BO1.1. History of Mathematics
Lecture II
Dissemination and development (600 BC – AD 1600)

MT24 Week 1

Summary

- Influence of the ancient world
- The European Renaissance (15th and 16th centuries)
- Rediscovery and transmission of ancient texts
- The 16th century
- A case study: Napier's invention of logarithms 1614

Ancient influences on early modern European mathematics

(Early modern = roughly 1400–1800)

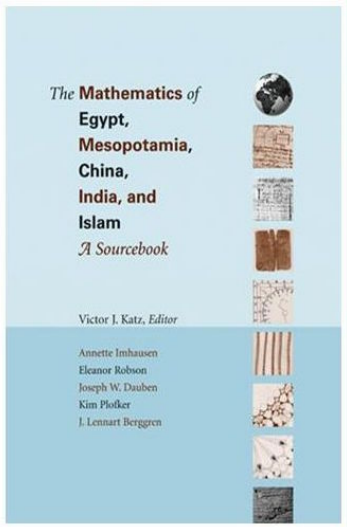
The prior mathematical accomplishments of

- China
- India
- Mesopotamia
- Egypt
- and most other places

were largely unknown in Europe until the 19th century

The single greatest influence on early modern European mathematics was the mathematics of ancient Greece, with that of mediaeval Islam (a little later) coming a close second

Ancient origins of mathematics



On the ancient origins of mathematics, see:

Victor J. Katz (ed.), *The mathematics of Egypt, Mesopotamia, China, India, and Islam: a sourcebook*, Princeton University Press, 2007

Ancient Greek mathematics

Earliest origins of Greek mathematics in 6th century BC

But what do we mean by 'Greek'?

500 BC – 300 BC	Collection of city-states in Greece
300 BC – AD 500	Greek-speaking peoples around the Mediterranean, especially in Alexandria

Some major figures of ‘Greek’ mathematics

Pythagoras	Samos (Greece)?	c. 600 BC
Euclid	Alexandria (Egypt)?	c. 300 (or 250?) BC
Archimedes	Syracuse (Sicily)	c. 250 BC
Apollonius	Perga (Turkey)	c. 180 BC
Diophantus	Alexandria (Egypt)	c. AD 200

Euclid's *Elements*

The 'elements of geometry' in 13 books, compiled around 300 (250?) BC from existing geometrical knowledge

Books I–VI	plane geometry	points, lines, angles, circles, ...
Books VII–X	properties of numbers	odd, even, square, triangular, prime, perfect, ...
Books XI–XIII	solid geometry	cube, tetrahedron, icosahedron, ...

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David Joyce's Java version of Euclid's *Elements*

Oliver Byrne's colour version of the first six books

Euclid's *Elements*, book I

23 definitions: point, line, surface, angle, circle, ...

5 postulates: what one can do
    e.g. a straight line may be drawn between any two points;
        a circle may be drawn with given centre and radius

5 'common notions': how one may reason
    e.g. if equals are added to equals, then the wholes are equal

48 propositions: each built only on what has gone before

The influence of Euclid's *Elements*

HUGE influence on Western mathematics:

- hundreds of editions and translations from renaissance onwards
- basis of mathematics teaching in schools until c. 1960
- style: definitions—axioms—theorems—proofs
- status of 'Parallel Postulate' led to much investigation and, ultimately, non-Euclidean geometries
- problems of 'ruler and compass' construction inspired much investigation and many new discoveries
- wider cultural importance: http://readingeuclid.org/

Other Greek authors

Archimedes	d. 212 BC:	method of exhaustion and much else
Apollonius	c. 180 BC:	conic sections
Diophantus	c. AD 250:	*Arithmetica* in 13 books (number problems)

Also had HUGE influence on Western mathematics

Remnants of the collapse of the ancient world

in Greek:	manuscripts preserved at Constantinople and in libraries or collections around the Mediterranean
in Latin:	writings by Boethius (c. 480–524) on philosophy, arithmetic, geometry, music

The spread of Islam and Islamic learning

632–732:	Islam spreads throughout Middle East, north Africa, and into Spain and Portugal
c. 820:	*Bayt al-Ḥikma*, the House of Wisdom, founded in Baghdad under Caliph al-Ma'mūn; it became a centre for translation into Arabic from Greek, Persian, Sanskrit
c. 825:	al-Khwārizmī active in Baghdad
9th century:	texts on arithmetic, algebra, astronomy reach Spain
12th century:	translations from Arabic to Latin

The mid-Renaissance (15th and 16th centuries)

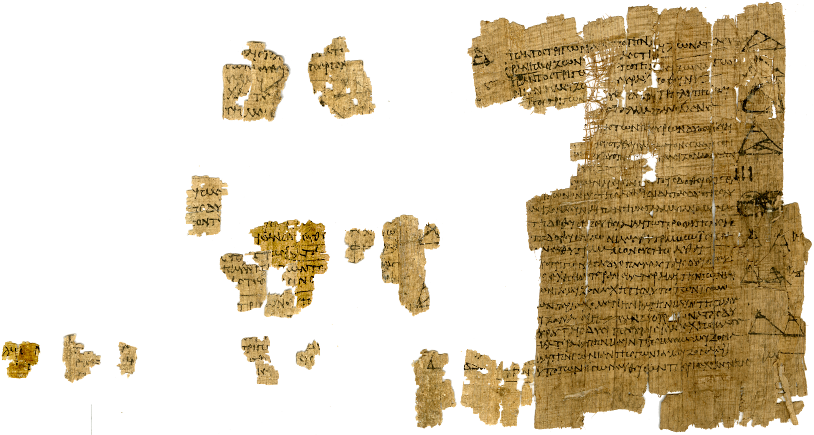
Classical mathematical texts more widely available due to:

- rediscovery of manuscripts
- revival of knowledge of Greek
- transmission of otherwise lost texts via Arabic versions
- (Western) invention of printing (Gutenberg, c. 1436)

Euclid's *Elements*: transmission history

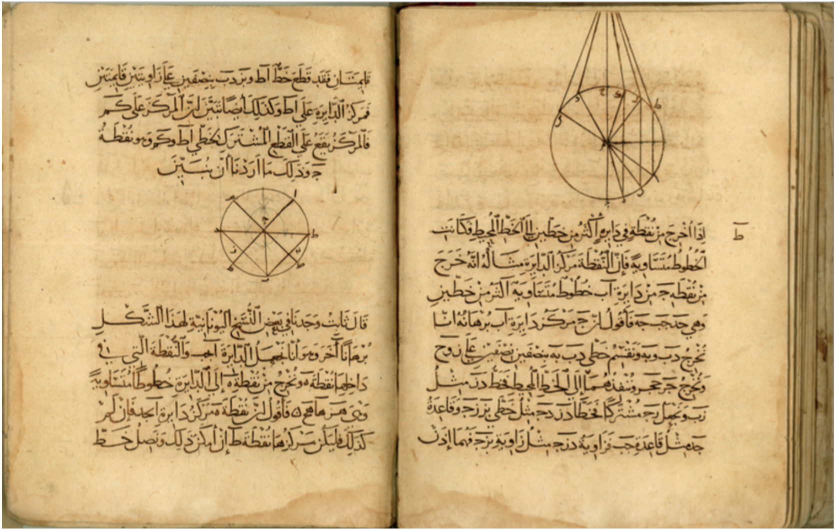
- commentaries written by Pappus (c. AD 320), Theon (c. AD 380), Proclus (c. AD 450)
- a few propositions in Boethius (c. AD 500)
- copies in Greek (earliest from Constantinople, AD 888)
- many translations or commentaries in Arabic (AD 750–1250)
- mediaeval translations from Arabic to Latin: Adelard of Bath (1130), Robert of Chester (1145), Gerard of Cremona (mid-12th century)
- printed editions in Latin or Greek from 1482 onwards

Early fragments of Euclid



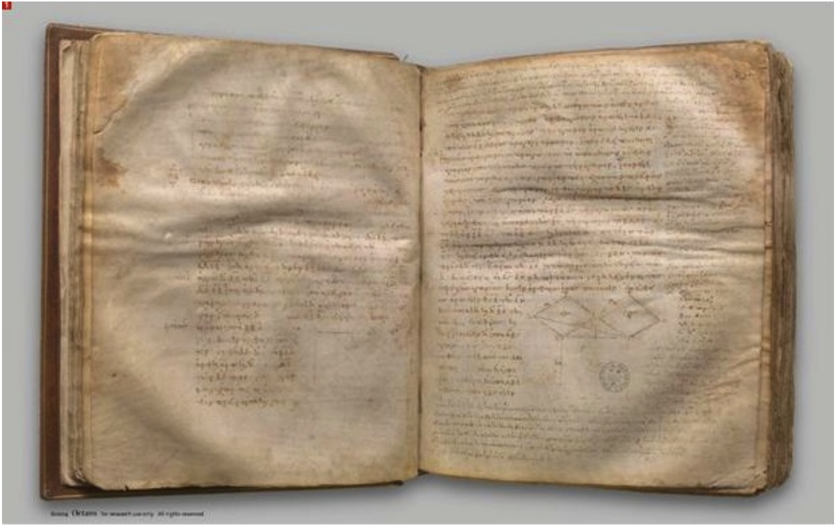
Fragments of Book I among the Oxyrhynchus Papyri (3rd century)

Euclid in Arabic



Translated from the Greek by Ishaq ibn Hunayn, AD 1466

Euclid I.47 from Bodleian MS dated 888



Whole manuscript is digitised:
http://www.claymath.org/library/historical/euclid/

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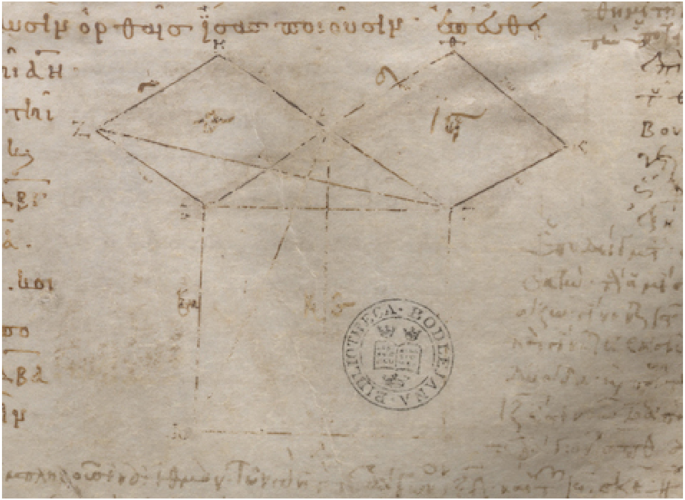

http://www.claymath.org/library/historical/euclid/files/elem.1.47.html

Treatises by Archimedes: transmission history

- quoted or explained by Pappus (c. 320 AD), Theon (c. 380 AD), Eutocius (c. 520 AD)
- 6th-century Byzantine ‘collected works’ (Isidore of Miletus)
- several translations of individual treatises into Arabic
- translations from Arabic into Latin
- a new find in the twentieth century: www.archimedespalimpsest.org/

Apollonius' *Conics* (c. 180 BC): transmission history

- Books I–IV survived in Greek
- Books V–VII survived only in Arabic
- Book VIII is lost, known only from commentaries
- early (Latin) printed edition, 1566

(See: *Mathematics emerging*, §1.2.4.)

16th century change

New forces at work in the 16th century:

- global exploration
- growth of international commerce
- new technology (in printing, shipping, military engineering, instrumentation, etc.)

A case study of a text from 1614

Napier's invention of logarithms:

- what did 17th-century mathematics look like?
- how can we begin to read historical texts?

Napier's definition of a logarithm (of a sine)

The **Logarithme** *therefore of any sine is a number very neerely expressing the line, which increased equally in the meane time, whiles the line of the whole sine decreased proportionally into that sine, both motions being equal-timed, and the beginning equally swift.*

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Context, content, significance

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Context: who? when? where? why?

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Significance: why did/does it matter?

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Context:	who? when? where? why?
Content:	what is it about? how is it written?
Significance:	why did/does it matter?
Historical Significance:	what new insight does this text offer us?

Context — who?

John Napier (1550–1617), Merchiston, Scotland

Scottish landowner with interests in:

- mining
- calculating aids
- astrology/astronomy
- The Book of Revelation

See *Oxford Dictionary of National Biography*: http://www.oxforddnb.com/view/article/19758

Context — why?

From Napier's preface to the English translation of 1616:

> *Seeing there is nothing (right well-beloved Students of the Mathematics) that is so troublesome to mathematical practice, nor that doth more molest and hinder calculators, than the multiplications, divisions, square and cubical extractions of great numbers, which besides the tedious expense of time are for the most part subject to many slippery errors, I began therefore to consider in my mind by what certain and ready art I might remove those hindrances.*

Context — why?

Inspired by the 16th-century technique of prosthaphaeresis:

the use of trigonometric identities such as

$$\cos x \cos y = \frac{1}{2}\left[\cos(x+y) + \cos(x-y)\right]$$
$$\sin x \sin y = \frac{1}{2}\left[\cos(x-y) - \cos(x+y)\right]$$

to convert multiplication into addition.

Context — in what form, and in which language?

Original Latin text of 1614:

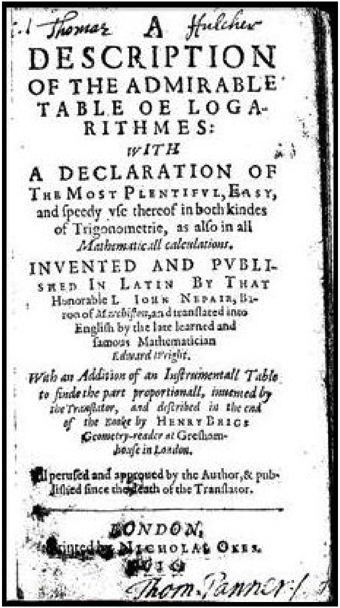
Mirifici logarithmorum canonis descriptio

translated into English by Edward Wright in 1616 as

A description of the admirable table of logarithms

Both the Latin original and English translation are available online

Napier's 1616 title-page decoded



Thomas A Hulcher

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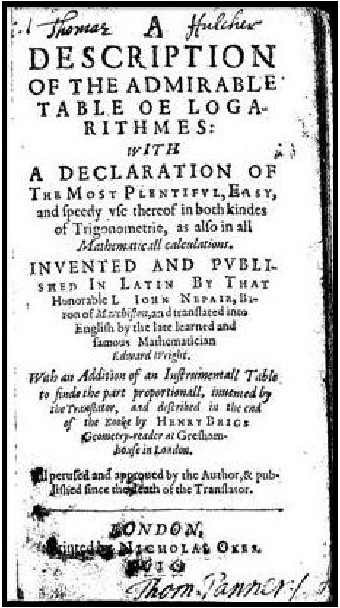
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Inventor:

John Napier (1550–1617)

Translator:

Edward Wright (?1558–1615) (interests: navigation, charts and tables)

Additional material:

Henry Briggs (1561–1630) Gresham Professor of Geometry, later Savilian Professor of Geometry at Oxford (interests: navigation)

Printer:

Nicholas Okes

Readers:

Thomas Hulcher,

Thomas Panner

Napier's logarithms: content

Recall:

> *The* **Logarithme** *therefore of any sine is a number very neerely expressing the line, which increased equally in the meane time, whiles the line of the whole sine decreased proportionally into that sine, both motions being equal-timed, and the beginning equally swift.*

Napier's logarithms

peare by the 19 *Prop.* 5. and 11. *Prop.* 7, *Euclid.*

3 *Def.* *Surd quantities, or vnexplicable by number, are ſaid to be defined, or expreſſed by numbers very neere, when they are defined or expreſſed by great numbers which differ not ſo much as one vnite from the true value of the Surd quantities.*

As for example. Let the ſemidiameter, or whole ſine be the rational number 10000000 the ſine of 45 degrees ſhall be the ſquare root of 50,000, 000,000,000, which is ſurd, or irrationall and inexplicable by any number, & is included between the limits of 7071067 the leſſe, and 7071068 the greater: therfore, it differeth not an vnite from either of theſe. Therefore that ſurd ſine of 45 degrees, is ſaid to be defined and expreſſed very neere, when it is expreſſed by the whole numbers, 7071067, or 7071068, not regarding the fractions. For in great numbers there ariſeth no ſenſible error, by neglecting the fragments, or parts of an vnite.

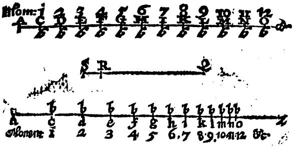
4 *Def.* *Equall-timed motions are thoſe which are made together, and in the ſame time.*

As in the figures following, admit that B be moued from A to C, in the ſame time, wherin *b* is moued from *a* to *c* the right lines A C & *a c*, ſhall be ſayd to be deſcribed with an equall-timed motion.

5 *Def.* *Seeing that there may bee a ſlower and a ſwifter motion giuen then any motion, it ſhall neceſſarily follow, that there may be a motion giuen of equall ſwiftneſſe to any motion (which wee define to be neither ſwifter nor ſlower.)*

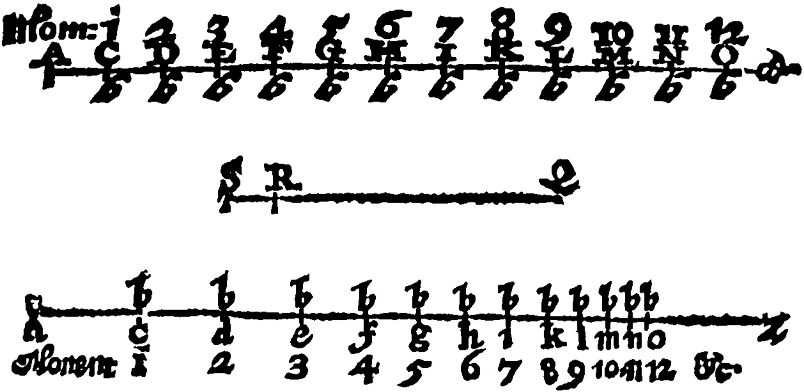
6 *Def.* The Logarithme *therfore of any ſine is a number very neerely expreſsing the line, which increaſed*

ſed equally in the meane time, whiles the line of the whole ſine decreaſed proportionally into that ſine, both motions being equal-timed, and the beginning equally ſwift.



As for example. Let the 2 figures going afore bee here repeated, and let B bee moued alwayes, and euery where with equall, or the ſame ſwiftneſſe wherewith *b* beganne to bee moued in the beginning, when it was in *a*. Then in the firſt moment let B proceed from A to C, and in the ſame time let *b* moue proportionally from *a* to *c*, the number defining or expreſſing A C ſhal be the *Logarithme* of the line, or ſine *c* Z. Then in the ſecond moment let B bee moued forward from C to D. And in the ſame moment or time let *b* be moued proportionally from *c* to *d*, the number definining A D. ſhall bee the *Logarithme* of the ſine *d* Z. So in the third moment let B go forward equally from D to E, and in the ſame moment let *b* be moued forward proportionally from *d* to *e*, the number expreſſing A E the *Logarithme* of the ſine *e* Z. Alſo in the fourth moment, let B pro-

Napier's logarithms



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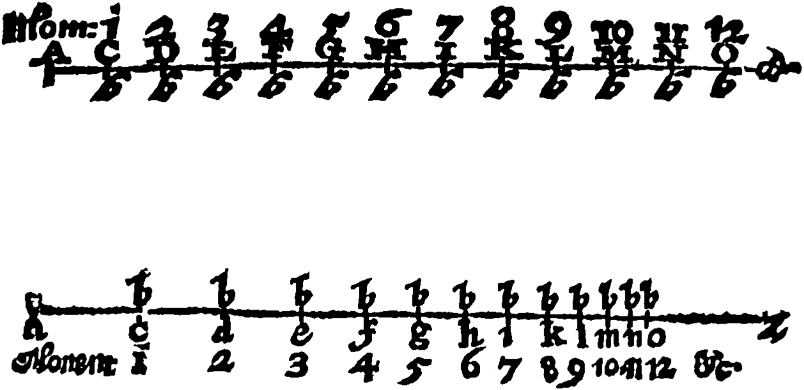

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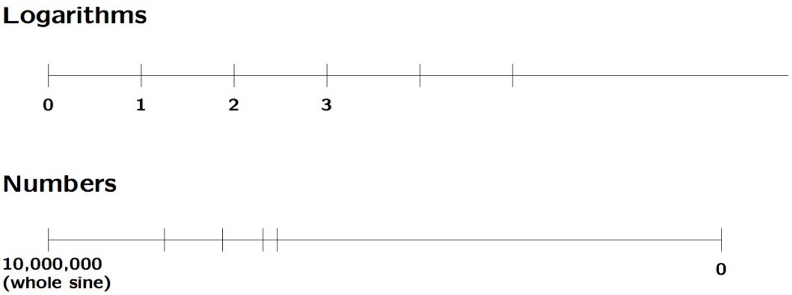

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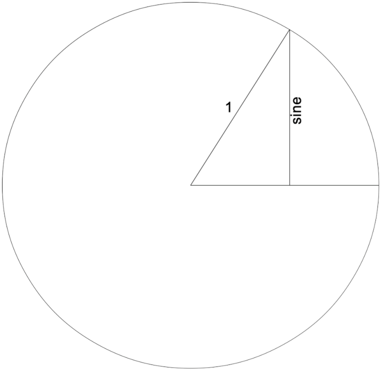

Sine of angle at centre varies between 0 and ± 1 as the labelled radius sweeps around the circle

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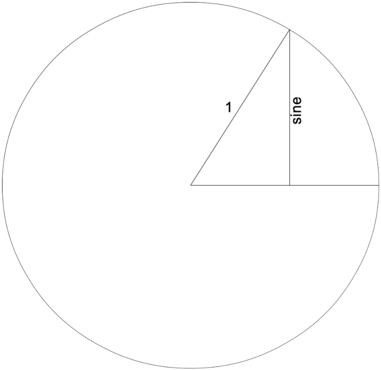

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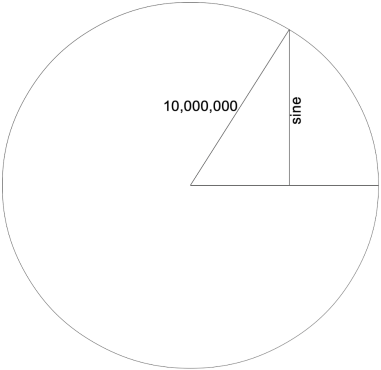
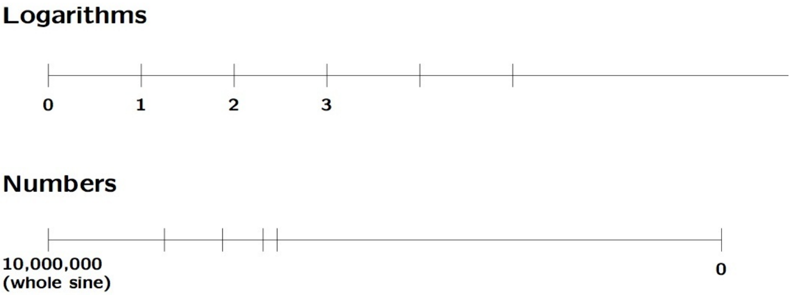


Sine of angle at centre varies between 0 and $\pm 10,000,000$ as the labelled radius sweeps around the circle

Napier's logarithms



Napier's logarithms (1614)

In modern terms (i.e., not Napier's):

$$\text{if } y = 10^7\left(1 - 10^{-7}\right)^x, \text{ then } \operatorname{Nap}\log y = x$$

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Note that $\text{Nap log}\, x = 10^7 \ln\left(\frac{10^7}{x}\right)$

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$$\text{Nap}\log\left(\frac{p \times q}{10^7}\right) = \text{Nap}\log p + \text{Nap}\log q$$

$$\text{Nap}\log(p \times q) = \text{Nap}\log p + \text{Nap}\log q - \text{Nap}\log 1$$

Note that $\text{Nap}\log x = 10^7 \ln\left(\frac{10^7}{x}\right)$

No notion of base, although Nap log 'nearly' has base $\frac{1}{e}$ — see Robin Wilson, *Euler's Pioneering Equation*, OUP, 2019, p. 101

Modifications by Napier and Briggs (1617)

Definition revised to remove the need to subtract $\operatorname{Nap}\log 1$

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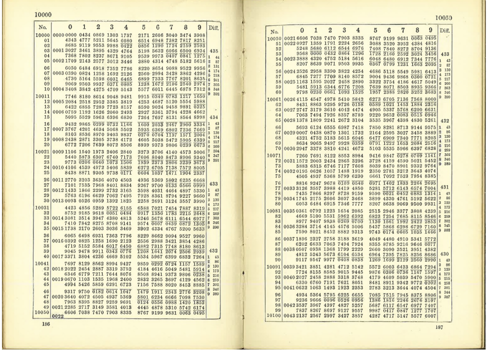
Briggs produced *Logarithmorum chilias prima* (*The first thousand logarithms*) in 1617, followed by his *Arithmetica logarithmica* in 1624, which contained logarithms of 1 to 20,000 and 90,000 to 100,000, all to 14 decimal places (calculated by hand); the gap in the table was filled by Adriaan Vlacq in 1628

Napier's logarithms

One last time:

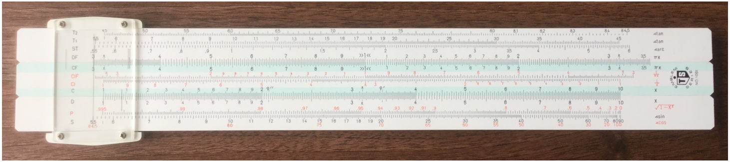
> *The* **Logarithme** *therefore of any sine is a number very neerely expressing the line, which increased equally in the meane time, whiles the line of the whole sine decreased proportionally into that sine, both motions being equal-timed, and the beginning equally swift.*

Significance



Napier's logarithms:

- caught on very quickly
- a calculating aid (until the 1980s)
- logarithms rapidly came to have other interpretations (as you know, and as we shall see)



Significance as a historical source

- Roles of translation in mathematics
- Concept of authorship in the 16th century
- Use of diagrams in mathematical texts
- Importance of informal/social communication, alongside published texts