String Theory 1

Lecture # 16

6 D-branes

Last lecture: we defined a Dp-brane as a $(p+1)$-dimensional subspace of target space where the ends of open strings can end

(We refer to this subspace as the D brane world volume)

We saw how D-branes appear from T-duality

strings with Neumann boundary conditions $\longleftrightarrow$ Dirichlet boundary conditions

Today: a number of observations about Dbranes (mostly without proofs $\rightsquigarrow$ just an idea of what these important objects are in the context of string theory; see eg Zwiebach)

In this lecture course we studied:

- quantised strings (open & closed) in $\mathbb{R}^{1,25}$

  a salient feature is that the massless sector includes

  a graviton (from the closed sector)

  gauge fields (from the open sector)

  Note that we discussed OS with Neumann bcs only

- We also discussed quantised strings in $\mathbb{R}^{1,24} \times S^1_R$

  $\hookrightarrow$ new features eg

  CS • states have quantised momentum along S^1_R and a winding quantum number

  • T duality

  OS more complicated; T-duality leads to the notion of D-branes

Last lecture:

OS - $\xleftrightarrow{\text{T-duality}}$

open string with **Neumann** boundary conditions compactified on S^1_R

dual open string with **Dirichlet** boundary conditions compactified on $S^1_{\hat{R}}$, $\hat{R} = \alpha'/R$

D25 space-filling brane
↳ open string ends are free to move on spacetime

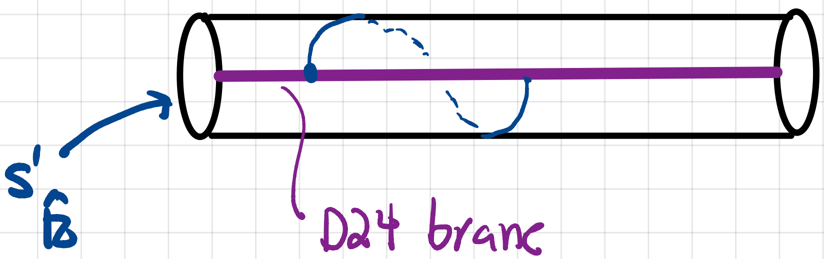
endpoints of the string live on a D24 brane

$p^{25} = \dfrac{m}{R}$ quantized

no translational symmetry along $S^1_{\hat{R}}$

no winding

string can wind around $S^1_{\hat{R}}$

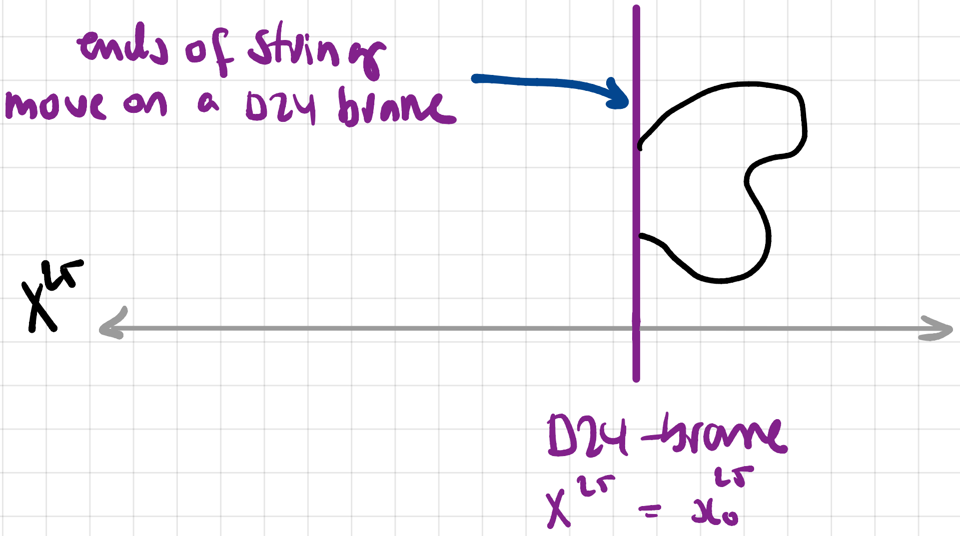


massless sector: (both sides)
25 dimensional U(1) gauge field

$\boxed{6.1}$ Open strings with Dirichlet b.cs in flat $\mathbb{R}^{1,25}$

(no compactification)

Consider an open string on $\mathbb{R}^{1,25}$ with Dirichlet boundary conditions in $\boxed{\text{one}}$ direction (X^{25}) and Neumann boundary conditions in all other directions (X^i $i=0,\dots,24$).



no translational symmetry along X^{25}

space-time symmetry

$$SO(1,25) \longrightarrow SO(1,24)$$

[More generally, one can consider an open string with Dirichlet boundary conditions in $26-(p+1)$ directions and Neumann boundary conditions in $(p+1)$ directions. In this case string ends move on a Dp brane and $SO(1,25) \to SO(1,p) \times SO(25-p)$]

Mode expansion for $X^\mu(\tau,\sigma)$:

Neumann boundary conditions

$$X^i(\tau,\sigma) = x^i + 2\alpha'\tau p^i + i\sqrt{2\alpha'}\sum_{n\neq 0}\frac{1}{n}\alpha_n^i \cos(n\sigma)\, e^{-in\tau} \qquad i=0,\ldots,24$$

Dirichlet boundary conditions

$$X^{25}(\tau,\sigma) = x_0^{25} + \frac{1}{\sqrt{2\alpha'}\,\pi}\left(x_1^{25}-x_0^{25}\right)\sigma + \sqrt{2\alpha'}\sum_{n\neq 0}\frac{1}{n}\alpha_n^{25}\, e^{-in\tau} \sin(n\sigma)\, e^{-in\tau}$$

$$X^{25}(\tau,0) = X^{25}(\tau,\pi) = x_0^{25}$$

$\boxed{\text{no } \alpha_0^{25} \text{ mode}}$.

$\Rightarrow$ no momentum along X^{25}

(a term $p^a\tau \Rightarrow$ endpoints would not stay at x_0^{25} when $\tau\neq 0$)

quantizing the string: mostly as before except

X_0^{25} remains a number

(X_0^{25} is not a parameter, it represents the location of a fixed Dbrane)

Virasoro operators as before.

all oscillators

Mass-shell condition: $L_0 - 1 = (\alpha' p^2 + N) - 1$

inner product on $\mathbb{R}^{1,24}$

becomes $\alpha' M_{25}^2 = -\alpha' |p|^2 = N-1$, $|p|^2 = p \cdot p$

Ground level $(N=0)$: tachyon on the D-brane $\alpha' M_{25}^2 = -1$

Massless spectrum: level $N=1$

$$|\xi,\eta;k\rangle = (\xi\cdot\alpha_{-1} + \eta\,\alpha_{-1}^{25})\,\underbrace{|0;k\rangle}_{\text{ground state }(N=0)}$$

k: 25 dim momentum

ξ: (1+24)-dim polarization vector

η: spacetime scalar

Imposing $\quad L_1|\xi,\eta;k\rangle = 0 \qquad$ ($\Rightarrow L_m|\phi\rangle=0,\ m\geq 2$)

we find that $|\xi,\eta;k\rangle$ is physical if $\xi\cdot k=0$
with η unconstrained.

$\underline{L_1|\xi,\eta;k\rangle} = \left(\xi_i\left([L_1,\alpha_{-1}^i]+\alpha_{-1}^i L_1\right)+\eta\left([L_1,\alpha_{-1}^{25}]+\alpha_{-1}^{25}L_1\right)\right)|0;k\rangle$

$= \left(\xi\cdot\alpha_0 + \cancel{\eta\,\alpha_0^{25}} + (\xi\cdot\alpha_{-1}+\eta\,\alpha_{-1}^{25})L_1\right)|0;k\rangle$

$= (\xi\cdot k + \cancel{(\xi\cdot\alpha_{-1}+\eta\,\alpha_{-1}^{25})(\alpha_1\cdot\alpha_0)})|0;k\rangle = \underline{(\xi\cdot k)|0;k\rangle}$

null states at level one of the form $L_{-1}|0;K\rangle$:

$$L_{-1}|0;K\rangle = K\cdot\alpha_{-1}|0;K\rangle \quad \text{with} \quad K\cdot K = 0$$

Then we have the massless physical states

- 25-dimensional photon $\quad \xi\cdot\alpha_{-1}|0;K\rangle \quad$ U(1) physical states
  $\xi\cdot K = 0$

so the Dbrane has a U(1) field on its world volume
(true for any Dp brane)

- scalar field $\quad \varphi = \eta\,\alpha^{25}_{-1}|0;K\rangle$

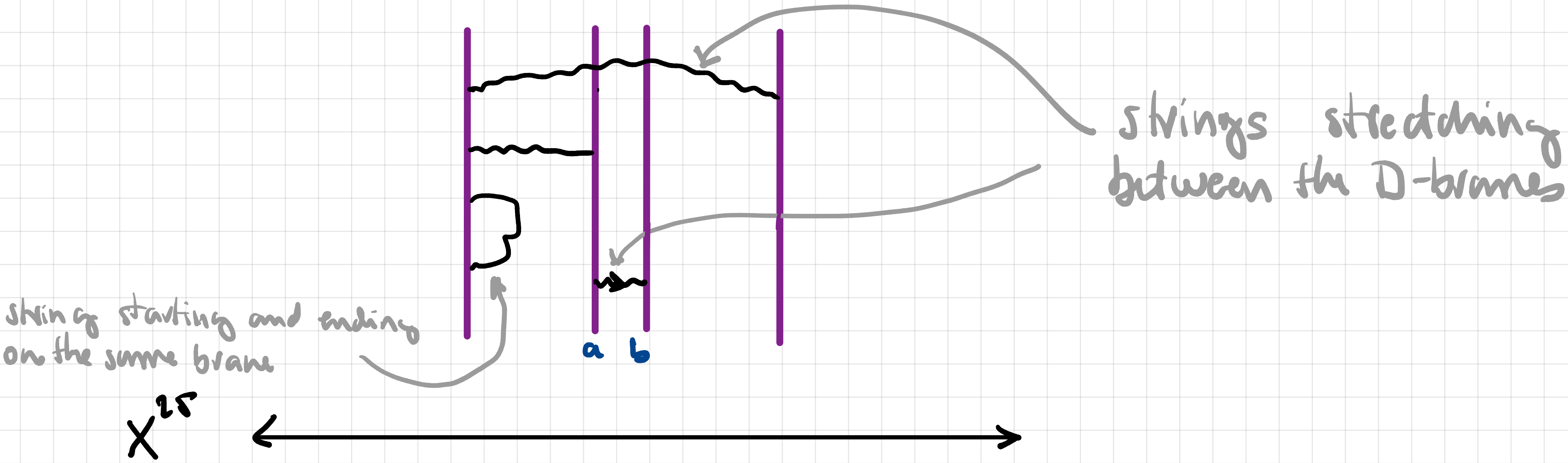
more generally a Dp brane has a massless scalar for each normal direction.

φ can be identified with fluctuations in the position of the D-brane along the transverse X^{25} direction (no proof here!)

see B Zwiebach

6.2 Stretched strings

One can also have systems of D-branes with different classes of open strings (open string sectors)



Consider a string stretched between two parallel D24 branes located at $x_0^{25} = x_a^{25}$ and $x_0 = x_b^{25}$

String endpoints $\quad X^{25}_{ab}(\tau, \sigma=0) = x_a \,, \quad X^{25}_{ab}(\tau, \sigma=\pi) = x_b$

$$X^{25}_{ab} = x_a^{25} + \frac{1}{\pi} \underbrace{(X_b^{25} - X_a^{25})}_{\Delta X_{ab}} \sigma + \sqrt{2\alpha'} \sum_{n\neq 0} \frac{1}{n} \alpha_n^{25} e^{-in\tau} \sin(n\sigma)$$

$$\alpha_0^{25} = \frac{1}{\sqrt{2\alpha'}\,\pi} (X_b^{25} - X_a^{25})$$

mass-shell condition: $\quad M^2_{ab} = -p \cdot p = \underbrace{\left(\frac{X_b^{25} - X_a^{25}}{2\pi\alpha'} \right)^2}_{} + \frac{1}{\alpha'}(N-1)$

shift of mass-levels : $\left(\frac{\Delta X}{2\pi\alpha'}\right)^2 = [T \, \Delta X]^2 =$ mass^2 of a string stretched between the branes

Spectrum of the stretched string:

· N=0 $\qquad |k, ab\rangle$

information about which Dbrane string ends live
Chan-Paton levels

labels $[a\, b]$

a denotes brane on which end $\sigma=0$ lives

b denotes brane on which end $\sigma=\pi$ lives

in our case a,b take values 1 or 2; more generally, for N Dbranes they take values 1,…,N

(For a string with ends on the same brane a=b)

mass shell condition: $$M_{ab}^2 = -\frac{1}{\alpha'} + \left(\frac{\Delta X_{ab}}{2\pi\alpha'}\right)^2$$

tachyon if $|\Delta X_{ab}| < 2\pi\sqrt{\alpha'}$

· $\underline{N=1}$

$$M_{ab}^2 = \left(\frac{\Delta X_{ab}}{2\pi\alpha'}\right)^2$$

$|S, K, ab\rangle = S \cdot \alpha_{-1} |K, ab\rangle \quad (K \cdot S = 0)$

$\eta\, \alpha_{-1}^{25} |K, ab\rangle$

massive vector on 25 dim space time

null states

$$L_{-1} | K; ab\rangle = K \cdot \alpha_{-1} | K; ab\rangle + \frac{\Delta X_{ab}}{\sqrt{2\alpha'}\,\pi} \alpha_{-1}^{25} | K; ab\rangle$$

no osc

with $K \cdot K = 0$

Longitudinal mode & scalar are null!

null states

$$L_{-1} | K; ab \rangle = \frac{1}{2} \sum_n \alpha_{-1-n} \cdot \alpha_n | \cdots \rangle$$

$$= \frac{1}{2} \Big(\alpha_{-1} \cdot \alpha_0 + \sum_{n \geq 0} \big(\cancel{\alpha_{-1-n} \cdot \alpha_n} + \alpha_{-1+n} \cdot \alpha_{-n} \big) \Big) | \cdots \rangle$$

$$= \frac{1}{2} \big(\alpha_{-1} \cdot \alpha_0 + \alpha_{-1} \cdot \alpha_0 \big] | \cdots \rangle = \big(\alpha_{-1} \cdot \alpha_0 \big) | \cdots \rangle$$

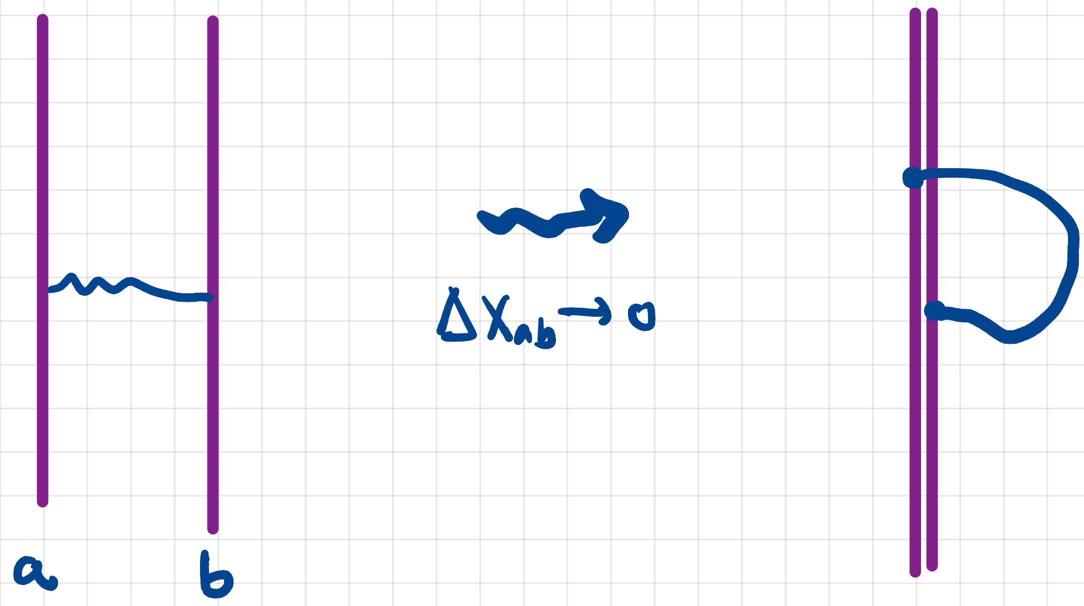
$$= \big(\alpha_{-1} \cdot K + \alpha_{-1}^{25} \alpha_0^{25} \big) | \cdots \rangle$$

$$= K \cdot \alpha_{-1} | \cdots \rangle + \frac{\Delta X}{\sqrt{2\alpha'} \pi} \alpha_{-1}^{25} | \cdots \rangle \qquad \Big(\alpha_0^{25} = \frac{1}{\sqrt{2\alpha'} \pi} \Delta X \Big)$$

with $K \cdot K = 0$

Coincident limit:

suppose we have N D-branes at $X^{25}_a, X^{25}_b, \dots$



stretched string state between Dbrane at x_a & D-brane at x_b

N massive vector fields

$|\zeta; K, ab\rangle \quad a,b = 1,\dots,N$

N^2 sectors

$\rightsquigarrow$ $\Delta X_{ab} \to 0$ massless gauge fields

$\zeta \cdot \alpha_{-1} |k; ab\rangle$ + scalars

$a,b = 1, \dots, N$

Chan-Paton labels

all (ab) strings have the same spectrum

One can show that the spectrum has a manifest $U(N)$ symmetry and that these (N^2) states transform in the adjoint representation of $U(N)$

One can choose a basis for these states

$$|\xi, k; A\rangle = \sum_{a,b} (t^A)^a{}_b \, |\xi, k; ab\rangle$$

$A = 1, \ldots, N^2$

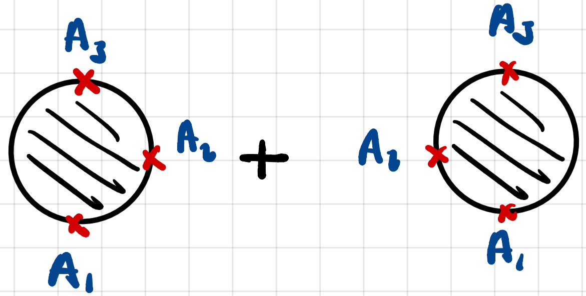
Chan-Paton factors

hermitian basis of $u(N)$

$$\mathrm{tr}(t^A t^B) = \delta^{AB}$$

(Null states: $L_{-1}|0, \underline{k}; ab\rangle = |\underline{k}; \underline{k}; ab\rangle + \frac{\Delta X_{ab}}{\sqrt{2\alpha'}\pi}|\eta; \underline{k}; ab\rangle$)

3-point coupling of massless vectors



$$\mathcal{A}(\zeta_1, k_1, A_1; \zeta_2, k_2, A_2; \zeta_3, k_3, A_3)$$

$$\sim g_o\, \delta(k_1+k_2+k_3) \{ \zeta_1 \cdot k_{23}\, \zeta_2 \cdot \zeta_3 + \zeta_2 \cdot k_{31}\, \zeta_1 \cdot \zeta_3 + \zeta_3 \cdot k_{12}\, \zeta_1 \cdot \zeta_2$$

$$+ \frac{\alpha'}{2} \zeta_1 \cdot k_{23}\, \zeta_2 \cdot k_{31}\, \zeta_3 \cdot k_{12} \} \times \mathrm{tr}\left(t^{a_1}[t^{a_2}, t^{a_3}]\right)$$

This is the 3 point vertex operator associated to the action

$$\mathcal{L} = \underbrace{-\frac{1}{4} \mathrm{Tr}(F_{\mu\nu}F^{\mu\nu})}_{\text{Yang-Mills}} - \frac{2i\alpha'}{3} \mathrm{Tr}(F_\mu{}^\nu F_\nu{}^\rho F_\rho{}^\mu) + \text{scalars}$$

EFT action on D24 brane

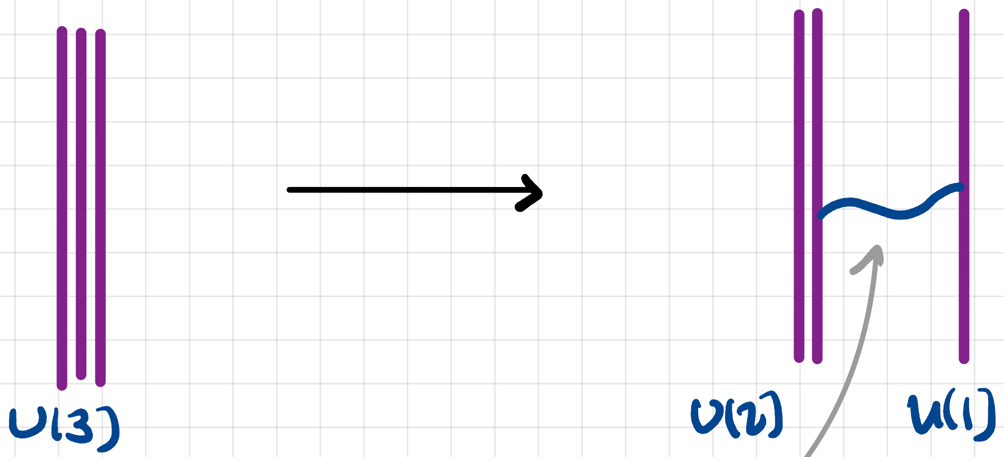
α'-correction

for a $U(N)$ non-abelian gauge theory

One can derive this using β-functions

(needs extra tools: boundary couplings, boundary renormalization flow!)

D-brane picture of non-abelian gauge theory leads to a rather nice picture of the Higgs mechanism.



stack of 3 Dbranes

$$M_{ab}^2 = \left(\frac{|\Delta X|}{2\pi\alpha'}\right)^2$$

mass of string stretched between branes

$$\Downarrow$$

massive W-bosons : $|\xi; \underline{k}; ab\rangle$

null states $|\underline{k}; \underline{k}; ab\rangle + \dfrac{\Delta X_{ab}}{\sqrt{2\alpha'}\,\pi}|m; \underline{k}; ab\rangle$

longitudinal W's

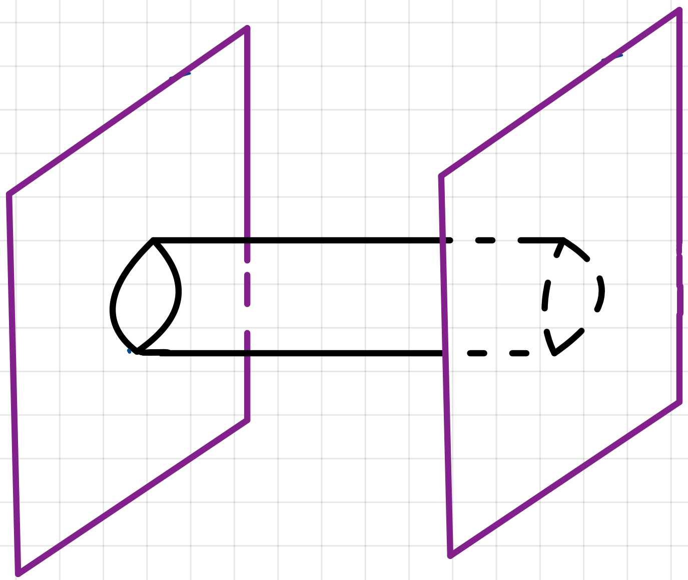
W's "eat" the Higgs

Epilogue: D-branes as dynamical objects?

If they are, maybe they need to be included in the perturbative description of strings? how?

Estimate mass scale relevant to D-branes by computing its tension

closed string exchange between two Dbranes



gravitational coupling $K \sim g_0^2$

$$A \sim K^2 \tau_p^2 \sim (g_0)^0 = 1$$

D-brane tension

$$\tau_p \sim \frac{1}{g_0^2} \Rightarrow$$

D-branes are massive "non perturbative" objects

Polchinski 1995, "duality revolution"

6.3 Final remarks:

We have seen that the theory of quantised string has a very rich structure

- quantised gravity (at low energies we obtain Einstein's gravity)
- gauge fields
- consistency of the theory $\rightsquigarrow$ fixes dimension of space time

- compactifications / strings in (nontrivial) background fields

↳ this course: $\mathbb{R}^{1,25}$, $\mathbb{R}^{1,24} \times S^1_{\mathbb{R}}$

More generally: • $\mathbb{R}^{1,d-1} \times M^{D-d}$ ← M Ricci flat to leading order in α', (from $\beta = 0$)

• $X^{1,d-1} \times M^{D-d}$ ← geometry dictated by $\beta = 0$

eg max symmetric

eg $AdS^3 \times S^7$, $AdS^5 \times S^5$, etc ...

• or even more general set ups

- T duality

- emergence of (non-perturbative) P branes

More to learn

- dualities (Mirror symmetry, AdS/CFT)
- emergence of non-perturbative Branes
- CFT & AdS/CFT
- Strong coupling regime
- Black hole physics
- realistic phenomenology
- mathematical structures
  ↳ geometry (differential, algebraic, ...)
  topology, number theory, algebra ...

Improvements: ST2

- remove tachyons $\rightsquigarrow$ superstrings
  (fermions in 2dim NLσM: supersymmetric WS theory)
- spacetime fermions
- superstring theory $\rightsquigarrow$ spacetime dim $= 10$

  ⋮
- dualities

End of String Theory I

Thanks!