - = $(3 + 28 + 48(a-2)) L_{1} 141>$
- $L_1 | \psi \rangle = 0 \iff \chi = \frac{3}{2(3-2\alpha)}$ (for $\alpha = 1$ we have $\chi = \frac{3}{a}$)
- $L+2(+) = L+2(L-2+8L_1)(k) = [L_2, L-2+8L_1](k)$
 - = (4 Lo + D. 6 + 8 ([Lz, L-1] L-1 + L-1 [Lz, L-1]) 1 x>
 - $= (4L_0 + \frac{1}{a}D + 3F[L_1, L_{-1}])V > = (4L_0 + \frac{1}{a}D + 3V \cdot 2L_0)V >$
 - $=\frac{1}{a}(4(2+38)(a-2)+D)1%>$
 - $L_{1}(4) = 0 \iff D = 4(2+38)(2-a)$

so the spinices state (4) is null iff

 $\gamma = \frac{3}{2(3-2\alpha)}$ $D = 4(2+30)(2-\alpha)$

critical bosonic string:

 $\alpha = 1$: 14) null if $\delta = \frac{3}{2} k$ D = 26critical dimension

- all somins states are null of high Ruch

os bry as D=26

ie states associated to residual going (contamul) mometrie generated by L-m (mo) decayle



Open strings: by studying the low level speatrum we formed

• a>1 D>2C : there are ghosts in flipmys

• a = 1 D = 2 c critical string we found infinité families of mulistates

D < 25 (suboritical case) • a < 1

no inconsistencies at tree-level (no ghosts)

but inconsistent at the level of string loops (need to look at one loop interactions)

OCQ: a=1 D=26 needs 1-popinteractions to prove (no proof at trached)

- (no goosts, mong null states)
- LCQ: a=1 D=26 follows by requiring borntz spacefime invariance . manifully ghost free

BIZST quantitation: a=1, D=26 required pr

quantum gauge (comprime) invariance.

Too bad we have no time to go own the modern (BET) quant. !

From mow on a=1 D=26

Next · cloud string spectrum

2.3 Physical states of the closed string (at least low level N=0,1)

Recall

states are of the firm $\lim_{i=1}^{K} \alpha_{-n_i}^{n_i} \prod_{j=1}^{K} \overline{\alpha}_{-m_j}^{v_j} [0, \overline{0}; K>, n_i, m_j>])$

Physical state conditions

- $\begin{cases} (L_0 \tilde{L}_0) [\phi] = 0 \iff N = \tilde{N} \\ (L_0 + \tilde{L}_0 2a) [\phi] = 0 \iff -d^1 K^2 = -4a + 2(N + \tilde{N}) \iff -d^2 K^2 = 4(N 1) \\ L_m [\phi] = 0 \quad k \quad \tilde{L}_m [\phi] \forall m \ge 1 \quad (\text{influent to place this for } m = 1, 2) \end{cases}$
- ground state $(N=\bar{N}=0)$: $10,\bar{0}$; K> with $-d^{2}K^{2}=-4$ (tackyon)

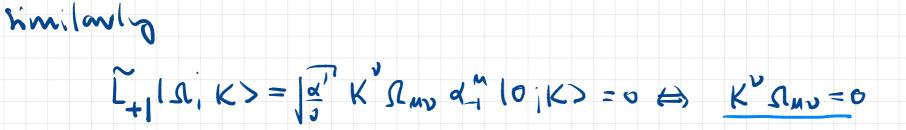
level ! states : N=N=1

General state $I \Omega_{,K} \times = \Omega_{,M} \vee d_{,i} \tilde{d}_{,i} I O, \tilde{O}; K \times Spacetime two-tensor$

mass shell condition: $-\alpha' K^2 = 4(1-\alpha) = 0 \implies \text{massless state for the oritical string}$

Impose Viravolo constraints : only need to impose $L_{+1}(\Lambda, K) = 0$ $L_{+1}(\Lambda, K) = 0$ $L_{+1}(\Lambda, K) = 0$

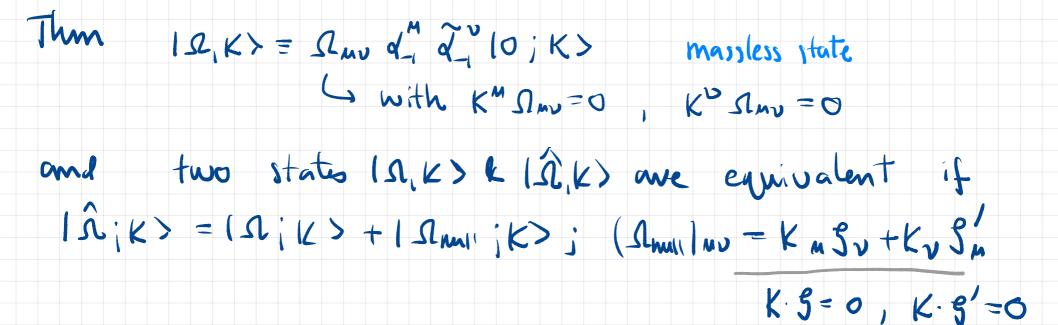
 $L_{+1}[\Omega, K\rangle = \Omega_{\mu\nu} L_{1} \mathcal{A}_{-1}^{\mu} \mathcal{A}_{-1}^{\nu} |0; K\rangle = \Omega_{\mu\nu} (\mathcal{A}_{0}^{\mu} \mathcal{A}_{-1}^{\nu}) |0; K\rangle$ $= \sqrt{\mathcal{A}_{0}^{\mu}} \mathcal{K}_{0}^{\mu} \Omega_{\mu\nu} \mathcal{A}_{-1}^{\nu} |0; K\rangle = 0 \quad (\Rightarrow \quad K^{\mu} \Omega_{\mu\nu} = 0)$



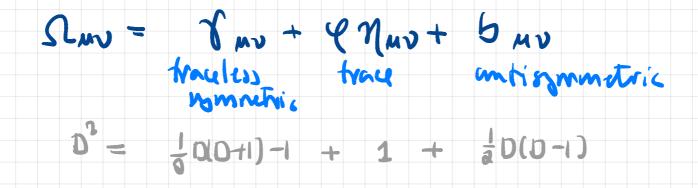
So for $I \Omega_{1} K = \Omega_{nv} d_{1}^{n} \tilde{d}_{-1}^{v} I O ; K$ massless stale with $K^{m} \Omega_{nv} = 0$, $K^{v} \Omega_{nv} = 0$

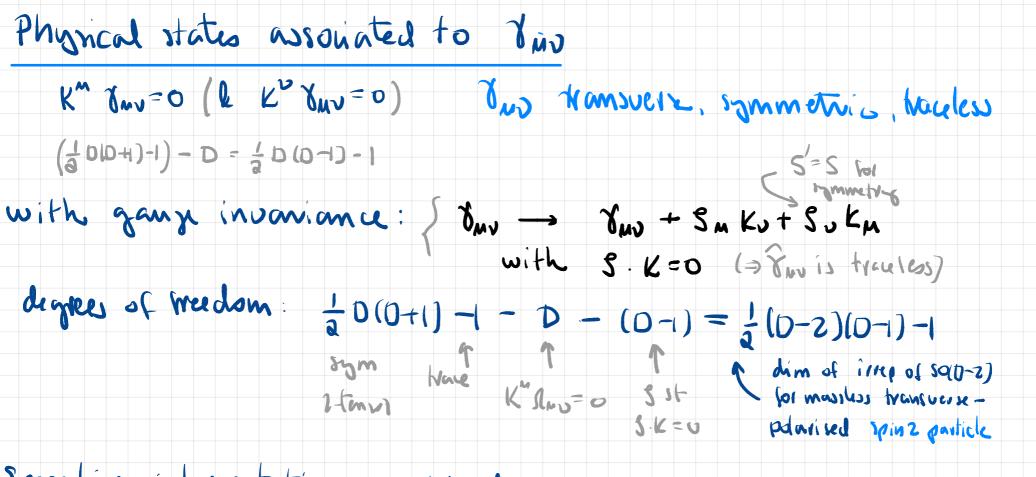
Null states: which degrees of predom of Miks are nul?

- $L_{-1}(X) > L_{0}(X) = 0 \qquad (1X) = g \cdot \tilde{a}_{-1}(0, K) \quad (N_{K}=0, \tilde{N}_{K}=1)$ $\tilde{L}_{-1}(\tilde{X}) > \tilde{L}_{0}(\tilde{X}) = 0 \qquad (\tilde{N}_{V}) = g' \cdot d_{-1}(0, K) \quad (N_{W}=1, \tilde{N}_{W}=0)$
- L-1XS, Z-1XS are spurius by construction
- Similarly $\tilde{L}_{-1}(K) = \int_{a}^{d} (S' \cdot a_{-1}) (K \cdot \tilde{a}_{-1}) (c_{-1}) K > 30 \quad \Delta w = \int_{a}^{d} S'_{M} K y$
- These one also physical when k Sur = 0 & K Sur = 0
 - ie $K \cdot S = 0$ $k \cdot K \cdot S' = 0$



To understand this better decompose the state into space-time corentz irreps





Spacetime intropretation: right degrees of weedom expected of a graviton in the trauless howmonic gauge

 $\begin{aligned} &\mathcal{J}_{\text{MN}}(X) = \mathcal{M}_{\text{MN}} + \mathcal{V}_{\text{MN}}(X) & \text{in finite innal diffeomorphisms parametrized by S} \\ &\mathcal{V}_{\text{MN}}(X) \sim \mathcal{V}_{\text{MN}} + \mathcal{D}_{\text{M}} S_{\text{M}}(X) + \partial_{\text{N}} S_{\text{M}}(X) & \text{which in momentum space become} \\ &\mathcal{V}_{\text{MN}}(X) \sim \mathcal{V}_{\text{MN}} + \mathcal{D}_{\text{M}} S_{\text{M}}(X) + \partial_{\text{N}} S_{\text{M}}(X) & \text{with in the second of t$

Physical states associated to bis Ramond-Kalb field

- K" bur=0 & K" bur=0 bus transverse, antisymmetric
- 5 = -5 for anti remnetives $5_{MV} + S_M K_V S_V K_M$, $S \cdot K = 0$ with ganza invorvionce: bas-
- Note that 3 has redundancy Su ~ Su + Ku 2
- dennes of freedom $\frac{1}{2}D(D-1) (D-2) = \frac{1}{2}(D-2)(D-3)$
- In spartime this is interpreted as a 2-form genry field
 - $b = \frac{1}{2} b m (x) d x^{n} d x^{n} \sim b + d s$, $s \sim s + d \lambda$ on form

La napt lecture ~ scalar state

