

Exercises for Practical 3

In this practical, you will practice more advanced plotting with Matplotlib in a Jupyter Notebook. Remember to enable an interactive backend (e.g., `%matplotlib widget`) to be able to rotate your 3D plots.

1. **Plotting Algorithm Convergence:** Modify your bisection algorithm from Practical 2 to solve for the root of $f(x) = x^{10} - 1/10$ on the interval $[0, 1]$. Your function should now also return a list or NumPy array of the error at each iteration (i.e., the width of the interval, $|b_n - a_n|$).

Create a plot of the error versus the iteration number. Use a logarithmic scale for the y-axis (`semilogy`) to show the linear convergence of the method. Properly label your axes and give the plot a title.

2. **3D Surface Plots:** Make interactive 3D surface plots of the following functions over the given ranges. Use `np.meshgrid` to create your coordinate matrices and `ax.plot_surface` to plot. Remember to label your axes.

(a) $f(x, y) = (x^2 + 3y^2)e^{-x^2 - y^2}, \quad -3 \leq x \leq 3, -3 \leq y \leq 3$

(b) $f(x, y) = \frac{-3y}{x^2 + y^2 + 1}, \quad |x| \leq 3, |y| \leq 4$

(c) $f(x, y) = |x| + |y|, \quad |x| \leq 1, |y| \leq 1$

(d) $f(x, y) = \sin(x) \cos(y)$ for all points (x, y) inside a circle of radius 1 centered at the origin. (Hint: Create a grid from -1 to 1 for both x and y. Then, for any points where $x^2 + y^2 > 1$, set the function's value to `np.nan` so they are not plotted.)

3. **Contour and Wireframe Plots:** For the function from question 2(a), create a new figure with two subplots side-by-side.

- In the left subplot, create a 2D contour plot of the function using `plt.contour`.
- In the right subplot, create a 3D wireframe plot using `ax.plot_wireframe`.

Ensure both plots have appropriate titles.