Introduction to schemes.
Schedule for presentations

Prof. Kevin McGerty

Hilary Term 2025.

From week 2 onwards, the reading course meeting will have student presentations on the topics assigned for the previous week.

Week 1 Introductory meeting.

Week 2 Speaker(s): Philipp Weidemann

The Spec of a ring, Zariski topology, comparison with classical algebraic geometry.

Week 3 Speaker(s):

Pre-sheaves and stalks, sheaves, sheafification. The abelian category of sheaves of abelian groups on a topological space. Direct and inverse images of sheaves. Sheaves defined on a topological basis.

Ringed spaces and morphisms of ringed spaces. Affine schemes, construction of the structure sheaf, the equivalence of categories defined by Spec.

Week 4 Speaker(s): Hao Cui

Schemes, closed subschemes. Global sections. The functor of points.

Properties of schemes: (locally) Noetherian, reduced, irreducible, and integral schemes. Properties of morphisms of schemes: finite type, open/closed immersions, flatness including simple examples of flat families of schemes arising from deformations.

Week 5 Speaker(s):

Gluing sheaves. Gluing schemes. Affine and projective n-space viewed as schemes.

Products, coproducts and fiber products in category theory. Existence of products of schemes. Fibers and pre-images of morphisms of schemes. Base change.

Week 6 Speaker(s):

Further properties of morphisms of schemes: separated, universally closed, and proper morphisms. Projective n-space and projective morphisms. Abstract varieties. Complete varieties. Scheme structure on a closed subset of a scheme.

Week 7 Speaker(s):

Sheaves of modules. Vector bundles and coherent sheaves. The abelian category of sheaves of modules over a scheme. Pullbacks.

Quasi-coherent sheaves. Gluing sheaves of modules. Classification of (quasi-)coherent sheaves on Spec of a ring.

Week 8 Speaker(s):

Čech cohomology. Vanishing of higher cohomology groups of quasi-coherent sheaves on affine schemes. Independence of Čech cohomology on the choice of open cover. Line bundles, examples on projective n-space.

Week 9*   Speaker(s): Ziyang Yang

      Sheaf cohomology. Acyclic resolutions. Comparison of sheaf cohomology and Čech cohomology.

      Quasi-coherent sheaves on projective n-space, graded modules, and Proj of a graded ring.

    *Note that the week 9 material will be discussed in a meeting the time and location of which have yet to be confirmed.