Numerical Linear Algebra and Approximation Theory

Introduction

Numerical Linear Algebra (NLA) and Approximation Theory (Approx) are two of the core branches of Numerical Analysis, or slightly more broadly, computational mathematics.

The key problems addressed in NLA are eigenvalue problems $Ax = \lambda x$ and linear systems $Ax = b$. An exciting recent area in NLA is randomised algorithms, where blending probability and matrix analysis can sometimes lead to algorithms that can solve problems with dramatically higher efficiency.

Approximation theory is the backbone of all sorts of computation, ranging from optimisation, signal processing, and design of numerical algorithms in scientific computing.

Project

While extensive research has been devoted to all these topics, a number of open problems remain, and many interesting questions have been identified with the rapid surge of data science, with significant ramifications in applications.

This project aims to explore, experiment, examine and potentially resolve problems in NLA or Approx.

Possible topics for the BSP in NLA include: (randomised) algorithms for solving linear systems and eigenvalue problems, estimation of trace (of functions of matrices), and perturbation analysis of matrices and functions of matrices.

There are a number of important problems to be explored also in Approx. These include approximation by composite functions (e.g. $f_2(f_1(x))$), solution of ODEs, and approximation of noisy functions.

Prerequisites

Prelims Linear Algebra. Part A Numerical Analysis is highly recommended. Part A Probability and Complex Analysis (for Approx) would be helpful.

Reading

Recommended textbook in NLA:

L. N. Trefethen and D. Bau, Numerical Linear Algebra, SIAM, 1997.

An excellent reference in randomised NLA is

P.-G. Martinsson and J. A. Tropp. Randomized numerical linear algebra: Foundations and algorithms. Acta Numerica, pages 403-572, 2020.

The recommended textbook in Approx is: L. N. Trefethen, Approximation Theory and Approximation Practice, SIAM, 2013.

For specific topics, a subset of these references could be of interest.

R. M. Gower and P. Richtarik. Randomized iterative methods for linear systems. SIAM J. Matrix Anal. Appl., 36(4):1660-1690, 2015.

Y. Nakatsukasa. Sharp error bounds for Ritz vectors and approximate singular vectors. Math. Comp., 89(324):1843–1866, 2020.

G. W. Stewart and J.-G. Sun. Matrix Perturbation Theory. Academic Press, 1990.