STRING THEORY II

Lecture III

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RNS STRING

$\alpha_m^\mu, b_r^\mu, p^\mu, x^\mu.$ $\qquad \mathcal{F} = \left\{ \prod \alpha_{-m_j}^{\mu_j} b_{-r_1}^{\nu_1} \cdots b_{-r_I}^{\nu_I} |0\rangle \right\}$

$|0\rangle =$ vacuum

$b_r^\mu |0\rangle = \alpha_m^\nu |0\rangle = 0 \quad m, r > 0.$

PHYSICAL STATES ARE A SUBSPACE OF $\mathcal{F}$, WHICH SATISFIES

$$(L_0 - a)|\phi\rangle = 0$$

$$\left.\begin{array}{ll} L_m |\phi\rangle = 0 & m > 0 \\ G_r |\phi\rangle = 0 & r > 0 \end{array}\right\} \Rightarrow \begin{array}{l} \langle\phi'|L_m|\phi\rangle \\ = \langle\phi'|G_r|\phi\rangle \\ = 0. \end{array}$$

NORMAL ORDERING CONSTANT:

$$\left.\begin{array}{l} a^R = -\frac{1}{24} \\ a^{NS} = \frac{1}{48} \end{array}\right\} + \begin{array}{c} a^{\text{Boson}} \\ \shortparallel \\ \frac{1}{24} \end{array} = \left\{\begin{array}{ll} 0 & = a_R^{\text{total}} \\ \frac{1}{16} & = a_{NS}^{\text{total}}. \end{array}\right.$$

TWO OPTIONS TO PROCEED:

- EITHER SOLVE THE PHYSICAL STATE CONDITIONS LEVEL BY LEVEL "OLD COVARIANT APPROACH".

- LIGHT-CONE GAUGE: (LC)
  SOLVE PSC IN LC AT THE COST OF NO MANIFEST LORENTZ INVARIANCE. $\rightsquigarrow$ NEED TO CHECK LORENTZ INVARIANCE.

$$X^\mu \to \begin{cases} X^\pm = \frac{1}{\sqrt{2}}(X^0 \pm X^1) \\ X^i \qquad i = 2 \ldots d-1 \end{cases}$$

$$X^+ = -X_- \qquad X^- = -X_+ \qquad X^i = X_i$$

LC GAUGE FOR THE BOSONIC STRING:
USE RESIDUAL REPARAMETRIZATION INV. $\sigma_\pm \to \sigma_\pm + f_\pm(\sigma_\pm)$
TO FIX $\quad X^+ = \alpha\,\tau \quad$ "along proper time"

$$p^+ = \frac{1}{2\pi\alpha'}\int_0^\ell d\sigma \underbrace{\partial_\tau X^+}_{\alpha} \Rightarrow \boxed{X^+ = \frac{2\pi\alpha'}{\ell}\,p^+\tau}$$

LIKEWISE: FOR ψ^μ WE HAVE RESIDUAL SUSY

$$\partial_+ \epsilon^- = 0 \qquad \partial_- \epsilon^+ = 0.$$

$\Longrightarrow$ SET $\psi^+ = 0$

$\mu = +$

$$\psi^\pm = \frac{1}{\sqrt{2}}\left(\psi^0 \pm \psi^1\right)$$

QUICK CHECK: CONSISTENCY W/ SUSY:

$$\delta X^+ = \bar{\epsilon}\psi^+ = 0 \ \checkmark.$$

RNS LC Gauge:

$$\boxed{\begin{aligned} \psi^+ &= 0 \\ X^+ &= \frac{2\pi\alpha'}{\ell} p^+ \tau \end{aligned}}$$

IDEA OF LC QUANTIZATION: SOLVE THE CONSTRAINTS

$$\boxed{T_{++}=0 \qquad T_{--}=0} \qquad \boxed{J_+=0 \qquad J_-=0}$$

IN TERMS OF $X^\pm$ & $\psi^\pm$ AND THEN QUANTIZE THE REMAINING TRANSVERSE DIRECTIONS X^i, ψ^i $\quad i=2\dots d-1$.

$\Rightarrow$ PHYSICAL STATES ARE CREATED BY α^i_{-m}, b^i_{-r}

$$T_{++} = -\frac{1}{\alpha'}\underbrace{\partial_+X\cdot\partial_+X}_{} - \frac{i}{2}\underbrace{\psi_+\cdot\partial_+\psi_+}_{} \doteq 0$$

$$\psi_+^+=0 \Rightarrow \; = \psi_+^i\partial_+\psi_+^i$$

$$= \partial_+X^i\partial_+X^i - 2\partial_+X^+\partial_+X^-$$

$$= \partial_+X^i\partial_+X^i - 2\frac{2\pi\alpha'}{\ell}p^+\partial_+X^-$$

$$X^+ = \frac{2\pi\alpha'}{\ell}p^+\tau$$

$\Rightarrow$ SOLVE $T_{\pm\pm}=0$ IN TERMS OF $\partial_\pm X^-$

(1) $$\boxed{\partial_\pm X^- = \frac{\ell}{p^+2\pi}\,\frac{1}{2}\left(\frac{2}{\alpha'}\partial_\pm X^i\,\partial_\pm X^i + i\,\psi_\pm^i\partial_\pm\psi_\pm^i\right)}$$

SUPERCURRENT: $J_\pm = 0 \Rightarrow \partial_\pm X\cdot\psi_\pm = 0$

$$\Rightarrow \boxed{\psi_\pm^- = \frac{2}{\alpha' p^+}\,\frac{\ell}{2\pi}\,\psi^i_\pm\,\partial_\pm\psi^i} \quad (2)$$

$\Rightarrow$ ① & ② SOLVE THE PSC IN TERMS OF THE LC $X^\pm, \psi^\pm_\pm$ AND THE TRANSVERSE FIELDS $X^i, \psi^i_\pm$ REMAIN UNCONSTRAINED.

$\Rightarrow$ QUANTIZE THE TRANSVERSE PROBLEM!

(N,N) OPEN RNS STRING:

MASS SHELL CONDITION: $\alpha' M^2 = N^X + N^\psi - a^{total}_{R/NS}$

includes X & ψ

$$N^X = \sum_{n=1}^{\infty} \alpha_{-n}\cdot\alpha_n \qquad N^\psi = \sum_{\substack{r\in\mathbb{Z}+\phi \\ r>0}} r\, b_{-r}\cdot b_r.$$

NS-SECTOR: EACH PAIR OF (X^i, ψ^i) CONTRIBUTES

$$\Delta a_{NS} = \frac{1}{16}. \qquad i = 2 \ldots d-1$$

$$\Rightarrow \qquad a_{NS} = \frac{1}{16}(d-2).$$

LEVEL 0: $|K\rangle_{NS}$ $\quad K^\mu$ = MOMENTUM.

$N^X = N^\psi = 0.$

$$\Rightarrow \boxed{\alpha' M^2 = -a_{NS}}$$

$a_{NS} > 0 \Rightarrow$ TACHYON

FIRST EXCITED LEVEL:

$$\zeta_i \, b^i_{-\frac{1}{2}} |k\rangle_{NS}$$

ζ_i = $d-2$ component vector.
↳ $SO(d-2)$ VECTOR REP.
$\triangleq$ LITTLE GROUP OF A MASSLESS VECTOR.

$$\Rightarrow \alpha' M^2 = \frac{1}{2} - a_{NS} \overset{!}{=} 0$$

$$\Rightarrow \quad a_{NS} = \frac{1}{2}$$

$$a_{NS} = \frac{1}{16}(d-2) \Rightarrow \boxed{d=10}$$

SPACETIME DIM.

i = TAKES 8 values.

SECOND EXCITED LEVEL:

$$\Big(\underbrace{\eta_i \, \alpha^i_{-1}}_{8} + \underbrace{t_{ij} \, b^i_{-\frac{1}{2}} \, b^j_{-\frac{1}{2}}}_{28}\Big) |k\rangle_{NS} \qquad t_{ij} = -t_{ji}$$

$\Rightarrow$ 36 COMPONENTS.
$= \binom{9}{2}$
$\Rightarrow$ MASSIVE TENSOR UNDER $SO(9)$.

SUMMARY: <u>NS- SECTOR</u> (OPEN STRING)

LEVEL 0 · TACHYONIC GROUND STATE ← WE NEED TO TALK ABOUT THIS AGAIN ⟶ GSO PROJECTION

1 · MASSLESS VECTOR

2 · MASSIVE TENSOR

<u>R- SECTOR.</u> $\alpha' M^2 = N^x + \underline{\underline{N}}^\psi + \underset{0}{\overset{\psi}{a_R}}$. $(a_{Bos} + a_R^\psi = 0)$.

RECALL: $r=0$: $\boxed{\{b_0^\mu, b_0^\nu\} = \eta^{\mu\nu}}$ Clifford $(SO(1,9))$ $(d=10)$.

LC GAUGE: b_0^i $i=1\ldots8$ Clifford $(SO(8))$.

⇒ LEVEL 0 STATES WILL HAVE TO BE REP. OF THE Clifford $(SO(8))$ ⟶ SPINORS OF $SO(8)$.

⇒ $SO(8)$ HAS TWO DISTINCT SPINORS: 8_s, 8_c

[SPINORS, REP: Polchinski II APPENDIX B].

SPINORS IN 10d & 8d

$b_0^M \sqrt{2} =: \Gamma^M$ $\qquad \boxed{\{\Gamma^M, \Gamma^\nu\} = 2\eta^{\mu\nu}}$ $\qquad \mu\nu$: $SO(1,9)$ vector indices.

$\Sigma^{\mu\nu} = -\frac{i}{4}[\Gamma^M, \Gamma^\nu]$ FORM A REPRESENTATION OF LORENTZ Algebra.

REPRENTATIONS ARE CONSTRUCTED AS A LOWEST WEIGHT REP:

$$\Gamma^{0\pm} = \frac{1}{2}(\pm\Gamma^0 + \Gamma^1)$$

$$\Gamma^{a\pm} = \frac{1}{2}(\Gamma^{2a} \pm i\Gamma^{2a+1}) \qquad a = 1..4 \quad (\text{for } SO(1,9)).$$

$$\Rightarrow \{\Gamma^{a+}, \Gamma^{b-}\} = \delta^{ab} \qquad \{\Gamma^{a+}, \Gamma^{b+}\} = \{\Gamma^{a-}, \Gamma^{b-}\} = 0$$

$$\Longrightarrow (\Gamma^{a\pm})^2 = 0.$$

$\rightarrow$ CREATION/ANNIHILATION OPERATORS $\Gamma^{a\pm}$.

$\Longrightarrow$ $|\Omega\rangle$ LOWEST WEIGHT STATE: $\quad \Gamma^{a-}|\Omega\rangle = 0 \ \forall a.$

$$|\underline{s} = (s_0\, s_1\, s_2\, s_3\, s_4)\rangle = (\Gamma^{4+})^{s_4+\frac{1}{2}} \cdots (\Gamma^{0+})^{s_0+\frac{1}{2}}|\Omega\rangle.$$

$$s_i = \pm\frac{1}{2}.$$

$$|\Omega\rangle = |(-\tfrac{1}{2} \ldots -\tfrac{1}{2})\rangle.$$

DEFINE $S'_a = \Gamma^{a+}\Gamma^{a-} - \frac{1}{2}$ THEN

$$S'_a \, |\underline{s}\rangle = s_a \, |\underline{s}\rangle.$$

$\Rightarrow$ THE SPACE CONSTRUCTED IN THIS WAY FROM Ω IS $2^5 = 32$ DIM. "<u>DIRAC</u> SPINOR" IN 10D.

THIS IS NOT IRREDUCIBLE $\Rightarrow$ FIND AN INTERTWINER (G s.t. $\{\Gamma^{a\pm}, G\} = 0$).

$\Gamma = \Gamma^0 \dots \Gamma^9$ (SIMILAR TO γ^5 in 4d).

$$= \underline{2^5 \, S'_0 \dots S'_4}$$

$\Rightarrow \{\Gamma, \Gamma^M\} = 0 \Rightarrow$ DECOMPOSE $\{|\underline{s}\rangle\}$ INTO EIGENSPACES OF Γ:

$$\Gamma \, |\underline{s}\rangle = \begin{cases} +1 & \# s_a = \frac{1}{2} \text{ even} \\ -1 & \# s_a = \frac{1}{2} \text{ odd}. \end{cases}$$

$\Rightarrow \underline{32} \longrightarrow \underline{16} \oplus \underline{16}'$. <u>WEYL</u> SPINORS.

NOTE: IN 2 MOD 8 DIMS WE CAN IMPOSE A REALITY CONDITION (SEE Lecture 1 in 2d!).

$\Rightarrow$ MAJORANA CONDITION:

$$B = \Gamma^1 \Gamma^3 \Gamma^5 \Gamma^7 \Gamma^9$$

PURELY IMAGINARY Γ's.

$$B \Gamma B^{-1} = \Gamma$$

$$B \Sigma^{\mu\nu} B^{-1} = -\Sigma^{\mu\nu *}$$

$$B^* B = \mathbb{1}.$$

$$\boxed{B \psi = \psi^*}$$

$$\Rightarrow \quad \underline{32}_{\substack{\text{DIRAC} \\ \mathbb{C}}} \xrightarrow{\text{WEYL}} \underline{16}_{\mathbb{C}} \oplus \underline{16}'_{\mathbb{C}} \xrightarrow{\text{MAJO}} \underline{16}_{\mathbb{R}} \oplus \underline{16}'_{\mathbb{R}}$$

FOR THE LC QUANTIZATION: $SO(1,9) \longrightarrow SO(8)$:

$$\underline{16}_{\mathbb{R}} \to \underline{8}_v \oplus \boxed{\underline{8}_s}$$

$$\underline{16}'_{\mathbb{R}} \to \underline{8}_v \oplus \boxed{\underline{8}_c}$$

$8_{s,c}$ ARE THE STATES $|s_1 \dots s_4\rangle$.

$SO(8)$: $8_v, 8_s, 8_c$ Triality of $SO(8)$.

R-SECTOR STATES (NN) BC OPEN STRING

$b_0^i \quad i = 1\dots 8$ $\underset{\text{as above}}{\sim}$ $|s_1 \dots s_4\rangle$

$\# \, s_i = \frac{1}{2}$ even 8_s $|a\rangle$

$\# \, s_i = \frac{1}{2}$ odd 8_c $|\dot{a}\rangle$

MASS SHELL: $\alpha' M^2 = N^X + N^4$

LEVEL 0: $M^2 = 0$: EITER $|a\rangle$ OR $|\dot{a}\rangle$

$\Rightarrow$ SPACETIME SPINORS, MASSLESS.

LEVEL 1: $(\underset{8_v}{\zeta_i} \alpha_{-1}^i + \underset{8_v}{\eta_i} b_{-1}^i) \underset{8_s}{|a\rangle}$

$+ (\underset{8_v}{\rho_i} \alpha_{-1}^i + \underset{8_v}{\delta_i} b_{-1}^i) \underset{8_c}{|\dot{a}\rangle}$

SPACETIME TRANSFORMATION PROPERTY:

$$\left.\begin{aligned} \underline{8}_v \otimes \underline{8}_s &= \underline{8}_c \oplus \underline{56}_c \\ \underline{8}_v \otimes \underline{8}_c &= \underline{8}_c \oplus \underline{56}_s \end{aligned}\right\} \text{massive fermions.}$$

LETS TAKE STOCK: OPEN (N,N) B.C.

	$SO(8)$	$\alpha' M^2$	SPACETIME STATS.
NS	$\underline{8}_v$	0	BOSON
NS	1	$-\frac{1}{2}$	BOSON (TACHYON)
R	$\underline{8}_s$	0	FERMION
R	$\underline{8}_c$	0	FERMION.

TWO ISSUES:

$\exists$ TACHYON.

NO APPARENT SPACETIME SUSY.

$\Rightarrow$ STRATEGY: GLIOZZI-SCHERK-OLIVE (GSO) PROJECTION.

PROJECT OUT TACHYON & ONE OF 8_c or 8_s.

$\Rightarrow$ NO TACHYON w/ SUSY.