Brownian microswimmers

Boundaries and particle shape

MMSC Case study project

Mondays @ 2pm

Motivation

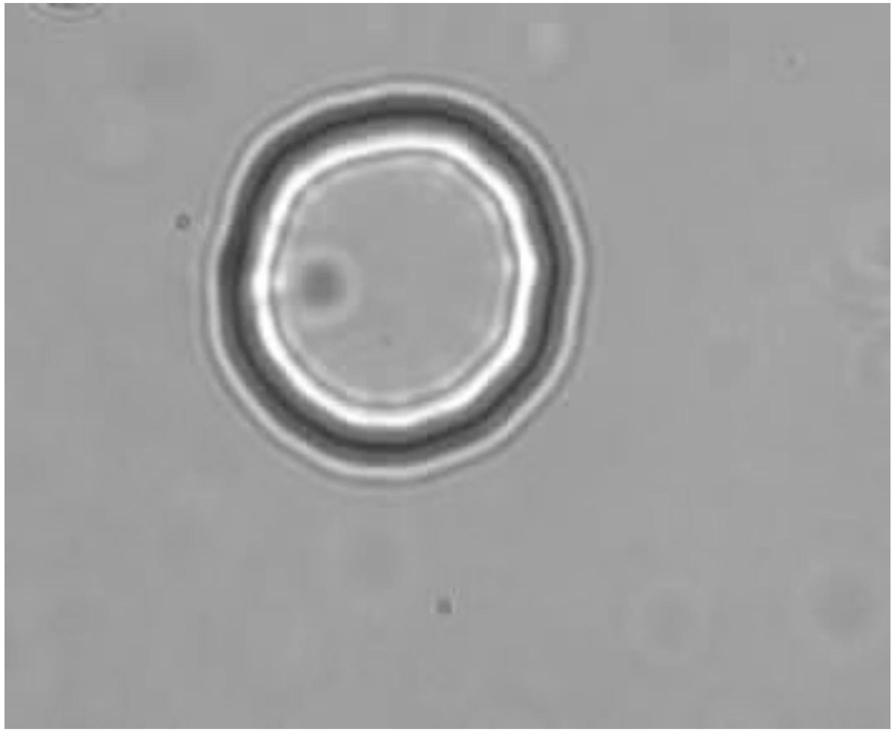
The interaction of active self-propelled particles with rigid boundaries under confinement plays a central role in many biological processes.

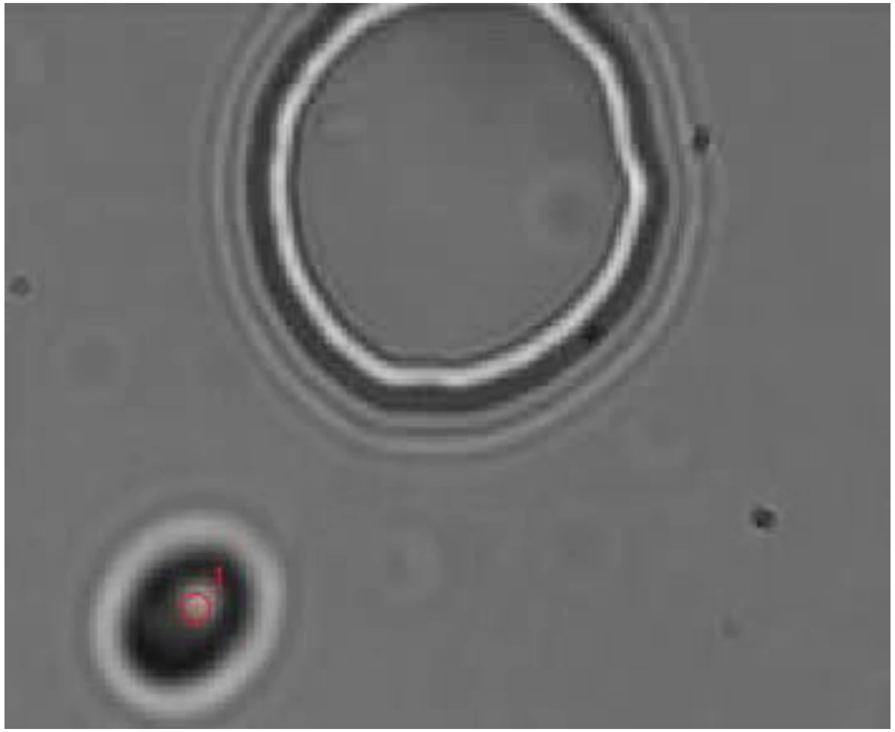
Examples include:

- Spermatozoa transport in the female tract
- Aggregation of bacteria near surfaces, biofilm formation.
- Control of bacteria in microfluidic devices.
- Lane formation in pedestrians and social insects.

In common across very different applications is the tendency to accumulate near boundaries.

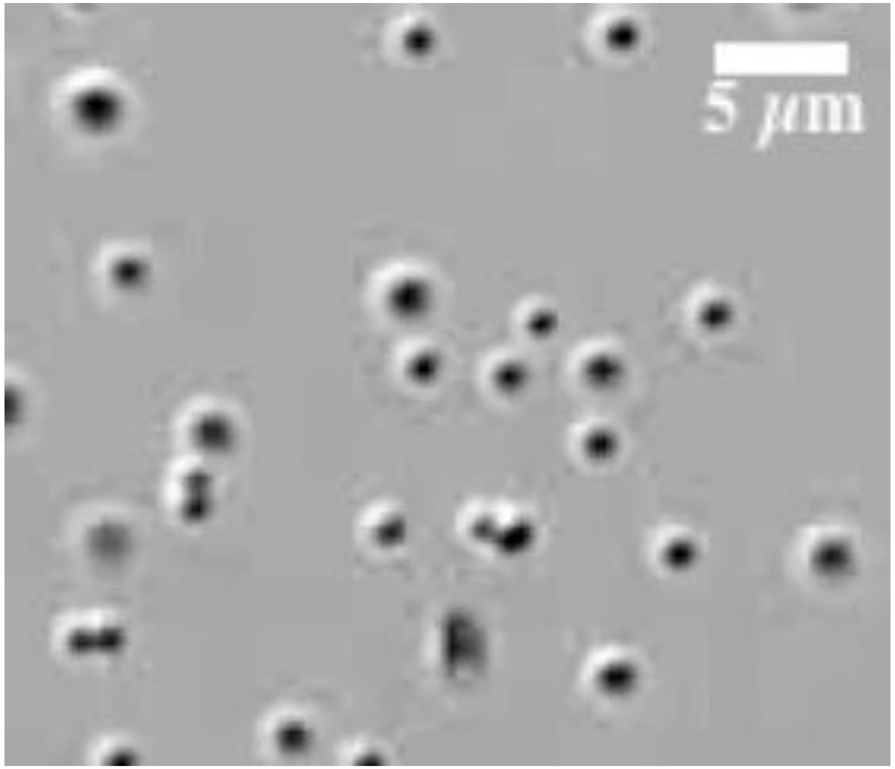
Biflagellate green alga *Chlamydomonas reinhardtii*

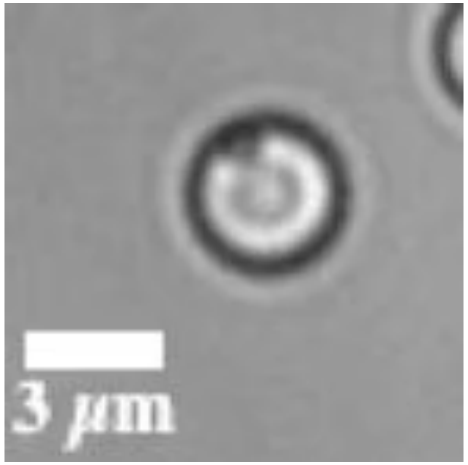


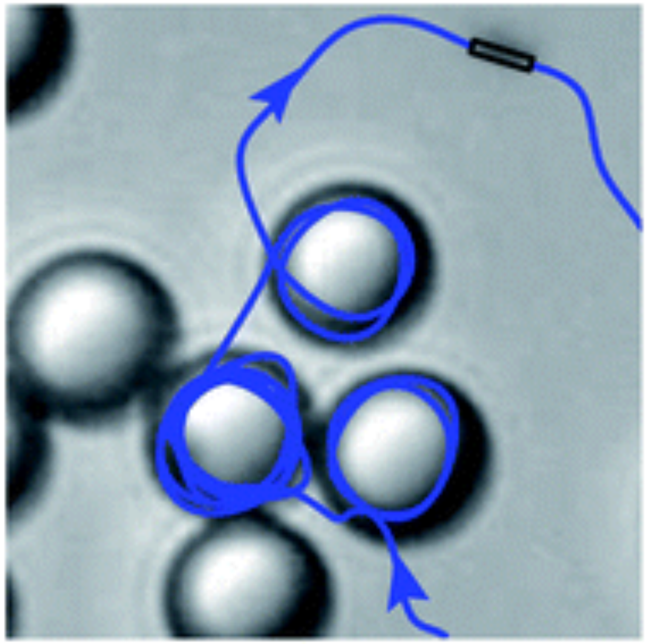


Contino et al., Phys. Rev. Lett. (2015)

Synthetic micro-rods







Takagi et al., Soft matter (2014)

Bull Spermatozoa

No. 4886 June 22, 1963 NATURE 1221

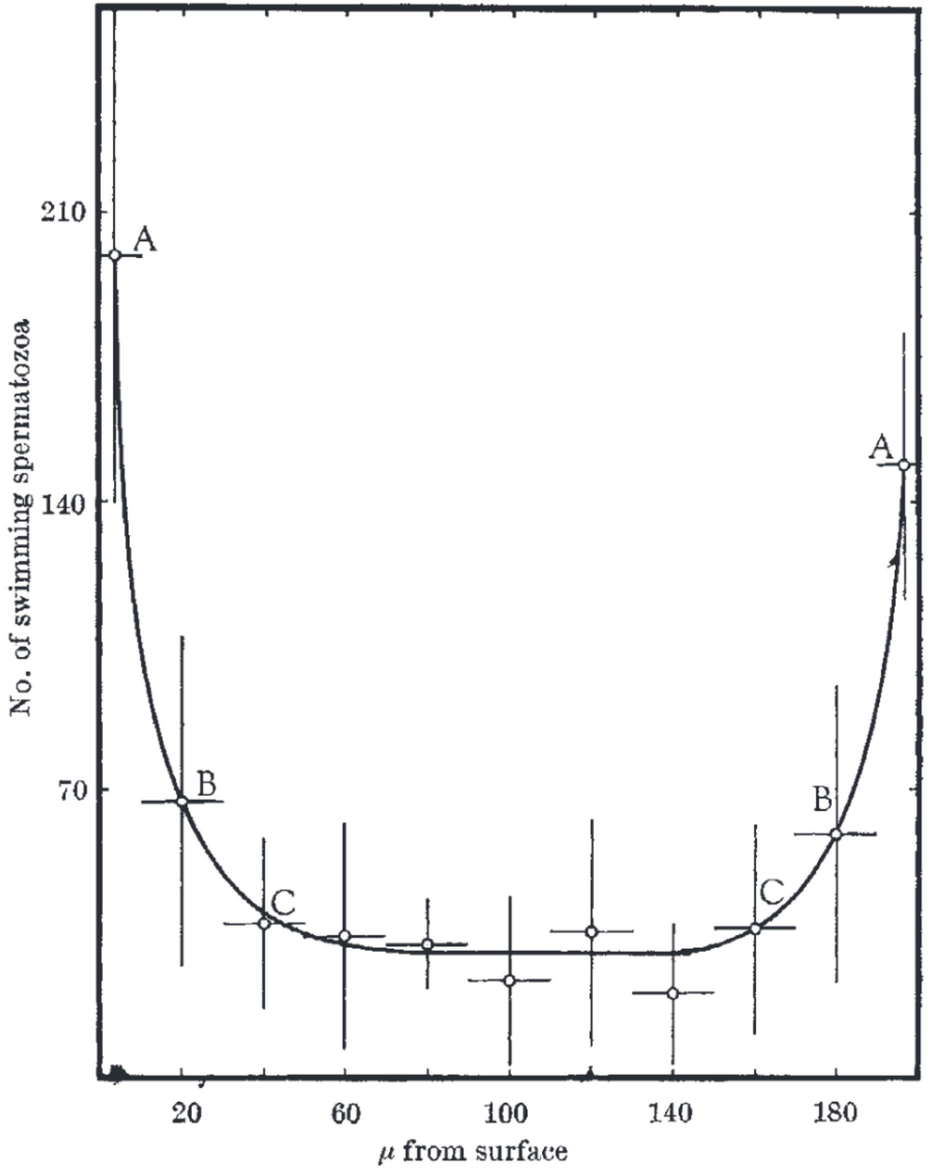


Fig. 1. Distribution of swimming bull spermatozoa (semen diluted 1/30) in a Hawksley hæmocytometer chamber 200μ deep, with the optical axis of the microscope horizontal. The vertical lines through each point are the 95 per cent fiducial limits. Each horizontal line is an estimate of the maximum error in determining the distance, in μ, from the coverslip

BIOLOGY

Non-random Distribution of Bull Spermatozoa in a Drop of Sperm Suspension

Spermatozoa are usually thought to swim in random directions in their suspending medium, seminal plasma, sea water, Krebs–Henseleit–Ringer solution, etc., unless, as in the spermatozoa of many plants and, possibly, some animals, chemotaxis occurs naturally or is induced. The hypothesis of random sperm movement has, in fact, been the basis for a quantitative study of the block to polyspermy in sea-urchin eggs, the spermatozoa being treated as gas molecules which obeyed the classical kinetic theory of gases[1].

Spermatozoa are examined visually in what is often described as 'a drop of suspension under the microscope'. I thought it might be of interest to study the hypothesis of random sperm movement in such a drop in more detail than before, this having last been done in 1949[1]. Sea-urchin spermatozoa were used then, but they were

Ant colonies

Poissonnier et al., eLife (2019)

Brownian particle

Consider a Brownian particle in $\mathbb{R}^2$ at position $(X(t), Y(t))$, with evolution

$$\begin{aligned} \mathrm{d}X(t) &= \sqrt{2D}\mathrm{d}W_1 + U_1\mathrm{d}t, \\ \mathrm{d}Y(t) &= \sqrt{2D}\mathrm{d}W_2 + U_2\mathrm{d}t, \end{aligned}$$

where D is the diffusion coefficient, (U_1, U_2) the drift vector and W_i independent standard Brownian motions.

Alternatively, we may consider the probability density $p(x, y, t)$ for the particle to be at position (x, y) at time t. By the Itô formula:

$$\partial_t p = \partial_x(D\partial_x p - U_1 p) + \partial_y(D\partial_y p - U_2 p).$$

Active Brownian particle (ABP)

A simple model for self-propelled particles is to introduce an orientation $\Theta(t)$ and make the drift Θ-dependent:

$$
\begin{aligned}
\mathrm{d}X(t) &= \sqrt{2D}\mathrm{d}W_1 + U\cos\Theta\mathrm{d}t, \\
\mathrm{d}Y(t) &= \sqrt{2D}\mathrm{d}W_2 + U\sin\Theta\mathrm{d}t, \\
\mathrm{d}\Theta(t) &= \sqrt{2D_\theta}\mathrm{d}W_3
\end{aligned}
$$

where D_Θ is the rotational diffusion coefficient and U is the (constant) self-propulsion speed.

Similarly, now we have $p(x, y, \theta, t)$ with:

$$
\partial_t p = \partial_x(D\partial_x p - U_1 p) + \partial_y(D\partial_y p - U_2 p) + D_\theta \partial_\theta^2 p,
$$

where $(U_1, U_2) = U(\cos\theta, \sin\theta)$.

Boundary interactions

Suppose the particle is constrained to $y \geq 0$. There are multiple ways to model the particle-wall interaction:

- attractive/repulsive forces.
- hydrodynamic interactions (fluid mediated).
- steric interactions.

Large experimental evidence shows that the shape and size of self-propelled particles can change the interaction. How do we model this?